Supplement to

Barotropic Instability During Eyewall Replacement

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S1. A Comparison of Numerics for the Non-divergent Barotropic Model

S1.1. The finite-difference and pseudo-spectral models

In our work, we present results from a finite difference version of the non-divergent barotropic model as represented in (29). For comparison purposes, we examine the non-linear evolution of vorticity for the Vortex A case (shown in Figure 11) using a pseudo-spectral model (Schubert *et al.* 1999). To do so, we compare the numerics for the finite-difference (Section S1.1a) and pseudo-spectral (Section S1.1b) models. We also provide a discussion of the implications that result from these numerical choices (Section S1.2) and commentary on the implementations of each model (Section S1.3).

a. Finite-difference model results

Figure 11 shows the non-linear evolution of the vorticity field using a finite-difference non-divergent barotropic model initialized with the smoothed version of Vortex A. As described, this finite-difference model uses the fourth-order Arakawa Jacobian (Arakawa 1966, 1970), a nine-point stencil Laplacian, multigrid methods via MUDPACK (Adams 1993), Dirichlet boundary conditions for the streamfunction, and a standard fourth-order Runge–Kutta scheme for time differencing with a 1.875 second time step. The model runs are performed on a domain of size 600 $\times$ 600 km with 2048 $\times$ 2048 equally spaced grid cells.

b. Pseudo-spectral model results

Here, we present the non-linear evolution of the vorticity field using a pseudo-spectral non-divergent barotropic model initialized with the same smoothed version of Vortex A. This is the same model used and described in Schubert *et al.* (1999). The pseudo-spectral model uses periodic boundary conditions and quadratic de-aliasing terms in (3.1). Although spectral modes up through 1024 are computed at a resolution of 0.293 km between the collocation points, only those modes up through 682 are kept, which results in a more realistic estimate of the resolution of the highest Fourier mode at 0.880 km. The model run shown in Figures S1.1–S1.3 uses the same run parameters (domain size, number of grid cells, time step, etc.) as those used in the finite-difference model, except the viscosity (coefficient of diffusion) is slightly larger ($\nu = 6.25$ m^2 s^{-1}) to account for the reduced effective resolution due to de-aliasing. This gives the $1/e$ damping times of 52 minutes for all modes having total wavenumber 682, while for the modes having total wavenumber 341, the damping time lengthens considerably to 209 minutes.

S1.2. Implications of the numerics

Since the very small initial asymmetric vorticity perturbations result from model discretization errors, the finite-difference and pseudo-spectral models have different onset times for barotropic instability. Comparing the results of the two models starting from their instability onset times, their general behavior is quite similar. Perhaps the most important difference is that the pseudo-spectral model is essentially free of computational dispersion errors. These errors are most pronounced in the finite-difference model where binary regions of high vorticity are advected by a nearly circular flow, as in Figure 11 at $t = 13$ and 14 hours. As discussed by Orszag (1971), computational dispersion produces a "wake of bad numbers" behind such binary regions of high vorticity. The problem is most severe with the second-order Arakawa Jacobian, but considerably less severe with the fourth-order Arakawa Jacobian used to produce Figure 11. As can be seen in Figures S1.1–S1.3 below, the problem is essentially eliminated with the pseudo-spectral method. Thus, to summarize, the pseudo-spectral discretization with quadratic nonlinearity computed using the transform method has two major advantages over the finite difference methods: (1) It eliminates aliasing error (and thus eliminates nonlinear computational instability) rather than just controlling it as in the Arakawa Jacobian methods; (2) It essentially eliminates computational dispersion errors. The fourth-order finite difference results shown in Figure 11 are roughly indicative of the potential vorticity fields that are obtained in full-physics models that use finite difference methods.

S1.3. Commentary on the models

While the finite-difference model is newer and more modern code that is better optimized, the Schubert *et al.* (1999) pseudo-spectral model is well-tested and has provided wide-ranging insight into tropical cyclone processes. This model must use de-aliasing, which reduces its effective resolution. But, the pseudo-spectral model approximates derivatives with higher accuracy than the finite-difference model. And, the pseudo-spectral model seems to be better at resolving the stretching (filamentation) of the vorticity field as the instability distorts and rearranges the vorticity field. As discussed, the differences in model numerics result in the instability taking about 45 minutes longer to develop in the pseudo-spectral model than in the finite-difference model, so the results from the two models are slightly out of phase. That being said, the finite-difference and pseudo-spectral models—two very different models—provide very similar results which provides good confirmation for the findings in this work.

