physical sciences forum

MDPI

Proceeding Paper

Muons: A Gateway to New Physics †

Jonathan Kriewald [1,2]

[1] Jožef Stefan Institut, Jamova Cesta 39, P.O. Box 3000, 1001 Ljubljana, Slovenia; jonathan.kriewald@ijs.si
[2] Laboratoire de Physique de Clermont (UMR 6533), CNRS/IN2P3, Universitè Clermont Auvergne, 4 Avenue Blaise Pascal, CEDEX, 63178 Aubière, France

† Presented at the 23rd International Workshop on Neutrinos from Accelerators, Salt Lake City, UT, USA, 30–31 July 2022.

Abstract: The discovery of neutrino oscillations is the first laboratory evidence of New Physics beyond the Standard Model. Oscillating neutrinos necessarily imply that neutrinos are massive and that (neutral) lepton flavour is violated. However, a signal of charged lepton flavour violation (cLFV) has so far eluded experimental discovery. In this proceeding, we review some phenomenological implications of the current experimental bounds (and future sensitivities) on observables related to muons, with particular attention to charged lepton flavour violating processes. In connection to neutrino masses, we also highlight some phenomenological implications of leptonic CP violation on cLFV observables.

Keywords: flavour physics; neutrino physics; muons; charged leptons; lepton flavour violation

Citation: Kriewald, J. Muons: A Gateway to New Physics. *Phys. Sci. Forum* **2023**, *8*, 28. https://doi.org/10.3390/psf2023008028

Academic Editor: Yue Zhao

Published: 1 August 2023

1. Introduction

The Standard Model of Particle Physics (SM) provides an extraordinarily successful and yet simple description of nature at its smallest scales. Despite its exceptional success, it is now firmly established that the SM cannot account for a certain number of observations, and one must thus envisage extensions of the model capable of accommodating experimental data. Moreover, a strong theoretical interest also fuels the study of "New Physics beyond the SM (BSM)", as the latter might provide a solution for some of the theoretical puzzles of the SM.

In its original formulation, the SM features strictly massless neutrinos, which implies the conservation of the total lepton number as well as individual lepton flavour [1,2]. Furthermore, in stark contrast to the SM quark sector, the SM lepton sector does not have an intrinsic source of CP violation such that electric dipole moments (EDMs) of charged leptons are immensely suppressed (generated at the 4-loop order) [3].

Up to the present moment, neutrino oscillations remain the only confirmed evidence for New Physics observed in a laboratory, implying that neutrinos have (tiny) masses and that neutral lepton flavour is violated in nature. Oscillating and massive neutrinos open the door to lepton flavour violation and new sources of CP violation.

In order to (indirectly) search for the New Physics that can potentially address the SM's observational and theoretical shortcomings, a useful approach is to test the (accidental) symmetries of the SM, which might not be present in New Physics models. In particular, lepton flavour conservation and lepton flavour universality (LFU) are broken by the presence of massive neutrinos. Thus, observables involving charged leptons, in particular, muons, offer uniquely versatile probes of various BSM constructions.

2. Muon Observables

In this section, we briefly review the current status and future prospects of several muon-related observables well-suited to indirectly searching for New Physics (NP). In particular, we focus on the muon anomalous magnetic moment, lepton flavour universality violation (LFUV), and charged lepton flavour violation (cLFV).

2.1. Lepton Moments: $(g-2)_\ell$

The anomalous magnetic moment of a charged lepton ℓ ($a_\ell \equiv (g_\ell - 2)/2$) allows probing numerous aspects of the SM and is also instrumental in determining some of its fundamental quantities.

Concerning muons, and following the release of the FNAL results [4], the experimental average and the latter SM prediction (following "Muon $g-2$ Theory Initiative [5]") lead to a tension between theory and observation, $\Delta a_\mu \equiv a_\mu^{\rm SM} - a_\mu^{\rm exp} = 251\,(59) \times 10^{-11}$, corresponding to a significance of $\sim 4.2\,\sigma$. Should this be confirmed, the need for an NP capable of accounting for such a sizeable discrepancy is manifest. Note that recent lattice QCD calculations [6] of the leading-order hadronic vacuum polarisation contribution might suggest a much lower significance of the anomaly, around $\sim 1.5\sigma$.

Depending on the value of α_e that is used for the computation of its SM prediction, the anomalous magnetic moment of the electron has also been at the origin of possible new tensions: extracting α_e from measurements using Cs atoms leads to $\Delta a_e^{\rm Cs} = -0.88\,(0.36) \times 10^{-12}$, corresponding to a deviation of $\sim -2.5\,\sigma$ [7,8]. Using a more recent estimation of α_e, which was obtained relying on measurements using Rubidium atoms (implying an overall deviation above the $5\,\sigma$ level for α_e), suggests a milder discrepancy between observation and theory prediction, $\Delta a_e^{\rm Rb} = 0.48\,(0.30) \times 10^{-12}$ corresponding to $\sim 1.7\,\sigma$ [9].

In addition to the (possible) deviations from the SM expectation themselves, it is interesting to consider the simultaneous potential impact of $\Delta a_e^{\rm Cs}$ and Δa_μ. On the one hand, the deviations are of opposite sign, while on the other hand, the ratio $\Delta a_\mu / \Delta a_e$ does not exhibit the naïve scaling $\sim m_\mu^2/m_e^2$, which would usually be expected from the magnetic dipole operator, due to a mass insertion of the SM lepton that is responsible for the required chirality flip. Thus, it is quite challenging to find a common explanation of both tensions in a New Physics model, leading to a departure from a minimal flavour violation hypothesis or necessitating non-minimal new field content (coupled to charged leptons) of SM extensions. Furthermore, we notice that this pattern in $\Delta a_e^{\rm Cs}$ and Δa_μ can also be a hint of the LFU violating New Physics.

