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On Extensions of the Starobinsky Model of Inflation †

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Abstract: We propose inflationary models that are one-parametric generalizations of the Starobinsky $R + R^2$ model. Using the conformal transformation, we obtain scalar field potentials in the Einstein frame that are one-parametric generalizations of the potential for the Starobinsky inflationary model. We restrict the form of the potentials by demanding that the corresponding function $F(R)$ is an elementary function. We obtain the inflationary parameters of the models proposed and show that the predictions of these models agree with current observational data.

Keywords: inflation; modified gravity; cosmology

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1. Introduction

Inflationary scenarios in the context of $F(R)$ gravity are being actively studied [1–18]. The historically first $F(R)$ gravity inflationary model is the purely geometric $R + R^2$ model [1], which is described by the following action:

$$S_{\text{Star.}}[g^J_{\mu\nu}] = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g^J}\left(R_J + \frac{1}{6m^2}R_J^2\right), \tag{1}$$

where the Ricci scalar R_J, the reduced Planck mass M_{Pl} and the inflaton mass m are introduced.

The Starobinsky inflationary model (1) is in good agreement with the Planck measurements of the cosmic microwave background (CMB) radiation [19,20]. The inflaton mass m is fixed by CMB measurements of the amplitude of scalar perturbations A_s. Note that the values of the scalar spectral index n_s and the tensor-to-scalar ratio r do not depend on m. This model is the simplest model of inflation that has the maximal predictive power. At the present time, the value of the tensor-to-scalar ratio r is not known, and will likely be observed in future experiments. If the observed value of the tensor-to-scalar ratio r is different from its value in the Starobinsky model, some corrections of this model will be required.

There are two pure geometric ways to generalize the Starobinsky model without adding scalar fields or other matter. One can either add string-theory-inspired terms [21–30] or construct new $F(R)$ gravity inflationary models connected to the Starobinsky model [2–7,9,11,13–15,18]. Furthermore, it is possible to combine these two methods, thereby, obtaining $f(R)$ models related to fundamental theories of gravity. For example, the supergravity models in some approximations can be considered as $F(R)$ gravity models [21–24].

The addition of the R^n term to the Starobinsky model with $n > 2$ does not allow the construction of realistic inflationary models because inflation demands fine-tuning of the initial values [2,14,15]. This discussion is based on our previous paper [15], where new one-parameter generalizations of the Starobinsky $R + R^2$ inflationary model were considered. In particular, it was shown that the adding of the $R^{3/2}$ term allows for the construction of a viable inflationary model with a tensor-to-scalar ratio r four times larger than in the original Starobinsky model.

At the same time, models with the $R^{3/2}$ term are ill-defined at $R \leqslant 0$, whereas the Starobinsky model is well-defined and has no ghost for all $R > -3m^2$. The inflationary models proposed in [18] include an $(R + R_0)^{3/2}$ term, where $R_0 > 0$; these models have useful properties with the $R^{3/2}$ term and are well-defined for some negative values of R. We also proposed a new method of generalization of the Starobinsky model based on suitable one-parametric generalizations of the corresponding scalar field potentials in the Einstein frame.

2. $F(R)$ Models and the Corresponding Scalar Potentials

The generic $F(R)$ gravity theories have the following action

$$S_F[g^J_{\mu\nu}] = \int d^4x \sqrt{-g^J} F(R_J) \tag{2}$$

with a differentiable function F.

To avoid a graviton as a ghost and scalaron (inflaton) as a tachyon, one should use the following conditions [31,32]:

$$\frac{dF}{dR_J} > 0 \quad \text{and} \quad \frac{d^2F}{dR_J^2} > 0 \tag{3}$$

which restrict the possible values of the parameters and R_J. In the Starobinsky model, the first condition in (3) is equivalent to $R_J > -3m^2$.

For any nonlinear function $F(R_J)$, action (2) can be rewritten as

$$S_J = \int d^4x \sqrt{-g^J} \left[F_{,\sigma}(R_J - \sigma) + F \right] , \tag{4}$$

where a new scalar field σ is introduced, and $F_{,\sigma} \equiv \frac{dF(\sigma)}{d\sigma}$. If $F_{,\sigma\sigma}(\sigma) \neq 0$, then, by eliminating σ via equation $R_J = \sigma$, one yields back action (2).