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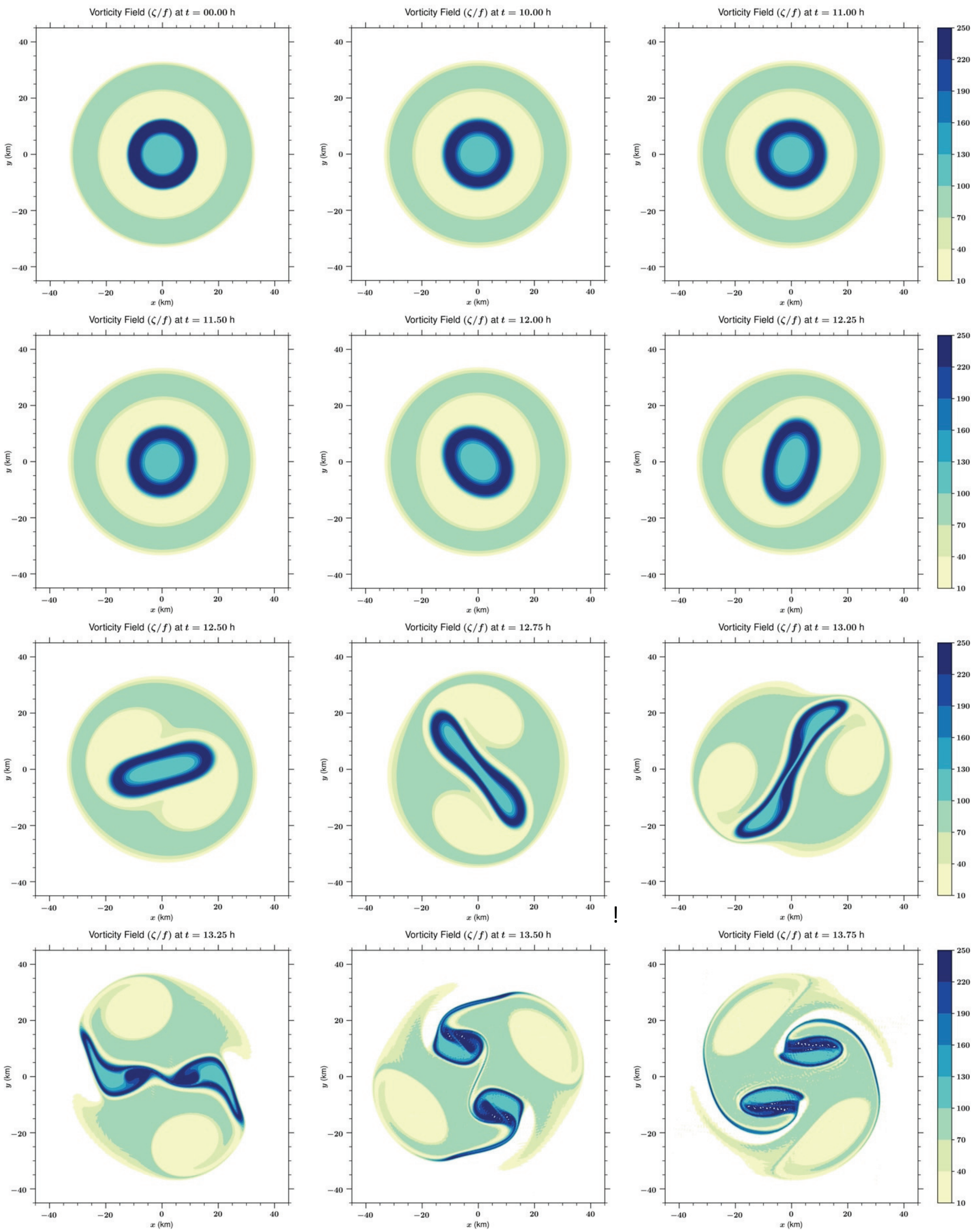

Figure S1.1. Evolution of the vorticity field in the pseudo-spectral nondivergent barotropic model initialized with the smoothed version (30) of Vortex A, using the parameters $(r_1, r_2, r_3, r_4) = (7.5, 12.5, 22.5, 32.5)$ km and $d = 1$ km. The panels are for $t = 0$, 10–13.75 h from the top left to the bottom right.

