

Communication

Towards Precision Muonic X-ray Measurements of Charge Radii of Light Nuclei

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Abstract: We, the QUARTET Collaboration, propose an experiment to measure the nuclear charge radii of light elements with up to 20 times higher accuracy. These are essential both for understanding nuclear physics at low energies, and for experimental and theoretical applications in simple atomic systems. Such comparisons advance the understanding of bound-state quantum electrodynamics and are useful for searching for new physics beyond the Standard Model. The energy levels of muonic atoms are highly susceptible to nuclear structure, especially to the mean square charge radius. The radii of the lightest nuclei (with the atomic number, $Z = 1, 2$) have been determined with high accuracy using laser spectroscopy in muonic atoms, while those of medium mass and above were determined using X-ray spectroscopy with semiconductor detectors. In this communication, we present a new experiment, aiming to obtain precision measurements of the radii of light nuclei $3 \leq Z \leq 10$ using single-photon energy measurements with cryogenic microcalorimeters; a quantum-sensing technology capable of high efficiency with outstanding resolution for low-energy X-rays.

Keywords: muonic atoms; charge radius; X-ray; metallic magnetic calorimeter (MMC); nuclear structure; bound-state quantum electrodynamics (QED); simple atomic systems

1. Introduction

Muonic atoms are highly suitable systems for studying the nucleus. Due to the heavy mass of muons ($m_\mu \sim 200 m_e$, with m_e the electron mass), the Bohr radius of muonic atoms is approximately 200 times smaller than that of electronic atoms, and thus, for low angular momentum states, the muon wavefunction has a $200^3 \approx 10^6$ times larger overlap with that of the nucleus. The nuclear properties thus lead to measurable shifts in the atomic transition

energies, making muonic atom spectroscopy an effective probe of phenomena such as finite nuclear size effects [1–7], relativistic QED (quantum electrodynamics) contributions [4,7–9], and possible short-range interactions carried by new mediators [10–16]. These systems are particularly well-suited to accurately determining the RMS (root mean square) nuclear charge radius (the slope of the Sachs form factor at small momentum transfer, henceforth called ‘radius’ for brevity), which can be obtained using the spectroscopy of low-lying radiative transitions (mostly $2P - 1S$) [1,17,18]. Indeed, historically, the best measurements of absolute radii have been obtained using muonic atom spectroscopy, sometimes leading to unexpected results such as the ‘proton radius puzzle’ [19–25].

The radius is a fundamental property of the nucleus, and knowledge of it is not only significant in the development of a nuclear structure theory, but also for obtaining a reliable comparison between the experimental results and theoretical expectations at the accuracy frontier of Standard Model tests with atoms and nuclei [3]. Accordingly, the radius of the proton and deuteron are considered fundamental constants, on a similar footing as their masses [26], with those of heavier nuclei expected to be included in the next CODATA adjustment of fundamental constants.

In Table 1, we collect the most precise values for the radii of light and stable nuclei. It is immediately seen that the elements in the most critical need of improved radii are those with a nuclear charge, Z , just above helium, a region that is currently beyond the reach of laser spectroscopy, where solid-state detectors are the most unsuitable. Advantageously, these nuclei are also those whose structure can now be calculated by the most advanced ab initio nuclear theory methods, as detailed in what follows.

Table 1. Current status of the charge radii, r_c , of light stable nuclei. The method abbreviations are as follows: ‘ μ -Laser’—muonic atoms laser spectroscopy; ‘el. scat.’—elastic electron scattering; ‘OIS’—optical isotope shift; ‘ π^+ scat.’—elastic positive pion scattering; ‘ μ -X’—muonic atom X-ray spectroscopy. Numbers in the parentheses mark the uncertainty to the preceding digits, which is also denoted by σ_r .

	r_c , fm	$\sigma_r r_c^{-1}/10^{-3}$	Method	References
^1H	0.84060(39)	0.5	μ -Laser	[27,28]
^2H	2.12775(17)	0.1	OIS + $r_c(^1\text{H})$	[29,30]
^3He	1.97007(94)	0.5	μ -Laser	[28,31]
^4He	1.6786(12)	0.7	μ -Laser	[28,32]
^6Li	2.589(39)	15	el. scat.	[33–36]
^7Li	2.444(42)	17	OIS + $r_c(^6\text{Li})$	[36,37]
^9Be	2.519(32)	13	el. scat. ¹	[38]
^{10}B	2.510(31)	12	OIS + $r_c(^{11}\text{B})$	[39]
^{11}B	2.411(21)	8.7	π^+ scat. + $r_c(^{12}\text{C})$	[40]
^{12}C	2.4829(19)	0.8	μ -X	[41]
^{13}C	2.4628(39)	1.6	μ -X	[42]

¹ Estimated using a model-dependent analysis of an electron scattering experiment covering a narrow momentum transfer range. The same study quotes a radius for ^{12}C [38], which differs by three standard deviations from the radius modern value. A systematic uncertainty to this value was added in quadrature to the smaller experimental uncertainty.

2. Physics Cases

2.1. Nuclear Structure

In contemporary ab initio approaches, nucleon–nucleon and three-nucleon interactions are derived from chiral effective field theory (EFT) and used to calculate observables in quantum many-body systems with quantifiable uncertainties [43,44]. Due to a combinatorial increase in computational cost with mass number, A , high-precision calculations are limited to $A \approx 16$. The next-generation calculations also treat the coupling to the continuum and what is referred to as “open quantum systems” [45–47], which are crucial for accurately reproducing the structure of both light and exotic nuclei, especially their spatial extension.

While nuclear forces are fit for the properties of light, dilute nuclear systems, testing their ability to predict the properties of $A > 4$ bound systems (i.e., when nuclear density achieves saturation but A remains sufficiently low for highly precise calculations) provides a means to gauge the measurements accuracy. The test allows for an investigation of the precision of the chiral EFT itself, ultimately challenging the understanding of the strong force at low energies. Nuclear radii are a particularly interesting testing ground, as they can be calculated to a high level of precision when the coupling with continuum is included, and the measurements of absolute charge radii may even be used to obtain, e.g., electric quadrupole (E2) observables by applying the observed correlations [48].

As an example, we consider the radii of the lithium isotopic chain, whose isotopic differences relative to ${}^6\text{Li}$ were extracted with high precision from optical isotope shift measurements [36,49,50]. Different *ab initio* nuclear models have been put forward to reproduce the results. However, the results cannot distinguish, because of the dominating uncertainty in the ${}^6\text{Li}$ radius, to which the chain is referenced. The same happens in the beryllium [51] and boron [39] cases. The corresponding measurement goal is to distinguish well which model reproduces the measured radii within a few 10^{-3} , of its value, which can be reached at the early stage of an experiment.