2.2. Lepton Flavour Universality

In the Standard Model, charged leptons are only distinguishable due to their masses. In particular, all electroweak couplings to gauge bosons are blind to lepton flavour, leading to an accidental symmetry called lepton flavour universality (LFU), whose validity has been determined to a very high accuracy for instance in $Z \to \ell^+\ell^-$ and $W^\pm \to \ell^\pm \nu$ ($\ell = e, \mu, \tau$) decays [10].

Lepton flavour universality can also be tested in charged and neutral (semi-)leptonic meson decays, such as kaon and pion decays. In order to test the LFU, one constructs ratios of the helicity suppressed widths $R_K^{e\mu} \equiv \frac{\Gamma(K\to e\bar\nu)}{\Gamma(K\to\mu\bar\nu)}$, with $R_K^{e\mu\,\rm SM} = (2.477 \pm 0.001) \times 10^{-5}$ [11] and $R_K^{e\mu\,\rm exp} = (2.488 \pm 0.009) \times 10^{-5}$ [10], thus also being highly consistent with the SM prediction. Equivalent ratios can be constructed for pion decays, with $R_\pi^{e\mu\,\rm SM} = (1.2354 \pm 0.0002) \times 10^{-4}$ [11] and $R_\pi^{e\mu\,\rm exp} = (1.230 \pm 0.004) \times 10^{-4}$ [10], which also confirm lepton universality as predicted by the SM.

However, during the last decade, hints on the violation of the LFU in $b \to c\ell\nu$ and $b \to s\ell\ell$ decays have begun to emerge (and to fade away), in mounting tension with respect to the SM expectations. In particular, measurements of the "theoretically clean" ratios of the branching ratios $R_{D^{(*)}} = {\rm BR}(B \to D^{(*)}\tau\nu)/{\rm BR}(B \to D^{(*)}\ell\nu)$ [12] and $R_{K^{(*)}} = {\rm BR}(B \to K^{(*)}\mu\mu)/{\rm BR}(B \to K^{(*)}ee)$ [13,14] deviate around $2-3\,\sigma$ from their theoretical predictions, which are expected to be unity in the SM, up to phase space suppression. However, a recent update to the measurement of R_K and R_{K^*} of the LHCb collaboration [15] was consistent with the SM prediction. A hint on the LFUV, as currently present in the $b \to c\ell\nu$ system, can also be suggestive of lepton flavour violating New Physics [16].

2.3. Muon cLFV

Muons are possibly the best laboratory to look for cLFV, since they can be abundantly produced and have a comparatively long lifetime. Furthermore, due to their low mass, the number of kinematically allowed decay channels, flavour violating or not, is relatively small, and the final states can be studied with great precision. Very high intensity muon beams are possible (obtained at meson factories and proton accelerators), allowing for a great variety of muon dedicated experiments with extremely high sensitivities. In view of this, it comes as no surprise that the best available experimental sensitivities and, consequently, the best available bounds on cLFV processes, arise from rare muon processes.

The most minimal SM extension (adding three right-handed neutrinos ad hoc, in full analogy to the charged fermions [17]) that accommodates neutrino oscillation data, would in principle allow for cLFV transitions. However, due to the unitarity of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix and to the tiny differences of the neutrino masses, there is a strong GIM cancellation taking place, so that the expected rates are vanishingly small. For instance, the prediction for the rate of $\mu \to e\gamma$ in this framework, using the current experimental constraints on neutrino mixing, is approximately given by [18,19]

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3\alpha_e}{32\pi}\left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\frac{m^2_{\nu_i}}{M^2_W}\right|^2 \simeq \mathcal{O}(10^{-55})\,, \tag{1}$$

clearly lying beyond the reach of any experimental sensitivity. Similar (extremely small) values are found for processes such as $\mu \to eee$ decays and the analogous lepton flavour violating τ decays. The observation of such cLFV signals would thus imply that more involved BSM extensions are needed in order to simultaneously explain the origin of neutrino masses and to interpret a possible cLFV signal. Any observation of cLFV would imply new degrees of freedom: the SM must be nontrivially extended.

In addition to the radiative and three-body decays ($\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$), several facilities are dedicated to studying muonic atoms. Muonic atoms are formed when a muon is "stopped" in some target material (N), usually very pure elements. The bound muon decays via interactions with the target nucleus, either exchanging a virtual photon or, in the presence of New Physics, undergoing some non-electromagnetic interaction. In the SM, there are two possible outcomes: either the muon decays in orbit (DIO) into an electron and two neutrinos, or it is captured by the target nucleus via inverse β decay. In the presence of New Physics, the exotic process of neutrinoless muon capture can occur in which the electron is produced with sufficient kinetic energy to escape the Coulomb potential of the target nucleus, which can be left in the ground state or in an excited one. Usually dominating, and from an experimental point of view the most advantageous, is the first case, called "coherent capture". This process is usually referred to as "$\mu - e$ conversion", and the associated observable is defined as

$$\mathrm{CR}(\mu - e, \mathrm{N}) = \frac{\Gamma(\mu^- + \mathrm{N} \to e^- + \mathrm{N})}{\Gamma(\mu^- + \mathrm{N} \to \text{all captures})}\,, \tag{2}$$

which, from a theoretical point of view, has the additional advantage that most of the nuclear form factors cancel out, and only the overlap integrals between the nuclear and leptonic wave function remain to be computed [20].