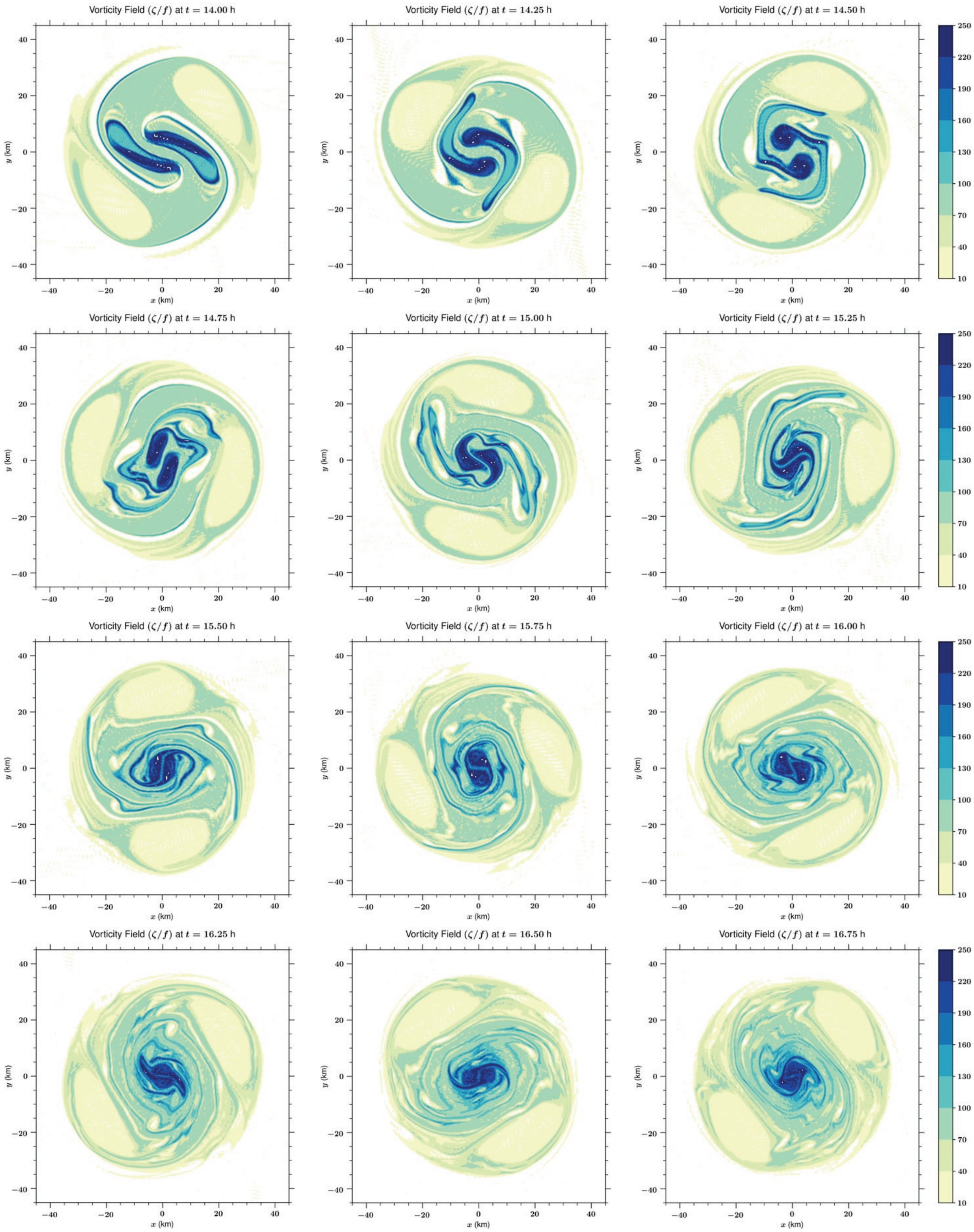

Figure S1.2. Same as Figure S1.1, but with panels for $t = 14$–16.75 h from the top left to the bottom right.