A more demanding accuracy is required when considering differential observables, such as the differences between the radii of mirror nuclei (the nuclei with neutron and proton numbers being interchanged): Δr_{mir} , which are a focus of contemporary studies of nuclear structure (see e.g., [52–57]). A linear relationship between Δr_{mir} and neutron skins [58–60], which are particularly complicated to directly measure, has been found. Measuring Δr_{mir} can thus contribute to the understanding of the variations in neutron skin with isospin, while any deviations from a linearity would indicate the role that continuum degrees of freedom play in exotic nuclei structure evolution. Light mirror pairs such as ${}^7\text{Li}$ – ${}^7\text{Be}$ and ${}^8\text{Li}$ – ${}^8\text{B}$ possess a large isospin asymmetry and are, hence, well-suited to testing these theoretical predictions. Differences in radii were measured with the optical isotope shifts in the Li [36,49,50] and Be [51] chains, while the measurement of ${}^8\text{B}$ [61] is ongoing. A need to measure Δr_{mir} accurately enough to not be limited by the reference radii sets a stringent demand to measure the radii with the accuracy as high as 10^{-3} for a stable isotope of each of the three elements.

2.2. QED and Beyond Standard Model (BSM)

The cases discussed in Section 2.1, are rooted in nuclear physics. Here, we show that high-precision measurements of the radii are necessary for testing the QED effects at the frontier of research in atomic physics. There are two main approaches to performing precision atomic structure calculations. The first is based on a perturbative expansion with respect to relativistic and QED effects in the Coulomb field, with electron–electron correlations being treated non-perturbatively. This approach is best adapted for low- Z systems such as hydrogen and helium. The second approach treats the QED and relativistic effects of all orders in the Coulomb field. It is needed for high- Z atoms, but loses accuracy in the low- Z region, where the correlations are more pronounced. The intermediate region, $Z \approx 6$ is particularly interesting, as both approaches have each maximum uncertainty. There is therefore an interest in studying QED for that light, but not too light, few-electron systems (see, e.g., [62–64] and the discussion therein).

Precise nuclear radii are needed to disentangle potentially missing QED contributions as a function of Z . To illustrate this point, let us consider $2^3S_1 - 2^3P_j$ transitions in ${}^7\text{Li}^+$. Their hyperfine-averaged value was determined to within 0.4 MHz [65]. From this measurement, the two-electron Lamb shift could be experimentally determined with 1.5 MHz precision, dominated by the uncertainty in the nuclear radius. Considering the ongoing experiments aim to obtain an accuracy of the order of 100 kHz [66], an order-of-magnitude improvement in the radius would allow for the missing α^8 (α denotes the fine structure constant) theory contribution of the order 3 MHz, to be determined with an uncertainty of 10% [64]. Moreover, this effect can be downscaled to the analogous

transition in the helium atom and to shed light on the observed, and recently confirmed, deviations between experiment and theory [67,68].

Above lithium, a new experimental program at Technical University Darmstadt, Germany, focuses on transitions in helium-like ions (HLIs) from beryllium to nitrogen [69–72], with the measurements in ^{12}C already completed [73,74]. Improved radii are crucial for confronting the results of the Darmstadt campaign with the state-of-the-art calculations, especially if one considers using measurements at a high Z to determine the missing QED contributions, and using these calculations in the measurements with lower Z .

At the accuracy frontier of BSM physics searches, new interactions are hunted through their manifestation as significant differences between experiment and theory. Currently, the strongest bound on fifth forces between charged leptons and neutrons derives from a combination of muonic and electronic isotope shifts in the hydrogen–deuterium pair [13]. When the mediating bosons are heavy, the sensitivity scale is Z^3 , favoring highly charged systems [75]. To utilize high-precision measurements of optical isotope shifts in simple enough electronic systems for BSM tests, one needs to considerably improve muonic isotope shifts. Accordingly, and as both the nuclear theory and experimental uncertainties associated with calibration largely cancel in the difference, we consider to measure these isotope shifts with suitable precision to determine differential radii with an accuracy above 10^{-3} fm, limited by the residual nuclear theory uncertainty.

We conclude this Section by noting that improved measurements of transitions to the ground level in a muonic atom directly translate to a better prediction of the muonic atom Lamb shift, which is accessible to laser spectroscopy. This statement is true irrespective of the nuclear-structure uncertainty. Quantitatively, a few parts per million measurement of the $2P - 1S$ energy in muonic lithium translates to a few-meV prediction of the muonic lithium Lamb shift. Such a narrow search region greatly reduces the time needed to conduct a successful high-precision laser spectroscopy measurement, thus increasing the feasibility of the experiments suggested in Ref. [76]. The resolution afforded by laser spectroscopy in muonic atoms would, in turn, enable the hyperfine structure to be resolved and determine the Zemach radius (a convolution of the electric and magnetic distributions) of lithium isotopes. This determination is highly demanded due to the sharp disagreement between this value, as calculated by nuclear theory and as determined by the electronic measurements [77–80]. Moreover, ongoing work suggests that the redundancy between X-ray and laser measurements in the same muonic species constitutes a powerful platform to search for new physics carried by new medium-mass (of the order of MeV) bosons.

3. Theory Considerations

The energy of the atomic transition between principle quantum numbers can be written as

$$E = E_D + \delta E_{\text{QED}} + \delta E_{\text{FNS}} + \delta E_{\text{TPE}} + \dots, \quad (1)$$

where E_D is the Dirac energy for a point-like nucleus, δE_{QED} is the sum of leading quantum electrodynamics effects, $\delta E_{\text{FNS}} \propto r_c^2$ is the leading order correction due to the finite nuclear size, from which the charge radius is extracted, and δE_{TPE} is the sum of corrections stemming from the two-photon exchange, which depends on the nuclear structure, namely the nuclear polarizability and higher charge moments of the nucleus. At the precision level that is foreseen for the project considered here, the uncertainty in point-nucleus QED corrections is negligible [81,82]. Accordingly, once the experimental accuracy of the transition energies is improved, the uncertainty in δE_{TPE} and higher-order nuclear structure contributions [29] is expected to dominate the derived radii. Based on the calculations for the lightest nuclei (see [28] and references therein), preliminary results for ^6Li [83] and the recent studies on heavier systems [84], we estimate that a 5 to 10% uncertainty in the calculated δE_{TPE} is achievable, resulting in an absolute radii with an accuracy of the order of a few times 10^{-4} , similar to that for the neighboring nuclei (see Table 1). Further calculations for the nuclei of interest are in progress. These could be achieved by applying the no-core shell model [85] with the Lanczos method [86,87]. In addition to

the calculations, the helpful information on the nuclear shape can, in some cases (notably ^{12}C [88]), be incorporated from elastic electron scattering measurements.