In the presence of lepton number violating interactions, another neutrinoless $\mu - e$ conversion can take place, given by

$$\mu^- \to (A, Z) \to e^+(A, Z-2)^{(*)}\,, \tag{3}$$

in which the the final state nucleus can be in its ground state or an excited one. Here, contrary to the $\mu^- - e^-$ conversion, no coherent enhancement is possible since the final and initial state nuclei are necessarily different from each other. Due to its LNV nature, this process is closely related to neutrinoless double-beta decay. However, from the theoretical perspective, there is a caveat; all but one of the nuclear form factors are presently unknown [21–23].

Another cLFV process in muonic atoms was proposed in [24]. It consists of a bound 1s muon and a bound 1s electron converting into a pair of electrons and has been identified as potentially complementary to other cLFV muon processes:

$$\mu^- e^- \to e^- e^- \,. \tag{4}$$

As pointed out in [24], it offers several experimental advantages. On the one hand, the experimental signal consists of two (almost) back-to-back emitted electrons with the same energy. On the other hand, this process is enhanced by the Coulomb potential of the nucleus with respect to other observables in muonic atoms. So far, this process has not been experimentally investigated, but it would potentially offer complementary information of the flavour structure of lepton flavour violating NP.

Further interesting observables concern Muonium (Mu). Muonium is a Coulomb bound state consisting of an electron and an anti-muon ($e^-\mu^+$), which is formed when a μ^+ slows down inside matter and captures an electron. Being free of hadronic uncertainties, this hydrogen-like bound state is well described by electroweak interactions and is used to study the fundamental constants of the SM or to search for deviations from the SM induced by the presence of possible New Physics interactions.

Concerning cLFV transitions, one can study the spontaneous conversion of Muonium into anti-Muonium ($\overline{\mathrm{Mu}} = e^+\mu^-$) and the cLFV decay of Muonium, $\mathrm{Mu} \to e^+e^-$. An observation of these would again be a clear signal of New Physics.

Many experiments have searched for signals of cLFV in the decays of an extensive array of neutral and charged mesons. These processes probe $q \to q^{(\prime)}\ell_\alpha\ell_\beta$ contact interactions, possibly accompanied by another final state meson. The most stringent bounds have been obtained for K_L decays, but the results for heavy meson decays have nevertheless reached an impressive level. Similar $q \to q^{(\prime)}\ell_\alpha\ell_\beta$ contact interactions can also be probed with measurements of high-p_T di-lepton tails in pp collisions at hadron colliders, with charged lepton flavour violating the final states (see, for instance, [25]).

At higher energies, and in the presence of cLFV New Physics, Z- and Higgs bosons can also undergo cLFV decays. Furthermore, cLFV transitions of charged leptons are often (depending on the underlying model) at least partly mediated via cLFV Z-penguin (also Higgs-penguin) diagrams, so that studying cLFV Z and Higgs decays offer important complementary information, which might help to disentangle New Physics scenarios.

Currently, there is a vast worldwide array of dedicated experiments and searches, at different energy scales, aiming to discover cLFV transitions. In Table 1, we list the current experimental bounds and future sensitivities for some of the muon observables.

Table 1. Current experimental bounds and future sensitivities on some muon cLFV observables. All limits are given at 90% C.L.

Observable	Current Bound	Future Sensitivity
$\mathrm{BR}(\mu \to e\gamma)$	$<4.2 \times 10^{-13}$ (MEG [26])	6×10^{-14} (MEG II [27])
$\mathrm{BR}(\mu \to 3e)$	$<1.0 \times 10^{-12}$ (SINDRUM [28])	$10^{-15(-16)}$ (Mu3e [29])
$\mathrm{CR}(\mu - e, \mathrm{N})$	$<7 \times 10^{-13}$ (Au, SINDRUM [30])	10^{-14} (SiC, DeeMe [31]) 2.6×10^{-17} (Al, COMET [32,33]) 8×10^{-17} (Al, Mu2e [34])
$\mathrm{BR}(Z \to e^\pm\mu^\mp)$	$<4.2 \times 10^{-7}$ (ATLAS [35])	$\mathcal{O}(10^{-10})$ (FCC-ee [36])
$\mathrm{BR}(h \to e^\pm\mu^\mp)$	$<6.1 \times 10^{-5}$ [10]	1.2×10^{-5} (FCC-ee [36])
$\mathrm{BR}(K_L \to \mu^\pm e^\mp)$	$<4.7 \times 10^{-12}$ [10]	—

3. The Probing Power of Muon cLFV

The non-observation of cLFV signals and the implied experimental upper bounds on the associated processes consequently lead to tight constraints on the parameters of New Physics models that could in principle predict sizeable rates for cLFV transitions. From a model-independent perspective, one can argue that the inherent scale of New Physics that can be probed with current and future cLFV dedicated experiments is up to thousands of TeV, far beyond the direct reach of current and future colliders [37]. An overview of this is shown in Figure 1, where one has the inherent New Physics scales to be indirectly probed by several flavour observables.