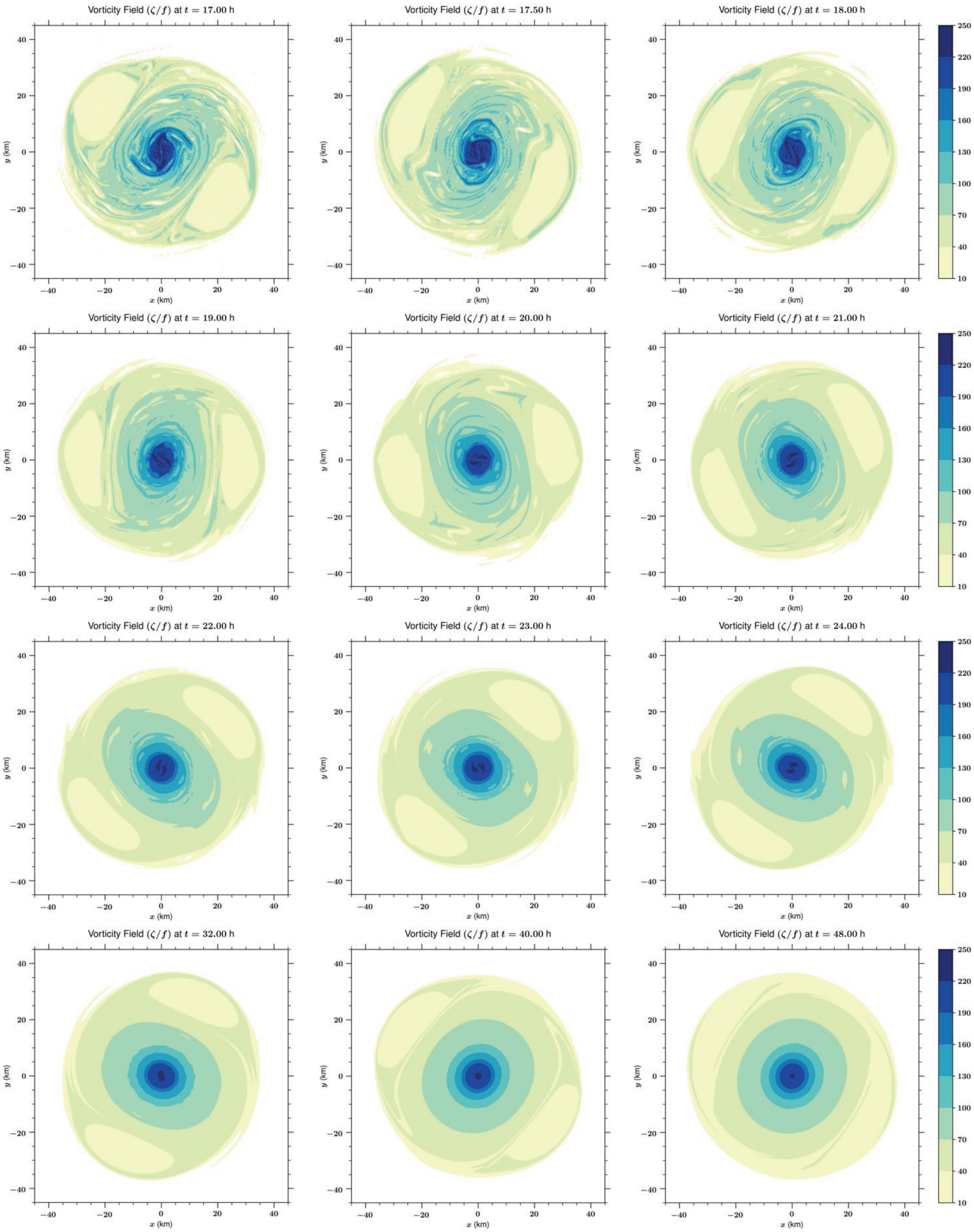

Figure S1.3. Same as Figure S1.1, but with panels for $t = 17$–48 h from the top left to the bottom right.