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Gauged (Super)Conformal Models †

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Abstract: Superconformal mechanics describes superparticle dynamics in near-horizon geometries of supersymmetric black holes. We systematically study the minimal compatible set of constraints required for a gauged superconformal symmetry. Our study uncovers classes of sigma models, which are only scale invariant in their ungauged form and become fully conformal invariant only after gauging.

Keywords: extended supersymmetry; scale and conformal symmetries; supersymmetric black holes

1. Introduction

(Super)conformal mechanics are believed to describe the radial motion of (super)particles in the near-horizon (AdS) geometry of (supersymmetric) black holes [1]. We investigate one-dimensional gauged superconformal sigma models that admit the exceptional one-parameter supergroup $D(2,1;\alpha)$ as their symmetry group. This is the most general $\mathcal{N}=4$ superconformal group in one dimension [2]. In particular, we determine the set of structural and geometric conditions required by the Lagrangian invariance and closure of the superconformal algebra. As a consequence of gauging, some of these conditions undergo deformations compared with their well-known ungauged version [3]. More interestingly, our investigation reveals classes of one-dimensional sigma models, which are only scale invariant before gauging. They become fully conformal invariant only after gauging a certain isometry group. For a full discussion on the gauging procedure in canonical formalism and the quantization of these models, as well as a more comprehensive list of references, please check the original paper [4].

Among our gauged superconformal sigma models with various numbers of supersymmetries, the $\mathcal{N}=4B$ cases are particularly interesting, as they include a physically relevant subclass [4]. For $\alpha=0$, the model effectively describes an n-node Coulomb branch quiver quantum mechanics [5,6]. This $D(2,1;0)$-invariant gauged sigma model corresponds to a supergravity counterpart consisting of a number of dyonic BPS black holes in an asymptotic $AdS_2\times S^2$ space–time [7,8].

2. Conformal Invariant Bosonic Sigma Models with Gauged Isometries

The sigma model we are interested in starting with describes the one-dimensional motion of a bosonic particle in a d-dimensional Riemannian target space with metric $G_{AB}(x)$. There is also a coupling to a background gauge field $A_A(x)$. For later convenience, we separate the Lagrangian into linear and quadratic terms in velocity

$$L_B = L_B^{(1)} + L_B^{(2)}; \qquad L_B^{(1)} = A_A\dot{x}^A, \qquad L_B^{(2)} = \frac{1}{2}G_{AB}\dot{x}^A\dot{x}^B \tag{1}$$

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One realizes that the model is invariant under the global shift transformations $\delta x^A = \lambda^I k_I^A$ generated by vector fields $k_I^A(x)$ provided that

$$\mathcal{L}_{k_I} k_J = f_{IJ}{}^K k_K, \qquad i_{k_I} F = dv_I, \qquad \mathcal{L}_{k_I} G_{AB} = 0. \tag{2}$$

Here, $f_{IJ}{}^K$ are the structure constants of a Lie algebra, $F = dA$, and $v_I(x)$ are some potentials on the target space.

2.1. Gauging Procedure

We now gauge the global shift transformation by considering a time-dependent transformation parameter $\lambda(t)$. We then need to introduce a gauge field a^I. The new transformation laws for the fields become

$$\delta_\lambda x^A = \lambda^I(t) k_I^A, \qquad \delta_\lambda a^I = \dot{\lambda}^I + f_{JK}{}^I a^J \lambda^K. \tag{3}$$