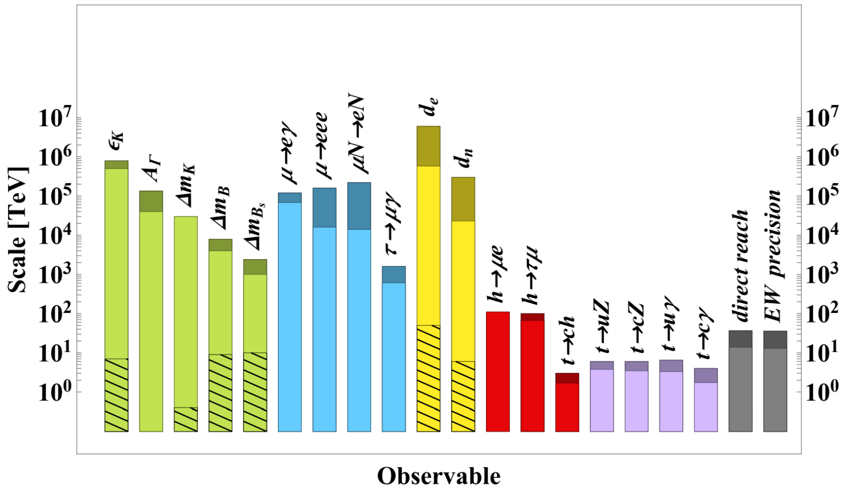

Figure 1. New Physics scales to be indirectly probed by the indicated observables. The darkened areas are the "naïve" New Physics scales, by assuming the Wilson coefficients of order one, the coloured bars indicate the inherent New Physics scales assuming weak interaction strengths, and the hatched areas account for loop suppression due to higher-order effects. Figure taken from ref. [37].

3.1. Correlations Matter

As extensively argued, the observation of one (or several) cLFV processes would be a clear unambiguous sign of New Physics. However, it is important to stress that although neutrino oscillations imply that lepton flavour is violated in nature, a possible observation of charged lepton flavour violating processes is not necessarily associated with neutrino oscillation phenomena; the cLFV can emerge as an independent process, without any connection to the mechanism of neutrino mass generation.

In order to disentangle the origin of the cLFV and to constrain the flavour structure of the New Physics responsible for it, it is often useful to study the correlations between different cLFV transitions. While for radiative decays, the dipole operator at its origin is necessarily realised at loop level, $\mu \to 3e$ decays and neutrinoless $\mu - e$ conversion in nuclei can stem from both higher-order (photon-, Z- or Higgs-mediated diagrams and boxes) or even from tree-level processes. Depending on the "nature" of the New Physics mediator(s) (scalar, vector, fermion, ...), certain operators might be enhanced with respect to others, while depending on the flavour structure of the couplings, certain flavour transitions can be enhanced. On the one hand, the synergy of identical transitions between different flavours (e.g., the comparison of $\tau \to \mu\gamma$ with $\mu \to e\gamma$ decays) offers insight into the flavour structure of common cLFV interactions; on the other hand, the comparison of different processes requiring similar flavour violation (e.g., $\mu \to e\gamma$, $\mu \to 3e$, and $\mu - e$ conversion) could shed light on the nature of the new mediator.

To illustrate this, we focus on neutrino mass models. Several mechanisms of neutrino mass generation are known to be associated with peculiar cLFV signatures, which have been extensively investigated as powerful tools to disentangle (and further give insight into) certain mechanisms of neutrino mass generation. This is particularly true for the different seesaw constructions. While in type I seesaw models, for propagator masses aroung the

TeV-scale, one finds $\mathrm{BR}(\mu \to e\gamma)/\mathrm{BR}(\mu \to 3e) \sim 5-10$, in type III seesaw constructions, this ratio is $\sim$4 orders of magnitude smaller, around $\mathrm{BR}(\mu \to e\gamma)/\mathrm{BR}(\mu \to 3e) \sim 10^{-3}$, since the cLFV three-body decay occurs at tree level in these types of models (see e.g., [38]). However, the presence of CP violating phases (Dirac and/or Majorana) in association with the new lepton mixings can strongly impact such predictive scenarios, as we discuss in the next section.

As a further illustrative example of the probing power of the correlations of cLFV observables, we consider scotogenic models, in which the SM is extended via inert scalar doublets and right-handed neutrinos [39]. In these models, neutrino masses are generated radiatively, and either the lightest neutral component of the inert scalar or the lightest right-handed neutrino can be considered as a dark matter candidate, often stabilised by a discrete symmetry. Measurements of several cLFV observables, especially $\mathrm{BR}(\mu \to 3e)$ and $\mathrm{BR}(\mu \to e\gamma)$, can then shed light on the nature of the dark matter candidate (either the lightest fermion or the lightest neutral scalar), as shown in [40]. Furthermore, a future measurement of the ratio of $\mu - e$ flavour violating observables could further constrain the absolute scale of neutrino masses (see ref. [40] for details).

3.2. The Role of CP Violation

The presence of leptonic CP violating (CPV) phases is known to have a strong impact on the predictions of several lepton number violating processes, such as neutrinoless double beta decays, or (semi-) leptonic meson decays [41]. Concerning cLFV observables, we performed a thorough study of the effects of Dirac and Majorana phases on numerous cLFV transitions in [42], and in the following, we summarise the most relevant results.

