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*Abstract*

# Simple Methods for Traveling Salesman Problems †

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Here we will focus on an ongoing project on approximation algorithms for the Euclidean Traveling Salesman Problems (TSP). Given $n$ points and the distances between them, the aim is to construct a tour that visits each point exactly once and has the minimum total cost/distance (the total cost of a tour is the sum of the distances defined by each pair of points from that tour). The Euclidean version of the TSP, in which the distances between the objects (points or cities) are distances in a two-dimensional Euclidean space, is strongly NP-hard. However, compared to the general setting, the Euclidean version is still more treatable since it naturally allows us to employ simple geometric tolls in the solution process, so that a "blind" enumeration of all feasible tours can be replaced by a more rational and transparent kind of selection of reasonable tours. Our method for solving Euclidean TSP initially, at Phase 1, constructs a girding polygon, a boundary that includes all the given points. The convex boundary (hull) of the polygon, consisting of its edges, already defines a partial tour, which is optimal for the set of nodes that this boundary contains. The insertion heuristic of Phase 2 iteratively augments the partial tour of Phase 1 to a complete feasible tour using the cheapest insertion strategy: iteratively, the current partial tour is augmented with a new point, which yields the minimal increase in the cost. The tour improvement heuristic of Phase 3 improves the tour of Phase 2 using some local optimality conditions. Thanks to simple geometry in the decision-making process at Phases 2 and 3, our algorithm is extremely fast and requires little computer memory, whereas the quality of the delivered solutions is comparable with that of the state-of-the-art algorithms, see [1]. We are currently working on another approximation algorithm for the Euclidean TSP which also starts with the girding polygon. A special kind of inner and outer convex boundaries (convex hulls) that include subsets of points are iteratively constructed. Each convex hull defines an optimal tour for the points of that hull. Two successively generated convex hulls are unified into a partial feasible tour that covers the nodes from both boundaries. The algorithm halts when it constructs a complete feasible tour. In Multiprocessor TSP there is one distinguished point called the depot, and $k$ salesmen. Each salesman has to build its own tour that starts from the depot, ends in the depot and only visits one or more additional points once. All points are to be visited by a salesman. The aim is to minimize the total cost of all $k$ tours. In the bounded version of Multiprocessor TSP lower and upper bounds determine the minimum and maximum number of points in a tour, i.e., a feasible tour respects these restrictions. Our algorithm for the Bounded Multiprocessor TSP initially partitions the set of vertices into $k$ disjoint subsets at Phase 1. Then, at Phase 2, it constructs the initial $k$ tours using the abovementioned three-phase algorithm for TSP. The feasible solution of Phase 2 is further improved at Phase 3: iteratively, a vertex from a tour is moved from its current position to another specially determined position within the same or another tour so that the resultant solution remains feasible. The destiny vertex and its new position are selected so that the accomplished rearrangement provides the maximum decrease of the current cost. We obtained preliminary experimental results for the 22 known benchmark instances. The approximation rate provided by the proposed heuristic is comparable to the state-of-the-art results, but it requires considerably less memory and CPU time.

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## Reference

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