Moreover, one needs to replace normal time derivatives in (1) with their gauge covariant version given by

$$D_t x^A = \dot{x}^A - a^I k_I^A. \tag{4}$$

Gauging $L_B^{(1)}$ needs to be performed via the Nöether procedure, which requires adding a new term to the first-order Lagrangian as

$$L_B^{(1)} = A_A \dot{x}^A + a^I v_I. \tag{5}$$

For later convenience, we now list the Lagrangian (L), gauge symmetries (GS), structural conditions (SC), algebra (A), and geometric conditions (GC) for the gauged nonlinear sigma models

$$\mathrm{L}: \quad L_B^{(1)} = A_A \dot{x}^A + a^I v_I, \qquad L_B^{(2)} = \frac{1}{2} G_{AB} D_t x^A D_t x^B \tag{6}$$

$$D_t x^A := \dot{x}^A - a^I k_I^A$$

$$\mathrm{GS}: \quad \delta_\lambda x^A = \lambda^I k_I^A, \qquad \delta_\lambda a^I = \dot{\lambda}^I + f_{JK}{}^I a^J \lambda^K \tag{7}$$

$$\mathrm{A}: \quad [\delta_{\lambda_1}, \delta_{\lambda_2}] = \delta_{\lambda_3} \qquad \lambda_3^I = f_{JK}{}^I \lambda_1^J \lambda_2^K \tag{8}$$

$$\mathrm{SC}: \quad \mathcal{L}_{k_I} k_J = f_{IJ}{}^K k_K \tag{9}$$

$$\mathrm{GC}: \quad i_{k_I} F = dv_I, \qquad \mathcal{L}_{k_I} G_{AB} = 0, \qquad \mathcal{L}_{k_I} v_J = f_{IJ}{}^K v_K. \tag{10}$$

The last condition in (10) is derived from the middle relation in (2). More precisely, one obtains $d(\mathcal{L}_{k_I} v_J) = d(f_{IJ}{}^K v_K)$, which in general has the following solution with constant $w_{IJ} = w_{[IJ]}$

$$\mathcal{L}_{k_I} v_J = f_{IJ}{}^K v_K + w_{IJ}. \tag{11}$$

Here, we restrict ourselves to the cases where v^I exists and satisfies (11) with $w_{IJ} = 0$.

2.2. Conditions for Conformal Invariance

The next symmetry invariance we require for the gauged sigma model is conformal symmetry. The symmetry transformations form the $PSL(2, \mathbb{R})$ subgroup of the time reparametrizations parameterized by $P = u + vt + wt^2$

$$t' = \frac{at + b}{ct + d}, \qquad \delta t = -P(t), \tag{12}$$

Accordingly, the covariant form of transformations of the fields is given by

$$\begin{aligned} \delta_P x^A &= P\dot{x}^A + \dot{P}\xi^A \\ \delta_P a^I &= P\dot{a}^I + \dot{P}(\delta^I_J + \gamma^I{}_J)a^J + \ddot{P}h^I \end{aligned} \tag{13}$$

where ξ^A is a vector in the target space. Additionally, we needed to introduce a constant matrix γ, and potentials $h^I(x)$ whose expression is determined by demanding the conformal invariance of $L^{(2)}$. We obtain

$$h^I = G^{IJ}\xi_A k^A_J. \tag{14}$$

In fact, these potentials parameterize the deformation of the special conformal transformation of the gauge-covariant velocities

$$\delta D_t x^A = \frac{d}{dt}(D_t x^A)P + (\delta^A_B + \partial_B \xi^A)D_t x^B \dot{P} + \xi^A_\perp \ddot{P} \tag{15}$$

where we defined $\xi^A_\perp := \xi^A - h^I k^A_I$. This definition, together with the expression (14), indicates that $\xi_\perp$ should be seen as the projection of ξ orthogonal the Killing vectors k_I, i.e.

$$\xi^A_\perp = P_\perp{}^A{}_B \xi^B \,; \qquad P_\perp{}^A{}_B := \delta^A_B - G^{IJ}k_{IB}k^A_J. \tag{16}$$

The conditions imposed by the conformal invariance on the background are interesting in particular, as they reveal the role of the vectors $\xi^A_\perp$. One finds

$$\mathcal{L}_\xi G_{AB} = -G_{AB}, \qquad \xi_{\perp A} = -\frac{1}{2}\partial_A K. \tag{17}$$

The first condition indicates that ξ has to be a conformal Killing vector, whereas the second condition shows that the one-form associated with $\xi_\perp$ has to be exact. It is, in particular, required by the invariance under special conformal transformations and is in contrast to the corresponding condition for the ungauged model [9,10]. The function $K(x)$ in (17) turns out to be the special conformal Nöether charge given by $K = 2\xi_{\perp A}\xi^A_\perp$.