In our study, in order to stay as general as possible, we considered an effective "3 + 2 toy model". We made no assumption on the actual mechanism of neutrino mass generation and just explored the low-energy effects of two heavy neutral leptons (HNL) added to the SM content, called upon by many extensions of the SM. The neutral lepton spectrum thus consisted of five massive Majorana states, and the leptonic mixings were incoded in a 5×5 unitary matrix, which was parametrised via 10 rotations around the mixing angles $\theta_{\alpha j}$, with 10 physical CPV phases (six Dirac $\delta_{\alpha j}$ and four Majorana φ_j). Taking the limit of small mixing angles, the active-sterile mixings can be approximated as

$$\mathcal{U}_{\alpha(4,5)} \approx \begin{pmatrix} s_{14}e^{-i(\delta_{14}-\varphi_4)} & s_{15}e^{-i(\delta_{15}-\varphi_5)} \\ s_{24}e^{-i(\delta_{24}-\varphi_4)} & s_{25}e^{-i(\delta_{25}-\varphi_5)} \\ s_{34}e^{-i(\delta_{34}-\varphi_4)} & s_{35}e^{-i(\delta_{35}-\varphi_5)} \end{pmatrix}, \tag{5}$$

with $s_{\alpha i} = \sin\theta_{\alpha i}$. We emphasise here that the would-be PMNS matrix is no longer unitary, leading to modified charged and neutral lepton currents. This can have (potentially) significant impacts to several lepton observables, and one might also expect significant contributions to SM-forbiddden transitions.

As an illustrative example to highlight the impact of CPV phases on cLFV observables, let us consider the case of the radiative $\mu \to e\gamma$ decays, which are mediated by W bosons and involve both light and heavy neutrino contributions. The associated branching fraction (see [42] and references therein) can be written as

$$\mathrm{BR}(\mu \to e\gamma) \propto |G_\gamma^{\mu e}|^2\,, \text{with} \quad G_\gamma^{\mu e} = \sum_{i=4,5} \mathcal{U}_{ei}\,\mathcal{U}_{\mu i}^*\,G_\gamma(m_{N_i}^2/M_W^2)\,, \tag{6}$$

in which $G_\gamma(x_i)$, with $x_i = m_{N_i}^2/m_W^2$, is a dimensionless loop function (see [42]). Taking the limits $m_4 \approx m_5$ and $\sin\theta_{\alpha 4} \approx \sin\theta_{\alpha 5} \ll 1$, upon inserting the elements of $\mathcal{U}$, the form factor can be written as

$$|G_\gamma^{\mu e}|^2 \approx 4 s_{14}^2 s_{24}^2 \cos^2\left(\frac{\delta_{14}+\delta_{25}-\delta_{15}-\delta_{24}}{2}\right) G_\gamma^2(x_{4,5})\,. \tag{7}$$

The decay rate clearly depends on the Dirac phases, and in the case of $\delta_{14} + \delta_{25} - \delta_{15} - \delta_{24} = \pi$, a full cancellation occurs.

Following the initial simplified approach described above, we now proceed with a comprehensive exploration of the impact of CPV phases on cLFV observables. Our study involves thorough scans of the parameter space, encompassing the mixing angles and all phases, while taking into account all available relevant constraints. In addition to various cLFV constraints, we consider experimental results and limits on SM extensions via TeV-scale HNL. The constraints we consider include electroweak precision observables (M_W, G_F, invisible Z width, ...), lepton universality tests, (leptonic W and Z decays, ratios of leptonic meson decays, ratios of (semi)leptonic tau decays, ...), neutrinoless double beta decays, and finally perturbative unitarity constraints ($\Gamma_{N_{4,5}}/m_{4,5} \leq 1/2$); for a detailed description, and corresponding references, see [42].

In Figure 2, the left plot demonstrates the influence of the CP-violating (CPV) phases on the correlation between two observables in the $\mu - e$ sector, namely CR($\mu - e$, N) and BR($\mu \to 3e$). The results presented in the plot were obtained through a random scan conducted within a semi-constrained parameter space. Specifically, the condition imposed was $\theta_{\alpha 4} \approx \pm\theta_{\alpha 5}$. The heavy states were assumed to be degenerate, with $m_4 = m_5 = 1$ TeV. For each point in the scan, the CPV phases $\delta_{\alpha 4}$ and φ_4 were set as follows: vanishing (represented by blue points), randomly varied (indicated by orange points), or varied systematically on a grid (represented by green points). The inclusion of the latter possibility aimed to ensure that special cases involving "cancellation" effects were adequately considered. In the current regime of HNL masses, both observables, CR($\mu - e$, N) and BR($\mu \to 3e$), are primarily influenced by dominant contributions from Z-penguins. As a result, it is expected that these rates exhibit a correlation, as demonstrated by the thick blue line in the plot of CR($\mu - e$, N) vs. BR($\mu \to 3e$). However, when non-vanishing CPV phases are introduced, a loss of correlation is observed, particularly for the "special" values of the phases: $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$, represented by the green points. This loss of correlation becomes particularly pronounced under these phase values.

Given this behavior, it is crucial to emphasize that the presence of a single cLFV signal should not lead to the immediate dismissal of HNL extensions of the SM. For instance, if future collider searches strongly indicate the existence of sterile states with masses around 1 TeV, and a measurement of BR($\mu \to 3e$) yields approximately 10^{-15}, one need not expect the observation of CR($\mu - e$, Al). In the absence of CPV phases, CR($\mu - e$, Al) would be expected to be of the order of $\approx\mathcal{O}(10^{-14})$. However, in the presence of CPV phases, the expected range becomes considerably broader, with CR($\mu - e$, Al) potentially reaching as low as 10^{-18}. Therefore, the range of possible values for CR($\mu - e$, Al) expands significantly, when the CPV phases are taken into account.