More accurate atomic theory considerations are needed in order to account for the unresolved fine and hyperfine structure features, mixed finite size and QED corrections, and shifts from spectator electrons, which screen the nuclear potential seen by the muon [89,90]. Accordingly, we calculated the atomic structure of the targeted systems using the Multiconfiguration Dirac-Fock General Matrix Elements (MCDFGME) code, which is able to evaluate the energies, transition probabilities, and hyperfine structure for exotic atoms composed of a nucleus, an arbitrary number of electrons, and an additional fermion or boson [91–95]. The energies are obtained using a full-atomic wave function composed of a determinant with all the electrons, multiplied by the muon wave function, and by solving the full coupled system of differential equations. The electron–electron and muon–electron interactions are chosen to represent the full Breit operator with Coulomb, magnetic, and retardation in the Coulomb gauge. Nuclear deformation effects could also be of importance [96–100]. They are not explicitly included in the atomic structure calculation and will be evaluated separately for this work.

Based on successful studies in the lightest systems, we are confident that precision experiments with these heavier muonic atoms will instigate new activity in the relevant atomic and nuclear theories, potentially contributing to related fields, such as studies with antiprotonic atoms [101] and highly charged ions [102].

4. Experimental Considerations

The radii of most of the stable nuclei were measured using traditional muonic X-ray spectroscopy with semiconductor detectors [103]. However, due to their moderate resolving power (fractional resolution) below 200 keV, compounded with the Z^2 scaling of the fractional contribution of the radius to the energy levels, this approach is insufficient to precisely determine finite-size effects in light nuclei. In contrast with the semiconducting detectors, crystal spectrometers offer high resolution in the multi-keV regime [104]. This detection method was used to determine the $2P - 1S$ transition energy in $\mu^{12}\text{C}$ to 5 ppm, and derive the radius with an accuracy of 2×10^{-3} fm [41,105]. This demonstrates that an X-ray detector with a resolving power of a few thousand enables the radii measurements in light nuclei with a precision better than 10^{-3} . However, this method suffers from low efficiency and a narrow bandwidth, making it impractical to extend to the entire series of light muonic elements in the available facilities considering beamtime constraints.

In order to measure the relevant transitions in light elements with sufficient accuracy and within a reasonable time, we use a different technology: metallic magnetic calorimeters (MMCs) [106], operated at exceptionally low cryogenic temperatures (about 20 mK). This quantum-sensing single-photon energy-detection technique achieves a high resolving power of few thousand, well-understood nonlinearity, and high quantum efficiency, as follows: above 50% up to 40 keV, and above 5% up to 180 keV [107]. It is thus ideally suited to tackling the problem of measuring the charge radii of light elements ($Z \leq 10$) using X-ray spectroscopy in muonic atoms. The principle of detection is that an X-ray is absorbed in a metallic absorber and its complete energy is converted to a temperature increase. This leads to a magnetization change in a paramagnetic material connecting the absorber to a thermal bath. This change in magnetization is high-sensitively detected using a superconducting quantum interference device (SQUID).

The first proof-of-concept measurement is currently undergoing its preparation. We transported an existing micro-calorimeter array for X-ray spectroscopy (maXs) [108,109] in a sidearm of a dilution refrigerator, from the Kirchhoff Institute for Physics in Heidelberg, to a secondary muon beamline at the Paul Scherrer Institute (PSI), Villigen, Switzerland. The detector is planned to be integrated with the existing muon, electron, and photon detectors from the muX experiment [110], which is expected to allow identifying and suppressing different sources of background. For detailed information on the detector, its performance, and its integration with the beamline, see [107].

Precise absolute X-ray measurements not only require a high resolution, ample statistics, and effective background suppression, but also a robust calibration strategy. The calibration function of the MMC detectors is to be determined periodically by using readily available sources [111]. To complement this method, a commercial X-ray tube is designed to be placed in the experimental system. There, electrons excite various metallic targets, emitting their characteristic X-rays, some of which have energies that are known to sub-ppm accuracy [112,113], and traceable to the International System of Units' second [114]. Similarly, the energy of muonic X-ray lines that do not involve an S-state can be reliably calculated to sub-ppm accuracy [115]. In this way, higher-lying lines in heavier muonic atoms could calibrate $2P - 1S$ lines in lighter systems.

Finally, let us note that the experiment considered in this paper and other next-generation experiments on exotic atoms (e.g., [116,117]), could significantly benefit from the ppm-level absolute gamma-ray energy measurements in the 20–200 keV range, especially from readily available long-lived commercial sources.

5. Summary

QUARTET is a new Collaboration that seeks to significantly improve the experimental values of the charge radii from lithium to neon by means of a precision muonic X-ray spectroscopy with metallic magnetic calorimeters, filling the gap between laser spectroscopy- and semiconductor-based X-ray spectroscopy for the elements that are beyond reach of both the methods. It will be the first time that those detectors are used with exotic atoms. The expected ten-fold improvement in precision will significantly impact nuclear structure and QED tests, and pave the way for moving BSM physics searches through the combinations of the muonic and electronic isotope shifts on one side and the laser and X-ray spectroscopy, on the other side.

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References

1. Wu, C.S.; Wilets, L. Muonic atoms and nuclear structure. *Annu. Rev. Nucl. Sci.* **1969**, *19*, 527–606. [[CrossRef](#)]
2. Friar, J.L. Nuclear finite-size effects in light muonic atoms. *Ann. Phys.* **1979**, *122*, 151–196. [[CrossRef](#)]
3. Friar, J.L. The structure of light nuclei and its effect on precise atomic measurements. In *Precision Physics of Simple Atomic Systems*; Karshenboim, S.G., Smirnov, V.B., Eds.; Springer: Berlin/Heidelberg, Germany, 2003; pp. 59–79. [[CrossRef](#)]

