



Dark Matter in the Milky Way as the F-Type of Vacuum Polarization [†]

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Abstract: Dark matter in the Milky Way is explained by the F-type of vacuum polarization, which could represent dark radiation. A nonsingular solution for dark radiation exists in the presence of eicheon (i.e., black hole in old terminology) in the galaxy's center. The model is spherically symmetric, but an approximate surface density of a baryonic galaxy disk is taken into account by smearing the disk over a sphere.

Keywords: dark matter; vacuum polarization; rotational curve; galaxy nuclei



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Observation of the stellar orbits around the center of the Milky Way [1] is considered as evidence of an extremely compact astrophysical object existence with a radius of an order of a Schwarzschild one. Conversely, an exact Schwarzschild solution of the general relativity (GR) equations exists [2]. A principal question is whether the Schwarzschild solution describes reality. This question is also related to the need for dark matter to explain the galactic rotational curves [3]. Modification of GR explaining rotational curves without dark matter as the MOND was suggested [4]. However, could we go without extraordinary physics but take vacuum polarization into account correctly [5]? The answer is no in the frame of *renormalizable* quantum field theory on a curved background. Still, this approach demands covariance of the mean value of the energy-momentum tensor over the vacuum state. This demand has no hard background because it is known that vacuum state invariant relative general transformation of coordinates does not exist. On the contrary, an argument is put forward that the preferred reference frame exists based on the conformally unimodular metric for describing vacuum polarization [6].

The conformally unimodular metric for a spherically symmetric space-time is written as

$$s^2 = a^2 \left(d\eta^2 - \tilde{\gamma}_{ij} dx^i dx^j \right) = e^{2\alpha} \left(d\eta^2 - e^{-2\lambda} (d\mathbf{x})^2 - \left(e^{4\lambda} - e^{-2\lambda} \right) (\mathbf{x} d\mathbf{x})^2 / r^2 \right), \quad (1)$$

where $r = |\mathbf{x}|$, $a = e^\alpha$, and λ are the functions of η , r . The matrix $\tilde{\gamma}_{ij}$ with the unit determinant expressed through $\lambda(\eta, r)$. The interval (1) rewritten in the spherical coordinates is [7]:

$$ds^2 = e^{2\alpha} \left(d\eta^2 - e^{4\lambda} dr^2 - e^{-2\lambda} r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right) \quad (2)$$

As it was shown [5,7], a nonsingular solution exists in this metric for the compact object of any mass (see, e.g., Figure 1a), for example, consisting of incompressible fluid. In the metric of the Schwarzschild type of

$$ds^2 = B(R) dt^2 - A(R) dR^2 - R^2 d\Omega^2. \quad (3)$$

These objects look like hollow spheres that prevent the appearance of infinite pressure. The inner R_i and the outer R_f radiuses (see Figure 1b) of this spherical shell exceed the Schwarzschild radius and the Buchdahl's bound [8] $m < 4R/9G$ is not reached.

Considering vacuum polarization for the arbitrary curved space-time background is a highly complex problem. Instead, one could consider scalar perturbations of the conformally unimodular metric:

$$ds^2 = (1 + \Phi(\eta, \mathbf{x}))^2 \left(d\eta^2 - \left(\left(1 + \frac{1}{3} \sum_{m=1}^3 \partial_m^2 F(\eta, \mathbf{x}) \right) \delta_{ij} - \partial_i \partial_j F(\eta, \mathbf{x}) \right) dx^i dx^j \right) \quad (4)$$

and calculate a spatially nonuniform energy density and pressure arising due to vacuum polarization in the *eikonal approximation* [5].

As was shown [5], this energy density and pressure of vacuum polarization corresponding to the F -type of metric perturbations (4) have the radiation equation of state. That gives a hypothetical possibility to use dark radiation in some nonlinear models. One could use it in the Volkov–Tolman–Oppenheimer (TOV) equation as a heuristic picture. For a radiation substance alone, a singular solution of a TOV equation exists, thus, having no physical meaning [9].