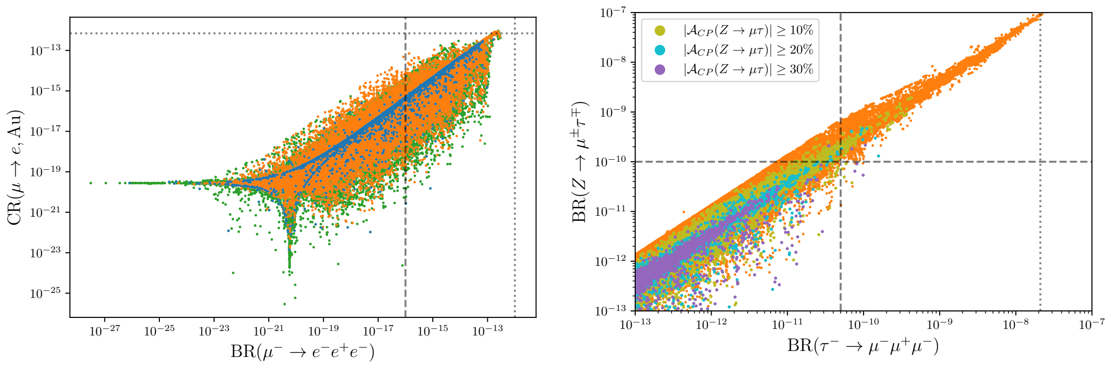

Figure 2. Left Correlation of $\mu - e$ cLFV observables, for varying values of the CPV Dirac and Majorana phases: vanishing values (blue), non-vanishing (orange), and "special grid" (green); see the description in the text. **Right** BR($Z \to \mu^\pm \tau^\mp$) vs. BR($\tau \to \mu\mu\mu$). The olive-green, cyan, and purple points denote an associated CP asymmetry $|\mathcal{A}_{CP}(Z \to \mu\tau)| \geq 10\%$, 20% and 30%, respectively, while the orange points denote CP asymmetries $\leq 10\%$. From [42,43].

Additionally, we also considered CP asymmetries in cLFV Z-boson decays [43],

$$\mathcal{A}_{CP}(Z \to \ell_\alpha \ell_\beta) \equiv \frac{\mathrm{BR}(Z \to \ell_\alpha^+ \ell_\beta^-) - \mathrm{BR}(Z \to \ell_\alpha^- \ell_\beta^+)}{\mathrm{BR}(Z \to \ell_\alpha^+ \ell_\beta^-) + \mathrm{BR}(Z \to \ell_\alpha^- \ell_\beta^+)}\,. \tag{8}$$

Sizeable CP asymmetries in Z decays turn out to be a generic feature of HNL extensions encompassing at least two heavy states. For final states composed of a tau and a light charged lepton, with decay rates potentially within future sensitivity, the CP asymmetries can be as large as 20–30% for the case of $\mathcal{A}_{CP}(Z \to \mu\tau)$, interestingly, in association with sizeable rates for $\tau \to 3\mu$ (also within future sensitivity). This is illustrated in the right plot of Figure 2.

If, on the one hand, it is clear that CP violating phases should be taken into account in general upon comparison between the prediction and the observation in the context of cLFV HNL extensions of the SM, $\mathcal{A}_{CP}(Z \to \ell_\alpha \ell_\beta)$ might hold the key to clearly establishing the presence of leptonic CP violation [43]; in turn, this might have strong implications regarding leptogenesis (relying on complete models including heavy sterile states). Thus, whenever possible, data from individual channels (i.e., $\ell_\alpha^+ \ell_\beta^-$ and $\ell_\alpha^- \ell_\beta^+$) should be separately analysed and compared.

In summary, the presence of leptonic CPV phases (both Dirac and Majorana) should be consistently included in the phenomenological analysis of the prospects of the HNL extensions of the SM in what concerns the cLFV.

4. Conclusions

Being the first laboratory evidence for New Physics, neutrino oscillations urgently call for extensions of the SM, in order to offer a viable mechanism of neutrino mass generation. Interestingly, due to offering a new source of CP violation and calling upon weakly interacting states, New Physics extensions aiming at providing an explanation for neutrino masses can often be connected to the baryon asymmetry of the universe and the dark matter problem. Consequently, the interest in high-intensity searches dedicated to the lepton sector has steadily increased.

The violation of accidental (lepton) symmetries of the SM, such as charged lepton flavour conservation and lepton flavour universality (both violated due to the presence of neutrino masses) opens many possible paths to search for New Physics. While massive neutrinos consist of only one possible source of lepton flavour and lepton flavour universality violation, indirect signals indicating the breaking of these symmetries in synergy with possible other indirect signals of New Physics will provide crucial guidelines for both experimental direct searches and theoretical efforts to describe New Physics interactions. With an unprecedented future of experimental prospects, the lepton sector, in particular, cLFV searches in the muon sector, is emerging as a powerful laboratory to search for New Physics.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Acknowledgments: The author wishes to thank the organising committee of NuFact 2022 for the kind invitation to present this review. The author acknowledges support from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 860881 (HIDDeν network) and from the IN2P3 (CNRS) Master Project, "Flavour probes: lepton sector and beyond" (16-PH-169).

Conflicts of Interest: The author declares no conflict of interest.