If $F_{,\sigma} > 0$, then the Weyl transformation of the metric $g_{\mu\nu} = \frac{2F_{,\sigma}(\sigma)}{M_{Pl}^2} g^J_{\mu\nu}$ allows one to obtain the following action in the Einstein frame [33]:

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{h(\sigma)}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_E(\sigma) \right], \tag{5}$$

where

$$h(\sigma) = \frac{3M_{Pl}^2}{2F_{,\sigma}^2} F_{,\sigma\sigma}^2 \quad \text{and} \quad V_E(\sigma) = M_{Pl}^4 \frac{F_{,\sigma}\sigma - F}{4F_{,\sigma}^2} . \tag{6}$$

It is easy to see that the following field transformation

$$\phi = \sqrt{\frac{3}{2}} M_{Pl} \ln \left[\frac{2}{M_{Pl}^2} F_{,\sigma} \right] . \tag{7}$$

gives the action S_E with the standard scalar field ϕ:

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_E(\sigma(\phi)) \right]. \tag{8}$$

It has been shown in [15] that the non-canonical dimensionless field

$$y \equiv \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right) = \frac{M_{Pl}^2}{2F_{,\sigma}} \tag{9}$$

is useful for considering generalization of the Starobinsky inflationary model as it is small during inflation.

For the Starobinsky model, the potential is

$$V_{\text{Star.}}(y) = V_0(1-y)^2\,, \quad \text{where} \quad V_0 = \frac{3}{4}m^2M_{Pl}^2\ . \tag{10}$$

If $V(y)$ is given, then the corresponding function $F(R_J)$ can be found in the parametric form (see [15] for details):

$$R_J(y) = \frac{2}{M_{Pl}^2}\left(2\frac{V}{y} - V_{,y}\right), \qquad F(y) = \frac{V}{y^2} - \frac{V_{,y}}{y}\,. \tag{11}$$

Equation (9) can be obtained as a consequence of Equation (11). The first stability condition $F_{,\sigma} > 0$ is equivalent to $y > 0$. Using Equations (9) and (11), we obtain the second stability condition in terms of y:

$$F_{,\sigma\sigma} = \frac{M_{Pl}^4}{4\left(2yV_{,y} - 2V - y^2V_{,yy}\right)} > 0. \tag{12}$$

3. One-Parametric Generalizations of $V_{\text{Star.}}(y)$ and the Corresponding $F(R)$ Models

In Ref. [15], we proposed a new method for the construction of $F(R)$ inflationary models based on the use of the potential $V(y)$.

The field y is small during slow-roll inflation; thus, the potential $V(y)$ can be written as follows,

$$V(y) = V_0\left[1 - 2y + \mathcal{O}\left(y^2\right)\right]\,, \tag{13}$$

where only the first two terms are essential for the CMB observables.

The potential

$$V(y) = V_0\left[1 - 2y + y^2\omega(y)\right] \tag{14}$$

with an arbitrary analytic function $\omega(y)$ that orders 1 does not essentially change the inflationary parameters predicted by the Starobinsky model. The Starobinsky model appears at $\omega(y) = 1$.

Equation (11) that defines $F(R_J)$ is given by

$$R_J = 3m^2\left(\frac{1}{y} - 1 - \frac{1}{2}y^2\frac{d\omega}{dy}\right)\,, \tag{15}$$

$$F = V_0\left(\frac{1}{y^2} - \omega - y\frac{d\omega}{dy}\right)\,. \tag{16}$$

If ω is an arbitrary constant, then

$$F(R_J) = F_{Star.}(R_J) - \Lambda\,, \tag{17}$$

where $\Lambda = V_0(1-\omega)$ is a cosmological constant.

A new model corresponds to

$$\omega(y) = \omega_0 + \omega_1 y\,, \tag{18}$$

where $\omega_0 \leqslant 1$ and $\omega_1 > 0$ are constants. The constant ω_1 should be positive for the potential V bounded from below. The inequality $\omega_0 \leqslant 1$ is needed for the positivity of a cosmological constant; see Equation (17).