Summarizing, conformal symmetry (CS) requires the following in addition to (6)–(10):

$$\begin{array}{rrlrl} \text{CS}: & \delta_P x^A &= P\dot{x}^A + \dot{P}\xi^A & \delta_P a^I &= P\dot{a}^I + \dot{P}(\gamma^I{}_J a^J + a^I) + \ddot{P}h^I \quad (18) \\ \text{A}: & [\delta_{P_1}, \delta_{P_2}] &= \delta_{P_3} & P_3 &= \dot{P}_1 P_2 - P_1 \dot{P}_2 \quad (19) \\ & [\delta_P, \delta_{\lambda_1}] &= \delta_{\lambda_2} & \lambda^I_2 &= -P\dot{\lambda}_1 - \dot{P}\gamma^I{}_J \lambda^J_1 \quad (20) \\ \text{SC}: & \mathcal{L}_\xi k^A_I &= -\gamma^J{}_I k^A_J & \gamma^I{}_L f_{JK}{}^L &= f_{LK}{}^I \gamma^L{}_J + f_{JL}{}^I \gamma^L{}_K \quad (21) \\ & \mathcal{L}_{k_J} h^I &= \gamma^I{}_J - f_{JK}{}^I h^K, & \mathcal{L}_\xi h^I &= \gamma^I{}_J h^J \quad (22) \\ \text{GC}: & i_\xi F &= d(h^I v_I) & \mathcal{L}_\xi G_{AB} &= -G_{AB} \quad (23) \\ & \mathcal{L}_\xi v_I &= -\gamma^J{}_I v_J & h^I &= G^{IJ}\xi_A k^A_J \quad (24) \\ & & & \xi_{\perp A} &:= \xi_A - h^I k_{IA} = -\frac{1}{2}\partial_A K. \quad (25) \end{array}$$

3. Supersymmetric Extension

We now move on to the supersymmetric extensions of our gauged bosonic sigma model. Requiring supersymmetry enhances the geometric structure of the target space and the symmetry algebra. We now need to deal with a torsionful covariant derivative appearing in the fermionic part of the Lagrangian. Moreover, the commutator of a special conformal transformation and a supersymmetry generates a new fermionic symmetry: a conformal supersymmetry. Here, we skip cases with $\mathcal{N} = 1B$ and $\mathcal{N} = 2B$ with one

$R-$symmetry as they are well explained in detail in [4]. We just remark that their superconformal generators obey the $osp(1|2)$ and $su(1,1|1)$ algebras, respectively.

3.1. The gauged $\mathcal{N} = 4B$ Supersymmetric Sigma Model

We directly start with the gauged $\mathcal{N} = 4B$ supersymmetric sigma model, referring to [11] for more details on the ungauged version. The new terms in the Lagrangian, in addition to (6), are

$$L_F^{(1)} = -\frac{i}{2}F_{AB}\chi^A\chi^B, \quad L_F^{(2)} = \frac{i}{2}G_{AB}\chi^A\check{D}_t\chi^B - \frac{1}{12}\partial_{[A}C_{BCD]}\chi^A\chi^B\chi^C\chi^D \tag{26}$$

where we defined the gauge-covariant derivative $\check{D}_t$ in terms of the torsionful covariant derivative $\check{\nabla}$ as

$$\begin{aligned}\check{D}_t\chi^A &:= \check{\nabla}_t\chi^A + a^I\left(\nabla^A k_{IB} + \frac{1}{2}C^A_{\ BC}k_I^C\right)\chi^B;\\ \check{\nabla}_t\chi^A &:= \dot{\chi}^A + \left(\Gamma^A_{\ BC} + \frac{1}{2}C^A_{\ BC}\right)\dot{x}^B\chi^C.\end{aligned} \tag{27}$$