4. Karshenboim, S.G. Precision physics of simple atoms: QED tests, nuclear structure and fundamental constants. *Phys. Rep.* **2005**, *422*, 1–63. [\[CrossRef\]](#)
5. Kanda, S.; Ishida, K.; Iwasaki, M.; Ma, Y.; Okada, S.; Takamine, A.; Ueno, H.; Midorikawa, K.; Saito, N.; Wada, S.; et al. Measurement of the proton Zemach radius from the hyperfine splitting in muonic hydrogen atom. *J. Phys. Conf. Ser.* **2018**, *1138*, 012009. [\[CrossRef\]](#)
6. Pizzolotto, C.; Adamczak, A.; Bakalov, D.; Baldazzi, G.; Baruzzo, M.; Benocci, R.; Bertoni, R.; Bonesini, M.; Bonvicini, V.; Cabrera, H.; et al. The FAMU experiment: Muonic hydrogen high precision spectroscopy studies. *Eur. Phys. J. A* **2020**, *56*, 185. [\[CrossRef\]](#)
7. Antognini, A.; Bacca, S.; Fleischmann, A.; Gastaldo, L.; Hagelstein, F.; Indelicato, P.; Knecht, A.; Lensky, V.; Ohayon, B.; Pascalutsa, V.; et al. Muonic-atom spectroscopy and impact on nuclear structure and precision QED theory. *arXiv* **2022**, arXiv:2210.16929. [\[CrossRef\]](#)
8. Eides, M.I.; Grotch, H.; Shelyuto, V.A. *Theory of Light Hydrogenic Bound States*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2007. [\[CrossRef\]](#)
9. Antognini, A.; Lin, Y.H.; Meißner, U.G. Precision calculation of the recoil–finite-size correction for the hyperfine splitting in muonic and electronic hydrogen. *Phys. Lett. B* **2022**, *835*, 137575. [\[CrossRef\]](#)
10. Karshenboim, S.G. Precision physics of simple atoms and constraints on a light boson with ultraweak coupling. *Phys. Rev. Lett.* **2010**, *104*, 220406. [\[CrossRef\]](#)
11. Barger, V.; Chiang, C.W.; Keung, W.Y.; Marfatia, D. Proton size anomaly. *Phys. Rev. Lett.* **2011**, *106*, 153001. [\[CrossRef\]](#) [\[PubMed\]](#)
12. Villalba-Chávez, S.; Golub, A.; Müller, C. Axion-modified photon propagator, Coulomb potential, and Lamb shift. *Phys. Rev. D* **2018**, *98*, 115008. [\[CrossRef\]](#)
13. Delaunay, C.; Fruguele, C.; Fuchs, E.; Soreq, Y. Probing new spin-independent interactions through precision spectroscopy in atoms with few electrons. *Phys. Rev. D* **2017**, *96*, 115002. [\[CrossRef\]](#)
14. Jentschura, U.D.; Nándori, I. Atomic physics constraints on the X boson. *Phys. Rev. A* **2018**, *97*, 042502. [\[CrossRef\]](#)
15. Karshenboim, S.G.; McKeen, D.; Pospelov, M. Constraints on muon-specific dark forces. *Phys. Rev. D* **2014**, *90*, 073004. [\[CrossRef\]](#)
16. Claudia, F.; Clara, P. Muonic vs electronic dark forces: A complete EFT treatment for atomic spectroscopy. *J. High Energy Phys.* **2022**, *2022*, 002. [\[CrossRef\]](#)
17. Engfer, R.; Schneuwly, H.; Vuilleumier, J.; Walter, H.; Zehnder, A. Charge-distribution parameters, isotope shifts, isomer shifts, and magnetic hyperfine constants from muonic atoms. *At. Data Nucl. Data Tables* **1974**, *14*, 509–597. [\[CrossRef\]](#)
18. Fricke, G.; Bernhardt, C.; Heilig, K.; Schaller, L.; Schellenberg, L.; Shera, E.; Dejager, C. Nuclear ground state charge radii from electromagnetic interactions. *At. Data Nucl. Data Tables* **1995**, *60*, 177–285. [\[CrossRef\]](#)
19. Lorenz, I.; Hammer, H.W.; Meißner, U.G. The size of the proton: Closing in on the radius puzzle. *Eur. Phys. J. A* **2012**, *48*, 151. [\[CrossRef\]](#)
20. Pohl, R.; Gilman, R.; Miller, G.A.; Pachucki, K. Muonic hydrogen and the proton radius puzzle. *Annu. Rev. Nucl. Part. Sci.* **2013**, *63*, 175–204. [\[CrossRef\]](#)
21. Carlson, C.E. The proton radius puzzle. *Prog. Part. Nucl. Phys.* **2015**, *82*, 59–77. [\[CrossRef\]](#)
22. Hammer, H.W.; Meißner, U.G. The proton radius: From a puzzle to precision. *Sci. Bull.* **2020**, *65*, 257–258. [\[CrossRef\]](#) [\[PubMed\]](#)
23. Peset, C.; Pineda, A.; Tomalak, O. The proton radius (puzzle?) and its relatives. *Prog. Part. Nucl. Phys.* **2021**, *121*, 103901. [\[CrossRef\]](#)
24. Jentschura, U.D. Proton radius: A puzzle or a solution? *J. Phys. Conf. Ser.* **2022**, *2391*, 012017. [\[CrossRef\]](#)
25. Meißner, U.G. The proton radius and its relatives—much ado about nothing? *J. Phys. Conf. Ser.* **2023**, *2586*, 012006. [\[CrossRef\]](#)
26. Tiesinga, E.; Mohr, P.J.; Newell, D.B.; Taylor, B.N. CODATA recommended values of the fundamental physical constants: 2018. *Rev. Mod. Phys.* **2021**, *93*, 025010. [\[CrossRef\]](#) [\[PubMed\]](#)
27. Antognini, A.; Nez, F.; Schuhmann, K.; Amaro, F.D.; Biraben, F.; Cardoso, J.M.R.; Covita, D.S.; Dax, A.; Dhawan, S.; Diepold, M.; et al. Proton structure from the measurement of 2S-2P transition frequencies of muonic hydrogen. *Science* **2013**, *339*, 417–420. [\[CrossRef\]](#) [\[PubMed\]](#)
28. Pachucki, K.; Lensky, V.; Hagelstein, F.; Li Muli, S.S.; Bacca, S.; Pohl, R. Comprehensive theory of the Lamb shift in light muonic atoms. *arXiv* **2022**, arXiv:2212.13782. [\[CrossRef\]](#)
29. Pachucki, K.; Patkós, V.; Yerokhin, V.A. Three-photon-exchange nuclear structure correction in hydrogenic systems. *Phys. Rev. A* **2018**, *97*, 062511. [\[CrossRef\]](#)
30. Parthey, C.G.; Matveev, A.; Alnis, J.; Pohl, R.; Udem, T.; Jentschura, U.D.; Kolachevsky, N.; Hänsch, T.W. Precision measurement of the hydrogen-deuterium 1S – 2S isotope shift. *Phys. Rev. Lett.* **2010**, *104*, 233001. [\[CrossRef\]](#) [\[PubMed\]](#)
31. Schuhmann, K.; et al. [The CREMA Collaboration]. The helion charge radius from laser spectroscopy of muonic helium-3 ions. *arXiv* **2023**, arXiv:2305.11679. [\[CrossRef\]](#)
32. Krauth, J.J.; Schuhmann, K.; Ahmed, M.A.; Amaro, F.D.; Amaro, P.; Biraben, F.; Chen, T.L.; Covita, D.S.; Dax, A.J.; Diepold, M.; et al. Measuring the α -particle charge radius with muonic helium-4 ions. *Nature* **2021**, *589*, 527–531. [\[CrossRef\]](#) [\[PubMed\]](#)
33. Suelzle, L.R.; Yearian, M.R.; Crannell, H. Elastic electron scattering from Li^6 and Li^7 . *Phys. Rev.* **1967**, *162*, 992–1005. [\[CrossRef\]](#)
34. Bumiller, F.A.; Buskirk, F.R.; Dyer, J.N.; Monson, W.A. Elastic electron scattering from ^6Li and ^7Li at low momentum transfer. *Phys. Rev. C* **1972**, *5*, 391–395. [\[CrossRef\]](#)
35. Li, G.; Sick, I.; Whitney, R.; Yearian, M. High-energy electron scattering from ^6Li . *Nucl. Phys. A* **1971**, *162*, 583–592. [\[CrossRef\]](#)