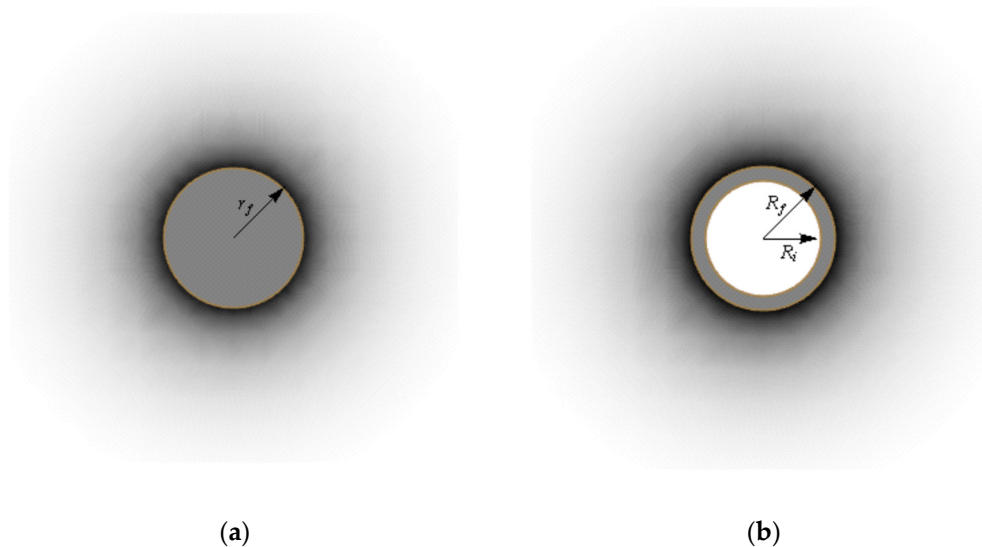


Figure 1. Nonsingular eicheon surrounded by dark radiation in conformally unimodular metric (2) has nonsingular core (a). In the Schwarzschild type metric (3), this core looks like a hollow sphere (b).

However, the situation changes cardinally in the conformally unimodular metric, in the presence of the nonsingular eicheon, which gives a possibility to set a boundary condition for radiation fluid at $r = 0$ and obtain nonsingular solution including the dark radiation. In the Schwarzschild type metric (3), the boundary condition is set at the radial coordinate of an inner shell $R = R_i$, which corresponds to the point $r = 0$ in the conformally unimodular metric. As a result, such radiation fluid models a dark matter in the Milky Way as it is shown in Figure 2a, where one could see the contribution of the eicheon at a small distance and the contribution of a dark radiation at large distances. This is spherically symmetric model, where an amount of dark radiation is adjusted to fit the observations. To take the baryonic matter into account, one could smear a baryonic galactic disk on a sphere and consider the resulting mass density as some external non-dynamical density in the TOV equations for the eicheon and dark radiation. This external density creates an additional gravitational potential.

Let us have a surface density of matter in a galactic disk:

$$\varrho = \frac{M_D}{2\pi R_D^2} e^{-R/R_D} \quad (5)$$

and write the mass dM corresponding to the radial distance dR

$$dM = \frac{M_D}{R_D^2} e^{-R/R_D} R dR = \frac{M_D}{R_D^2} e^{-R/R_D} R^2 dR. \quad (6)$$

According to (6), the smeared 3-dimensional density has the form

$$\rho = \frac{M_D}{4\pi R_D^2 R} e^{-R/R_D} \quad (7)$$

The result of the calculations for the Milky Way rotational curve is shown in Figure 2b.

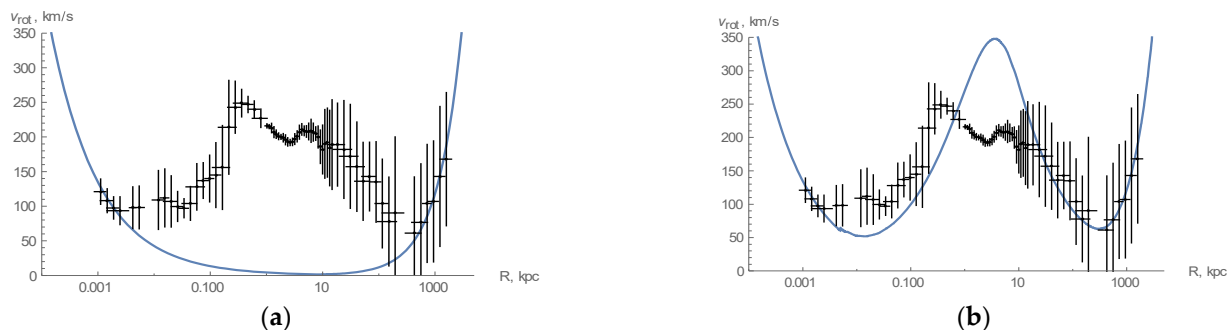


Figure 2. (a) Calculated rotational curve from Ref. [5], which includes contributions of the eicheon and dark radiation. (b) Rotational curve taking into account the baryonic matter by (5)–(7). The result of observations with the error bars are taken from Ref. [3].

As one can see, the simple model with smeared disk describes baryonic matter roughly, but the observed rotational curve has a more complicated structure. Let us remind the principles of calculation. We have considered the vacuum polarization of F-type in the conformally unimodular metric (4) and find that it has a radiation equation of state. Then, solve the TOV equation for incompressible fluid and dark radiation and obtain a nonsingular solution. To consider the baryonic matter, we smear a galactic disk and use the resulting density as some external density.

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