References

1. Bilenky, S.M.; Petcov, S.T. Massive Neutrinos and Neutrino Oscillations. *Rev. Mod. Phys.* **1987**, *59*, 671; Erratum in *Rev. Mod. Phys.* **1989**, *61*, 169; Erratum in *Rev. Mod. Phys.* **1988**, *60*, 575–575. [CrossRef]
2. Mohapatra, R.N.; Antusch, S.; Babu, K.S.; Barenboim, G.; Chen, M.C.; Davidson, S.; de Gouvêa, A.; de Holanda, P.; Dutta, B.; Grossman, Y.; et al. Theory of neutrinos: A White paper. *Rept. Prog. Phys.* **2007**, *70*, 1757–1867. [CrossRef]
3. Pospelov, M.; Ritz, A. Electric dipole moments as probes of new physics. *Ann. Phys.* **2005**, *318*, 119–169. [CrossRef]
4. Abi, B.; Albahri, T.; Al-Kilani, S.; Allspach, D.; Alonzi, L.P.; Anastasi, A.; Anisenkov, A.; Azfar, F.; Badgley, K.; Baeßler, S.; et al. Measurement of the positive muon anomalous magnetic moment to 0.46 ppm. *Phys. Rev. Lett.* **2021**, *126*, 141801. [CrossRef]
5. Aoyama, T.; Asmussen, N.; Benayoun, M.; Bijnens, J.; Blum, T.; Bruno, M.; Caprini, I.; Calame, C.M.C.; Cè, M.; Colangelo, G.; et al. The anomalous magnetic moment of the muon in the Standard Model. *Phys. Rep.* **2020**, *887*, 1–166. [CrossRef]
6. Borsanyi, S.; Fodor, Z.; Guenther, J.N.; Hoelbling, C.; Katz, S.D.; Lellouch, L.; Lippert, T.; Miura, K.; Parato, L.; Szabo, K.K.; et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature* **2021**, *593*, 51–55. [CrossRef]
7. Parker, R.H.; Yu, C.; Zhong, W.; Estey, B.; Müller, H. Measurement of the fine-structure constant as a test of the Standard Model. *Science* **2018**, *360*, 191–195. [CrossRef]
8. Yu, C.; Zhong, W.; Estey, B.; Kwan, J.; Parker, R.H.; Müller, H. Atom-interferometry measurement of the fine structure constant. *Ann. Phys.* **2019**, *531*, 1800346. [CrossRef]
9. Morel, L.; Yao, Z.; Cladé, P.; Guellati-Khélifa, S. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. *Nature* **2020**, *588*, 61–65. [CrossRef]
10. Zyla, P.A.; Particle Data Group. Review of Particle Physics. *Prog. Theor. Exp. Phys.* **2020**, *2020*, 083C01.
11. Cirigliano, V.; Rosell, I. Two-loop effective theory analysis of pi (K)—> e anti-nu/e [gamma] branching ratios. *Phys. Rev. Lett.* **2007**, *99*, 231801. [CrossRef]
12. Caria, G.; Urquijo, P.; Adachi, I.; Aihara, H.; Al Said, S.; Asner, D.M.; Atmacan, H.; Aushev, T.; Babu, V.; Badhrees, I.; et al. Measurement of R (D) and R (D*) with a Semileptonic Tagging Method. *Phys. Rev. Lett.* **2020**, *124*, 161803. [CrossRef]
13. Aaij, R.; Beteta, C.A.; Adeva, B.; Adinolfi, M.; Aidala, C.A.; Ajaltouni, Z.; Akar, S.; Albicocco, P.; Albrecht, J.; Alessio, F.; et al. Search for lepton-universality violation in $B^+ \to K^+\ell^+\ell^-$ decays. *Phys. Rev. Lett.* **2019**, *122*, 191801. [CrossRef]
14. LHCb Collaboration. Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays. *arXiv* **2017**, arXiv:1705.05802.
15. LHCb Collaboration. Measurement of lepton universality parameters in $B^+ \to K^+\ell^+\ell^-$ and $B^0 \to K^{*0}\ell^+\ell^-$ decays. *arXiv* **2022**, arXiv:2212.09153.
16. Glashow, S.L.; Guadagnoli, D.; Lane, K. Lepton flavor violation in B decays? *Phys. Rev. Lett.* **2015**, *114*, 091801. [CrossRef] [PubMed]
17. Bilenky, S.M.; Pontecorvo, B. Quark-lepton analogy and neutrino oscillations. *Phys. Lett. B* **1976**, *61*, 248. [CrossRef]
18. Petcov, S.T. The Processes $\mu \to e + \gamma, \mu \to e + \bar{e}, \nu' \to \nu + \gamma$ in the Weinberg-Salam Model with Neutrino Mixing. *Sov. J. Nucl. Phys.* **1977**, *25*, 340; Erratum in *Sov. J. Nucl. Phys.* **1977**, *25*, 698; Erratum in *Sov. J. Nucl. Phys.* **1977**, *25*, 1336.
19. Bilenky, S.M.; Petcov, S.T.; Pontecorvo, B. Lepton Mixing, mu –> e + gamma Decay and Neutrino Oscillations. *Phys. Lett. B* **1977**, *67*, 309. [CrossRef]
20. Kitano, R.; Koike, M.; Okada, Y. Detailed calculation of lepton flavor violating muon electron conversion rate for various nuclei. *Phys. Rev. D* **2002**, *66*, 096002; Erratum in *Phys. Rev. D* **2007**, *76*, 059902. [CrossRef]
21. Domin, P.; Kovalenko, S.; Faessler, A.; Simkovic, F. Nuclear (mu-, e+) conversion mediated by Majorana neutrinos. *Phys. Rev. C* **2004**, *70*, 065501. [CrossRef]
22. Geib, T.; Merle, A.; Zuber, K. $\mu^- - e^+$ conversion in upcoming LFV experiments. *Phys. Lett. B* **2017**, *764*, 157–162. [CrossRef]
23. Geib, T.; Merle, A. $\boldsymbol{\mu^- - e^+}$ Conversion from Short-Range Operators. *Phys. Rev. D* **2017**, *95*, 055009. [CrossRef]
24. Koike, M.; Kuno, Y.; Sato, J.; Yamanaka, M. A new idea to search for charged lepton flavor violation using a muonic atom. *Phys. Rev. Lett.* **2010**, *105*, 121601. [CrossRef]
25. Allwicher, L.; Faroughy, D.A.; Jaffredo, F.; Sumensari, O.; Wilsch, F. Drell-Yan tails beyond the Standard Model. *J. High Energy Phys.* **2023**, *3*, 64. [CrossRef]
26. MEG Collaboration. Search for the lepton flavour violating decay $\mu^+ \to \mathrm{e}^+\gamma$ with the full dataset of the MEG experiment. *Eur. Phys. J. C* **2016**, *76*, 434. [CrossRef]
27. MEG II Collaboration. The design of the MEG II experiment. *Eur. Phys. J. C* **2018**, *78*, 380. [CrossRef]
28. Bellgardt, U.; Otter, G.; Eichler, R.; Felawka, L.; Niebuhr, C.; Walter, H.K.; Bertl, W.; Lordong, N.; Martino, J.; Egli, S.; et al. Search for the Decay mu+ —> e+ e+ e-. *Nucl. Phys. B* **1988**, *299*, 1–6. [CrossRef]
29. Blondel, A.; Bravar, A.; Pohl, M.; Bachmann, S.; Berger, N.; Kiehn, M.; Schoning, A.; Wiedner, D.; Windelband, B.; Eckert, P.; et al. Research Proposal for an Experiment to Search for the Decay μ-> eee. *arXiv* **2013**, arXiv:1301.6113.
30. Bertl, W.; Engfer, R.; Hermes, E.A.; Kurz, G.; Kozlowski, T.; Kuth, J.; Otter, G.; Rosenbaum, F.; Ryskulov, N.M.; Van Der Schaaf, A.; et al. A search for μ-e conversion in muonic gold. *Eur. Phys. J. C* **2006**, *47*, 337–346 [CrossRef]
31. Nguyen, T.M. Search for μ − e conversion with DeeMe experiment at J-PARC MLF. In Proceedings of the Flavor Physics & CP Violation 2015, Nagoya, Japan, 25–29 May 2015. [CrossRef]
32. Krikler, B.E. An overview of the COMET experiment and its recent progress. *arXiv* **2015**, arXiv:1512.08564.
33. COMET Collaboration; Abramishvili, R.; Adamov, G.; Akhmetshin, R.R.; Allin, A.; Angélique, J.C.; Anishchik, V.; Aoki, M.; Aznabayev, D.; Bagaturia, I.; et al. COMET Phase-I technical design report. *Prog. Theor. Exp. Phys.* **2020**, *2020*, 033C01. [CrossRef]