36. Nörtershäuser, W.; Neff, T.; Sánchez, R.; Sick, I. Charge radii and ground state structure of lithium isotopes: Experiment and theory reexamined. *Phys. Rev. C* **2011**, *84*, 024307. [\[CrossRef\]](#)
37. Nörtershäuser, W.; Sánchez, R.; Ewald, G.; Dax, A.; Behr, J.; Bricault, P.; Bushaw, B.A.; Dilling, J.; Dombsky, M.; Drake, G.W.F.; et al. Isotope-shift measurements of stable and short-lived lithium isotopes for nuclear-charge-radii determination. *Phys. Rev. A* **2011**, *83*, 012516. [\[CrossRef\]](#)
38. Jansen, J.A.; Peerdeman, R.T.; De Vries, C. Nuclear charge radii of ^{12}C and ^9Be . *Nucl. Phys. A* **1972**, *188*, 337–352. [\[CrossRef\]](#)
39. Maaß, B.; Hüther, T.; König, K.; Krämer, J.; Krause, J.; Lovato, A.; Müller, P.; Pachucki, K.; Puchalski, M.; Roth, R.; et al. Nuclear charge radii of $^{10,11}\text{B}$. *Phys. Rev. Lett.* **2019**, *122*, 182501. [\[CrossRef\]](#) [\[PubMed\]](#)
40. Barnett, B.; Gyles, W.; Johnson, R.; Erdman, K.; Johnstone, J.; Kraushaar, J.; Lepp, S.; Masterson, T.; Rost, E.; Gill, D.; et al. Proton radii determinations from the ratio of π^+ elastic scattering from ^{11}B and ^{12}C . *Phys. Lett. B* **1980**, *97*, 45–49. [\[CrossRef\]](#)
41. Ruckstuhl, W.; Aas, B.; Beer, W.; Beltrami, I.; Bos, K.; Goudsmit, P.F.A.; Leisi, H.J.; Strassner, G.; Vacchi, A.; De Boer, F.W.N.; et al. Precision measurement of the 2p-1s transition in muonic ^{12}C : Search for new muon-nucleon interactions or accurate determination of the rms nuclear charge radius. *Nucl. Phys. A* **1984**, *430*, 685–712. [\[CrossRef\]](#)
42. de Boer, F.W.N.; Aas, B.; Baertschi, P.; Beer, W.; Beltrami, I.; Bos, K.; Goudsmit, P.F.A.; Kiebele, U.; Jeckelmann, B.; Leisi, H.J.; et al. Precision measurement of the 2p-1s transition wavelength in muonic ^{13}C . *Nucl. Phys. A* **1985**, *444*, 589–596. [\[CrossRef\]](#)
43. Epelbaum, E.; Hammer, H.-W.; Meißner, U.-G. Modern theory of nuclear forces. *Rev. Mod. Phys.* **2009**, *81*, 1773–1825. [\[CrossRef\]](#)
44. Hergert, H. A Guided tour of ab initio nuclear many-body theory. *Front. Phys.* **2020**, *8*, 379. [\[CrossRef\]](#)
45. Dohet-Eraly, J.; Navrátil, P.; Quaglioni, S.; Horiuchi, W.; Hupin, G.; Raimondi, F. $^3\text{He}(\alpha, \gamma)^7\text{Be}$ and $^3\text{H}(\alpha, \gamma)^7\text{Li}$ astrophysical S factors from the no-core shell model with continuum. *Phys. Lett. B* **2016**, *757*, 430–436. [\[CrossRef\]](#)
46. Navrátil, P.; Quaglioni, S.; Hupin, G.; Romero-Redondo, C.; Calci, A. Unified ab initio approaches to nuclear structure and reactions. *Phys. Scr.* **2016**, *91*, 053002. [\[CrossRef\]](#)
47. Vorabbi, M.; Navrátil, P.; Quaglioni, S.; Hupin, G. ^7Be and ^7Li nuclei within the no-core shell model with continuum. *Phys. Rev. C* **2019**, *100*, 024304. [\[CrossRef\]](#)
48. Caprio, M.A.; Fasano, P.J.; Maris, P. Robust ab initio prediction of nuclear electric quadrupole observables by scaling to the charge radius. *Phys. Rev. C* **2022**, *105*, L061302. [\[CrossRef\]](#)
49. Ewald, G.; Nörtershäuser, W.; Dax, A.; Götte, S.; Kirchner, R.; Kluge, H.-J.; Kühl, T.; Sanchez, R.; Wojtaszek, A.; Bushaw, B.A.; et al. Nuclear charge radii of $^8,9\text{Li}$ determined by laser spectroscopy. *Phys. Rev. Lett.* **2004**, *93*, 113002. [\[CrossRef\]](#)