The $\mathcal{N} = 4B$ Poincaré supersymmetry transformations parametrized by four real fermionic parameters $\epsilon^\rho, \rho = 1,\ldots,4$ and the $\tilde{R}$-symmetries, respectively, act as

$$\begin{aligned}\delta_\epsilon x^A &= -i(J^\rho)^A{}_B\epsilon_\rho\chi^B\\ \delta_\epsilon a^I &= 0\\ \delta_\epsilon\chi^A &= (\bar{J}^\rho)^A{}_B\epsilon_\rho D_t x^B + i\partial_C(J^\rho)^A{}_B\epsilon_\rho\chi^C\chi^B\\ \\ \delta_{\tilde{r}} x^A &= 0\\ \delta_{\tilde{r}} a^I &= 0\\ \delta_{\tilde{r}}\chi^A &= \frac{1}{2}\tilde{r}^i(J^i)^A{}_B\chi^B.\end{aligned} \tag{28}$$

where we have defined $J^\rho = (J^i, 1)$ and $\bar{J}^\rho = (-J^i, 1)$ for $i = 1,2,3$. In addition to the gauge transformation laws given in (7), we now introduce

$$\delta_\lambda\chi^A = \lambda^I\partial_B k_I^A\chi^B. \tag{29}$$

Summarized, $\mathcal{N} = 4B$ supersymmetry and gauge invariance require the following structural (SC) and geometric conditions (GC) on the target space in addition to (6)–(10)

$$\text{SC}: \quad (J^i)^A{}_C(J^j)^C{}_B = -\delta^{ij}\delta^A_B + \epsilon_{ijk}(J^k)^A{}_B, \qquad 0 = \mathcal{N}(J^i, J^j)^A_{BC} \tag{30}$$

$$0 = \mathcal{L}_{k_I}J^i \tag{31}$$

$$\text{GC}: \quad 0 = \mathcal{L}_{k_I}C_{ABC}, \qquad 0 = \check{\nabla}_A(J^i)^B{}_C \tag{32}$$

$$0 = G_{AC}(J^i)^C{}_B + G_{CB}(J^i)^C{}_A, \quad 0 = F_{AC}(J^i)^C{}_B + F_{CB}(J^i)^C{}_A \tag{33}$$

Let us briefly explain that condition (30) is required by the closure of the algebra demanding $J^i, i = 1,2,3$ to form an integrable quaternionic structure. We also introduce the Nijenhuis concomitant

$$\mathcal{N}(J^i, J^j)^A_{BC} \equiv (J^{(i})^D{}_{[B}\partial_{|D|}(J^{j)})^A{}_{C]} - (J^{(i})^A{}_D\partial_{[B}(J^{j)})^D{}_{C]}. \tag{34}$$

The second structural condition (31) requires k_I to be tri-holomorphic and follows from the closure of the combined algebra of gauge and supersymmetry transformations. The first condition in (32) is needed for the invariance of $L_F^{(2)}$ under global shift symmetries, whereas the second one, $\check{\nabla}_A(J^i)^B{}_C = 0$, means that the three different complex structures are covariantly constant with respect to the *same* torsionful covariant derivative $\check{\nabla}$. Using this relation, one can show the Hermiticity of the four-form dC, i.e., $\partial_{[E}C_{BCD]}(J^i)^E{}_A = 0$, which is needed for the invariance of the action. The two conditions in (33) state that the field strength F_{AB} and the metric G_{AB} are simultaneously Hermitian with respect to all three complex structures J^i. The second condition in (32), together with the first one in (32), define a weakly hyperKähler with torsion (wHKT) manifold [11]. These two, along with the Hermiticity condition of the field strength F_{AB}, are required by the action invariance under $\mathcal{N} = 4B$ supersymmetry and $\tilde{R}$-symmetries. Requiring invariance only under Poincaré supersymmetry without imposing the $\tilde{R}$-invariance leads to a slightly weaker set of conditions spelled out in [11].