34. Bartoszek, L. Mu2e technical design report. *arXiv* **2015**, arXiv:1501.05241.
35. Aad, G.; Abbott, B.; Abdallah, J.; Khalek, S.A.; Aben, R.; Abi, B.; Abolins, M.; AbouZeid, O.S.; Abramowicz, H.; Abreu, H.; et al. Search for the lepton flavor violating decay $Z \to e\mu$ in pp collisions at $\sqrt{s}$ TeV with the ATLAS detector. *Phys. Rev. D* **2014**, *90*, 072010. [CrossRef]
36. Abada, A.; Abbrescia, M.; AbdusSalam, S.S.; Abdyukhanov, I.; Abelleira Fernandez, J.; Abramov, A.; Aburaia, M.; Acar, A.O.; Adzic, P.R.; Agrawal, P.; et al. FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1. *Eur. Phys. J. C* **2019**, *79*, 474. [CrossRef]
37. Ellis, R.K.; Heinemann, B.; de Blas, J.; Cepeda, M.; Grojean, C.; Maltoni, F.; Nisati, A.; Petit, E.; Rattazzi, R.; Verkerke, W.; et al. Physics Briefing Book: Input for the European Strategy for Particle Physics Update 2020. *arXiv* **2019**, arXiv:1910.11775.
38. Hambye, T. CLFV and the origin of neutrino masses. *Nucl. Phys. B Proc. Suppl.* **2014**, *248–250*, 13–19. [CrossRef]
39. Ma, E. Verifiable radiative seesaw mechanism of neutrino mass and dark matter. *Phys. Rev. D* **2006**, *73*, 077301. [CrossRef]
40. Toma, T.; Vicente, A. Lepton flavor violation in the scotogenic model. *J. High Energy Phys.* **2014**, *1*, 160. [CrossRef]
41. Abada, A.; Hati, C.; Marcano, X.; Teixeira, A.M. Interference effects in LNV and LFV semileptonic decays: The Majorana hypothesis. *J. High Energy Phys.* **2019**, *9*, 17. [CrossRef]
42. Abada, A.; Kriewald, J.; Teixeira, A.M. On the role of leptonic CPV phases in cLFV observables. *Eur. Phys. J. C* **2021**, *81*, 1016. [CrossRef]
43. Abada, A.; Kriewald, J.; Pinsard, E.; Rosauro-Alcaraz, S.; Teixeira, A.M. LFV Higgs and Z-boson decays: Leptonic CPV phases and CP asymmetries. *Eur. Phys. J. C* **2023**, *83*, 494. [CrossRef]