50. Sánchez, R.; Nörtershäuser, W.; Ewald, G.; Albers, D.; Behr, J.; Bricault, P.; Bushaw, B.A.; Dax, A.; Dilling, J.; Dombsky, M.; et al. Nuclear charge radii of $^9,11\text{Li}$: The influence of halo neutrons. *Phys. Rev. Lett.* **2006**, *96*, 033002. [\[CrossRef\]](#)
51. Nörtershäuser, W.; Tiedemann, D.; Žáková, M.; Andjelkovic, Z.; Blaum, K.; Bissell, M.L.; Cazan, R.; Drake, G.W.F.; Geppert, C.; Kowalska, M.; et al. Nuclear charge radii of $^{7,9,10}\text{Be}$ and the one-neutron halo nucleus ^{11}Be . *Phys. Rev. Lett.* **2009**, *102*, 062503. [\[CrossRef\]](#)
52. Brown, B.A. Mirror charge radii and the neutron equation of state. *Phys. Rev. Lett.* **2017**, *119*, 122502. [\[CrossRef\]](#)
53. Brown, B.A.; Minamisono, K.; Piekarewicz, J.; Hergert, H.; Garand, D.; Klose, A.; König, K.; Lantis, J.D.; Liu, Y.; Maaß, B.; et al. Implications of the ^{36}Ca – ^{36}S and ^{38}Ca – ^{38}Ar difference in mirror charge radii on the neutron matter equation of state. *Phys. Rev. Res.* **2020**, *2*, 022035. [\[CrossRef\]](#)
54. Pineda, S.V.; König, K.; Rossi, D.M.; Brown, B.A.; Incorvati, A.; Lantis, J.; Minamisono, K.; Nörtershäuser, W.; Piekarewicz, J.; Powel, R.; et al. Charge radius of neutron-deficient ^{54}Ni and symmetry energy constraints using the difference in mirror pair charge radii. *Phys. Rev. Lett.* **2021**, *127*, 182503. [\[CrossRef\]](#) [\[PubMed\]](#)
55. Naito, T.; Roca-Maza, X.; Colò, G.; Liang, H.; Sagawa, H. Isospin symmetry breaking in the charge radius difference of mirror nuclei. *Phys. Rev. C* **2022**, *106*, L061306. [\[CrossRef\]](#)
56. Reinhard, P.G.; Nazarewicz, W. Information content of the differences in the charge radii of mirror nuclei. *Phys. Rev. C* **2022**, *105*, L021301. [\[CrossRef\]](#)
57. Bano, P.; Pattnaik, S.P.; Centelles, M.; Viñas, X.; Routray, T.R. Correlations between charge radii differences of mirror nuclei and stellar observables. *Phys. Rev. C* **2023**, *108*, 015802. [\[CrossRef\]](#)
58. Yang, J.; Piekarewicz, J. Difference in proton radii of mirror nuclei as a possible surrogate for the neutron skin. *Phys. Rev. C* **2018**, *97*, 014314. [\[CrossRef\]](#)
59. Gaidarov, M.; Moumène, I.; Antonov, A.; Kadrev, D.; Sarriguren, P.; Moya de Guerra, E. Proton and neutron skins and symmetry energy of mirror nuclei. *Nucl. Phys. A* **2020**, *1004*, 122061. [\[CrossRef\]](#)
60. Novario, S.J.; Lonardonì, D.; Gandolfi, S.; Hagen, G. Trends of neutron skins and radii of mirror nuclei from first principles. *Phys. Rev. Lett.* **2023**, *130*, 032501. [\[CrossRef\]](#)
61. Maaß, B.; Müller, P.; Nörtershäuser, W.; Clark, J.; Gorges, C.; Kaufmann, S.; König, K.; Krämer, J.; Levand, A.; Orford, R.; et al. Towards laser spectroscopy of the proton-halo candidate boron-8. *Hyperfine Interact.* **2017**, *238*, 25. [\[CrossRef\]](#)
62. Yerokhin, V.A.; Pachucki, K. Theoretical energies of low-lying states of light helium-like ions. *Phys. Rev. A* **2010**, *81*, 022507. [\[CrossRef\]](#)
63. Yerokhin, V.A.; Patkóš, V.; Pachucki, K. QED calculations of energy levels of heliumlike ions with $5 \leq Z \leq 30$. *Phys. Rev. A* **2022**, *106*, 022815. [\[CrossRef\]](#)
64. Yerokhin, V.A.; Patkóš, V.; Pachucki, K. QED $m\alpha^7$ effects for triplet states of heliumlike ions. *Phys. Rev. A* **2023**, *107*, 012810. [\[CrossRef\]](#)