Equation (15) leads to the depressed cubic equation

$$y^3 + \frac{2}{\omega_1}\left(1+\frac{R_J}{3m^2}\right)y - \frac{2}{\omega_1} = 0\,. \tag{19}$$

Equation (19) has a negative discriminant and, therefore, only one real root.

Equation (16) yields the following explicit $F(R_J)$ function:

$$\begin{aligned}
\frac{F}{V_0} &= \frac{1}{y^2} - \omega_0 - 2\omega_1 y \\
&= \omega_1^{2/3}\left[\left(1+\sqrt{1+\frac{8}{27\omega_1}\left(1+\frac{R_J}{3m^2}\right)^3}\right)^{1/3} + \left(1-\sqrt{1+\frac{8}{27\omega_1}\left(1+\frac{R_J}{3m^2}\right)^3}\right)^{1/3}\right]^{-2} \\
&- 2\omega_1^{2/3}\left[\left(1+\sqrt{1+\frac{8}{27\omega_1}\left(1+\frac{R_J}{3m^2}\right)^3}\right)^{1/3} + \left(1-\sqrt{1+\frac{8}{27\omega_1}\left(1+\frac{R_J}{3m^2}\right)^3}\right)^{1/3}\right] \\
&- \omega_1^{2/3}\left[\left(1+\sqrt{1+\frac{8}{27\omega_1}}\right)^{1/3} + \left(1-\sqrt{1+\frac{8}{27\omega_1}}\right)^{1/3}\right]^{-2} \\
&+ 2\omega_1^{2/3}\left[\left(1+\sqrt{1+\frac{8}{27\omega_1}}\right)^{1/3} + \left(1-\sqrt{1+\frac{8}{27\omega_1}}\right)^{1/3}\right].
\end{aligned}$$

The limit $\omega_1 \to 0$ is smooth and gives back the Starobinsky model (1).

From Equation (12), we find that

$$F_{,\sigma\sigma}(y) = \frac{M_{Pl}^2}{6m^2(1+\omega_1 y^3)}\ , \tag{20}$$

and thus the considered $F(R)$ gravity model satisfies the conditions (3) at $\omega_1 \geqslant 0$.

Our approach suggests another one-parametric deformation of the Starobinsky potential (10):

$$V(y) = V_0\left(1 - y - \zeta y^2\right)^2 \tag{21}$$

where the parameter $\zeta \geqslant 0$. This potential can be realized in supergravity [34], while the potential (10) is recovered at $\zeta = 0$. This possibility was investigated in Ref. [15] as well.

4. Conclusions

The accelerated expansion of the early universe, inflation, has been described in $F(R)$ gravity. The Starobinsky model of inflation [1], which was proposed more than 40 years ago, is in good agreement with the current observational data of the cosmic microwave background radiation [19,20]. New $F(R)$ inflationary models can be constructed as expansions of the Starobinsky model that smoothly connect to it. Such models will be in good agreement with the current observational data if the additional parameter is small enough (for the Starobinsky model, its value is equal to zero).

The main purpose of our previous paper [15] was to explore inflationary models that are one-parametric generalizations of the Starobinsky model and can be regarded as $F(R)$ models. We investigated two ways to modify the Starobinsky model. We can either initially modify the $F(R)$ function or the corresponding scalar field potential $V_E(\phi)$. When we use the second method, it is suitable to rewrite the potential in terms of the dimensionless variable y, which is small during inflation.

One of the restrictions of the inflaton scalar potential is the condition that the corresponding $F(R)$ should be an elementary function. More restrictions of the potential arise when one demands their minimal embedding into supergravity [22,24,35]. For instance, the R^3 term is excluded in supergravity, whereas $R^{3/2}$ and $(R+R_0)^{3/2}$ terms arise in certain versions of the chiral $F(\mathcal{R})$ supergravity [22]. The potential (21) is extendable in a minimal supergravity framework that requires the scalar potential to be a real function squared.

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