65. Riis, E.; Sinclair, A.G.; Poulsen, O.; Drake, G.W.F.; Rowley, W.R.C.; Levick, A.P. Lamb shifts and hyperfine structure in ${}^6\text{Li}^+$ and ${}^7\text{Li}^+$: Theory and experiment. *Phys. Rev. A* **1994**, *49*, 207–220. [\[CrossRef\]](#)
66. Guan, H.; Chen, S.; Qi, X.-Q.; Liang, S.; Sun, W.; Zhou, P.; Huang, Y.; Zhang, P.-P.; Zhong, Z.-X.; Yan, Z.-C.; et al. Probing atomic and nuclear properties with precision spectroscopy of fine and hyperfine structures in the ${}^7\text{Li}^+$ ion. *Phys. Rev. A* **2020**, *102*, 030801. [\[CrossRef\]](#)
67. Clausen, G.; Jansen, P.; Scheidegger, S.; Agner, J.A.; Schmutz, H.; Merkt, F. Ionization energy of the metastable $2\,{}^1\text{S}_0$ state of ${}^4\text{He}$ from Rydberg-series extrapolation. *Phys. Rev. Lett.* **2021**, *127*, 093001. [\[CrossRef\]](#) [\[PubMed\]](#)
68. Clausen, G.; Scheidegger, S.; Agner, J.A.; Schmutz, H.; Merkt, F. Imaging-assisted single-photon Doppler-free laser spectroscopy and the ionization energy of metastable triplet helium. *Phys. Rev. Lett.* **2023**, *131*, 103001. [\[CrossRef\]](#) [\[PubMed\]](#)
69. Maaß, B. Laser Spectroscopy of the Boron Isotopic Chain. Ph.D. Thesis, TU Darmstadt, Darmstadt, Germany, 2020. [\[CrossRef\]](#)
70. König, K.; Krämer, K.; Geppert, C.; Imgram, P.; Maaß, B.; Ratajczyk, T.; Nörtershäuser, W. A new Collinear Apparatus for Laser Spectroscopy and Applied Science (COALA). *Rev. Sci. Instrum.* **2020**, *91*, 081301. [\[CrossRef\]](#) [\[PubMed\]](#)
71. Mohr, K.; Buß, A.; Andelkovic, Z.; Hannen, V.; Horst, M.; Imgram, P.; König, K.; Maaß, B.; Nörtershäuser, W.; Rausch, S.; et al. Collinear laser spectroscopy of helium-like ${}^{11}\text{B}^{3+}$. *Atoms* **2023**, *11*, 11. [\[CrossRef\]](#)
72. Imgram, P. High-Precision Laser Spectroscopy of Helium-like Carbon ${}^{12}\text{C}^{4+}$. Ph.D. Thesis, TU Darmstadt, Darmstadt, Germany, 2023. [\[CrossRef\]](#)
73. Imgram, P.; König, K.; Maaß, B.; Müller, P.; Nörtershäuser, W. Collinear laser spectroscopy of highly charged ions produced with an electron-beam ion source. *Phys. Rev. A* **2023**, *108*, 062809. [\[CrossRef\]](#)
74. Imgram, P.; König, K.; Maaß, B.; Müller, P.; Nörtershäuser, W. Collinear laser spectroscopy of $2\,{}^3\text{S}_1 \rightarrow 2\,{}^3\text{P}_J$ transitions in helium-like ${}^{12}\text{C}^{4+}$. *Phys. Rev. Lett.* **2023**, *131*, 243001. [\[CrossRef\]](#) [\[PubMed\]](#)
75. Sailer, T.; Debierre, V.; Harman, Z.; Heiße, F.; König, C.; Morgner, J.; Tu, B.; Volotka, A.V.; Keitel, C.H.; Blaum, K.; et al. Measurement of the bound-electron g -factor difference in coupled ions. *Nature* **2022**, *606*, 479–483. [\[CrossRef\]](#)
76. Schmidt, S.; Willig, M.; Haack, J.; Horn, R.; Adamczak, A.; Abdou Ahmed, M.; Amaro, F.D.; Amaro, P.; Biraben, F.; Carvalho, P.; et al. The next generation of laser spectroscopy experiments using light muonic atoms. *J. Phys. Conf. Ser.* **2018**, *1138*, 012010. [\[CrossRef\]](#)
77. Yerokhin, V.A. Hyperfine structure of Li and Be^+ . *Phys. Rev. A* **2008**, *78*, 012513. [\[CrossRef\]](#)
78. Sun, W.; Zhang, P.-P.; Zhou, P.-P.; Chen, S.-L.; Zhou, Z.-Q.; Huang, Y.; Qi, X.-Q.; Yan, Z.-C.; Shi, T.-Y.; Drake, G.W.F.; et al. Measurement of hyperfine structure and the Zemach radius in ${}^6\text{Li}^+$ using optical Ramsey technique. *Phys. Rev. Lett.* **2023**, *131*, 103002. [\[CrossRef\]](#)
79. Pachucki, K.; Patkóš, V.; Yerokhin, V.A. Hyperfine splitting in ${}^{6,7}\text{Li}^+$. *Phys. Rev. A* **2023**, *108*, 052802. [\[CrossRef\]](#)
80. Pachucki, K.; Patkóš, V.; Yerokhin, V.A. Accurate determination of ${}^{6,7}\text{Li}$ nuclear magnetic moments. *Phys. Lett. B* **2023**, *846*, 138189. [\[CrossRef\]](#)
81. Krutov, A.A.; Martynenko, A.P.; Martynenko, F.A.; Sukhorukova, O.S. Lamb shift in muonic ions of lithium, beryllium, and boron. *Phys. Rev. A* **2016**, *94*, 062505. [\[CrossRef\]](#)
82. Dorokhov, A.E.; Faustov, R.N.; Martynenko, A.P.; Martynenko, F.A. Precision physics of muonic ions of lithium, beryllium and boron. *Int. J. Mod. Phys. A* **2021**, *36*, 2150022. [\[CrossRef\]](#)
83. Li Muli, S.S.; Poggialini, A.; Bacca, S. Muonic lithium atoms: Nuclear structure corrections to the Lamb shift. *SciPost Phys. Proc.* **2020**, *3*, 028. [\[CrossRef\]](#)
84. Valuev, I.A.; Colò, G.; Roca-Maza, X.; Keitel, C.H.; Oreshkina, N.S. Evidence against nuclear polarization as source of fine-structure anomalies in muonic atoms. *Phys. Rev. Lett.* **2022**, *128*, 203001. [\[CrossRef\]](#)
85. Barrett, B.R.; Navrátil, P.; Vary, J.P. Ab initio no core shell model. *Prog. Part. Nucl. Phys.* **2013**, *69*, 131–181. [\[CrossRef\]](#)
86. Lanczos, C. An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. *J. Res. Natl. Bur. Stand.* **1950**, *45*, 255–282. [\[CrossRef\]](#)
87. Haydock, R. The inverse of a linear operator. *J. Phys. Math. Nucl. Gen.* **1974**, *7*, 2120–2124. [\[CrossRef\]](#)
88. Reuter, W.; Fricke, G.; Merle, K.; Miska, H. Nuclear charge distribution and rms radius of ${}^{12}\text{C}$ from absolute elastic electron scattering measurements. *Phys. Rev. C* **1982**, *26*, 806–818. [\[CrossRef\]](#)
89. Schnewly, H.; Vogel, P. Electronic K X-ray energies in heavy muonic atoms. *Phys. Rev. A* **1980**, *22*, 2081–2087. [\[CrossRef\]](#)
90. Simons, L.M.; Abbot, D.; Bach, B.; Bacher, R.; Badertscher, A.; Blüm, P.; DeCecco, P.; Eades, J.; Egger, J.; Elsener, K.; et al. Exotic atoms and their electron shell. *Nucl. Instrum. Meth. Phys. Res. B Beam Interact. Mater. Atoms* **1994**, *87*, 293–300. [\[CrossRef\]](#)
91. Desclaux, J.P. A multiconfiguration relativistic DIRAC-FOCK program. *Comput. Phys. Commun.* **1975**, *9*, 31–45. [\[CrossRef\]](#)
92. Mallow, J.V.; Desclaux, J.P.; Freeman, A.J. Dirac-Fock method for muonic atoms : Transitions energies, wave functions, and charge densities. *Phys. Rev. A* **1978**, *17*, 1804–1809. [\[CrossRef\]](#)
93. Santos, J.P.; Parente, F.; Boucard, S.; Indelicato, P.; Desclaux, J.P. X-ray energies of circular transitions and electron screening in kaonic atoms. *Phys. Rev. A* **2005**, *71*, 032501. [\[CrossRef\]](#)
94. Trassinelli, M.; Indelicato, P. Relativistic calculations of pionic and kaonic atoms' hyperfine structure. *Phys. Rev. A* **2007**, *76*, 012510. [\[CrossRef\]](#)
95. Indelicato, P. Nonperturbative evaluation of some QED contributions to the muonic hydrogen $n = 2$ Lamb shift and hyperfine structure. *Phys. Rev. A* **2013**, *87*, 022501. [\[CrossRef\]](#)

96. De Wit, S.A.; Backenstoss, G.; Daum, C.; Sens, J.C.; Acker, H.L. Measurement and analysis of muonic X-ray spectra in deformed nuclei. *Nucl. Phys.* **1966**, *87*, 657–702. [[CrossRef](#)]
97. Pieper, W.; Greiner, W. The influence of nuclear dynamics on the X-ray spectrum of muonic atoms. *Phys. Lett. B* **1967**, *24*, 377–380. [[CrossRef](#)]
98. Foot, C.J.; Stacey, D.N.; Stacey, V.; Kloch, R.; Leś, Z. Isotope effects in the nuclear charge distribution in zinc. *Proc. R. Soc. A Math. Phys. Engin. Sci.* **1982**, *384*, 205–216. [[CrossRef](#)]
99. Sahoo, B.K.; Ohayon, B. All-optical differential radii in zinc. *Phys. Rev. Res.* **2023**, *5*, 043142. [[CrossRef](#)]
100. Çavuşoğlu, A.; Sikora, B. Impact of the nuclear charge distribution on the g-factors and ground state energies of bound muons. *arXiv* **2023**, arXiv:2311.16855. [[CrossRef](#)]
101. Paul, N.; Bian, G.; Azuma, T.; Okada, S.; Indelicato, P. Testing quantum electrodynamics with exotic atoms. *Phys. Rev. Lett.* **2021**, *126*, 173001. [[CrossRef](#)] [[PubMed](#)]
102. Indelicato, P. QED tests with highly charged ions. *J. Phys. B At. Mol. Opt. Phys.* **2019**, *52*, 232001. [[CrossRef](#)]
103. Fricke, G.; Heilig, K. *Nuclear Charge Radii*; Springer Nature Switzerland AG: Cham, Switzerland, 2004. [[CrossRef](#)]
104. Simons, L.M. X-ray spectroscopy at PSI. *Hyperfine Interact.* **1999**, *119*, 281–290. [[CrossRef](#)]
105. Ruckstuhl, W.; Aas, B.; Beer, W.; Beltrami, I.; de Boer, F.W.N.; Bos, K.; Goudsmit, P.F.A.; Kiebele, U.; Leisi, H.J.; Strassner, G.; et al. High-precision muonic X-ray measurement of the rms charge radius of ^{12}C with a crystal spectrometer. *Phys. Rev. Lett.* **1982**, *49*, 859–862. [[CrossRef](#)]
106. Fleischmann, A.; Enss, C.; Seidel, G. Metallic magnetic calorimeters. In *Cryogenic Particle Detection*; Enss, C., Ed.; Springer: Berlin/Heidelberg, Germany, 2005; pp. 151–216. [[CrossRef](#)]
107. Unger, D.; Abeln, A.; Cocolios, T.E.; Eizenberg, O.; Enss, C.; Fleischmann, A.; Gastaldo, L.; Godinho, C.; Heines, M.; Hengstler, D.; et al. MMC array to study X-ray transitions in muonic atoms. *arXiv* **2023**, arXiv:2311.12014. [[CrossRef](#)]
108. Hengstler, D.; Keller, M.; Schöltz, C.; Geist, J.; Krantz, M.; Kempf, S.; Gastaldo, L.; Fleischmann, A.; Gassner, T.; Weber, G.; et al. Towards FAIR: First measurements of metallic magnetic calorimeters for high-resolution X-ray spectroscopy at GSI. *Phys. Scr.* **2015**, *T166*, 014054. [[CrossRef](#)]
109. Unger, D.; Abeln, A.; Enss, C.; Fleischmann, A.; Hengstler, D.; Kempf, S.; Gastaldo, L. High-resolution for IAXO: MMC-based X-ray detectors. *J. Instrum.* **2021**, *16*, P06006. [[CrossRef](#)]
110. Wauters, F.; Knecht, A. The muX project. *SciPost Phys. Proc.* **2021**, *5*, 022. [[CrossRef](#)]
111. Boyd, S.; Kim, G.B.; Hall, J.; Cantor, R.; Friedrich, S. Metallic magnetic calorimeters for high-accuracy nuclear decay data. *J. Low Temp. Phys.* **2020**, *199*, 681–687. [[CrossRef](#)]
112. Deslattes, R.D.; Kessler, E.G.; Indelicato, P.; de Billy, L.; Lindroth, E.; Anton, J. X-ray transition energies: New approach to a comprehensive evaluation. *Rev. Mod. Phys.* **2003**, *75*, 35–99. [[CrossRef](#)]
113. Mendenhall, M.H.; Hudson, L.T.; Szabo, C.I.; Henins, A.; Cline, J.P. The molybdenum K-shell X-ray emission spectrum. *J. Phys. At. Mol. Opt. Phys.* **2019**, *52*, 215004. [[CrossRef](#)] [[PubMed](#)]
114. Hudson, L.T.; Cline, J.P.; Henins, A.; Mendenhall, M.H.; Szabo, C. Contemporary X-ray wavelength metrology and traceability. *Radiat. Phys. Chem.* **2020**, *167*, 108392. [[CrossRef](#)] [[PubMed](#)]
115. Trassinelli, M.; Anagnostopoulos, D.F.; Borchert, G.; Dax, A.; Egger, J.-P.; Gotta, D.; Hennebach, M.; Indelicato, P.; Liu, Y.-W.; Manil, B.; et al. Measurement of the charged pion mass using X-ray spectroscopy of exotic atoms. *Phys. Lett. B* **2016**, *759*, 583–588. [[CrossRef](#)]
116. Okumura, T.; Azuma, T.; Bennett, D.A.; Chiu, I.; Doriese, W.B.; Durkin, M.S.; Fowler, J.W.; Gard, J.D.; Hashimoto, T.; Hayakawa, R.; et al. Proof-of-principle experiment for testing strong-field quantum electrodynamics with exotic atoms: High precision X-ray spectroscopy of muonic neon. *Phys. Rev. Lett.* **2023**, *130*, 173001. [[CrossRef](#)] [[PubMed](#)]
117. Curceanu, C.; Abbene, L.; Amsler, C.; Bazzi, M.; Bettelli, M.; Borghi, G.; Bosnar, D.; Bragadireanu, M.; Buttacavoli, A.; Cargnelli, M.; et al. Kaonic atoms at the DAΦNE collider: A strangeness adventure. *Front. Phys.* **2023**, *11*, 1240250. [[CrossRef](#)]

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