3.2. $D(2,1;\alpha)$ Superconformal Invariance

The last step is to require conformal invariance for the gauged $\mathcal{N} = 4B$ sigma model. This will enhance the symmetry group to the one-parameter superconformal group $D(2,1;\alpha)$, where α is determined by the transformation of the supercharges under the second $R-$symmetry in the group. Under conformal transformations parameterized by $P(t)$, the fermionic fields transform as

$$\delta_P\chi^A = P\dot{\chi}^A + \dot{P}\left(\partial_B\xi^A\chi^B + \frac{1}{2}\chi^A\right). \tag{35}$$

For more convenience, let us also define vector fields ω_ρ^A and one forms $V_A^{\rho I}$ as following

$$\begin{aligned} \omega_\rho^A &= (\bar{J}_\rho)^A_B\xi_\perp^B, \qquad \rho = 1,\dots,4 \\ V_A^{\rho I} &= (J^\rho)^B_A\partial_B h^I. \end{aligned} \tag{36}$$

We furthermore combine supersymmetry and superconformal transformations parameterized by two time-independent Grassmann variables ϵ_ρ and η_ρ, respectively. The result will be a fermionic transformation parameterized by $\Sigma_\rho = \epsilon_\rho + \eta_\rho t$. The field transformation laws under this fermionic transformation and the second $su(2)$ $R-$symmetry generated by the commutators of fermionic transformations are determined as

$$\begin{aligned} \delta_\Sigma x^A &= -i(J^\rho)^A{}_B\Sigma_\rho\chi^B \\ \delta_\Sigma a^I &= 2iV_A^{\rho\,I}\dot{\Sigma}_\rho\chi^A \\ \delta_\Sigma\chi^A &= (\bar{J}^\rho)^A{}_B\left(\Sigma_\rho D_t x^B + 2\dot{\Sigma}_\rho\xi_\perp^B\right) + i\partial_C(J^\rho)^A{}_B\Sigma_\rho\chi^C\chi^B \\ \\ \delta_r x^A &= (1+\alpha)\,r^i\omega_i^A \\ \delta_r a^I &= -(1+\alpha)\,r^iV_A^{iI}D_t x^A \\ \delta_r\chi^A &= (1+\alpha)\,r^i\left(\partial_B\omega_i^A + V_B^{iI}k_I^A\right)\chi^B. \end{aligned} \tag{37}$$

Finally, assuming previous transformation rules and older conditions given by (6)–(10), (18)–(25), (26), (27), (30)–(33), (35), (37), the set of new commutators of the $D(2,1;\alpha)$ generators and new conditions required by the closure of the superconformal algebra are

$$\begin{aligned}
\mathrm{A}: \quad & [\delta_{r_1}, \delta_{r_2}] = \delta_{r_3} + \delta_\lambda && r_3^i = \epsilon_{ijk} r_1^j r_2^k, \\
& && \lambda^I = (1+\alpha)^2 \epsilon_{ijk} r_1^j r_2^k \mathcal{L}_{\omega_i} h^I && (38) \\
& [\delta_{\tilde{r}_1}, \delta_{\tilde{r}_2}] = \delta_{\tilde{r}_3} && \tilde{r}_3^i = -\epsilon_{ijk} \tilde{r}_1^j \tilde{r}_2^k && (39) \\
& [\delta_{r_1}, \delta_{\tilde{r}_2}] = \delta_{\tilde{r}_3} && \tilde{r}_3^i = \epsilon_{ijk} r_1^j \tilde{r}_2^k && (40) \\
& [\delta_P, \delta_{\Sigma_1}] = \delta_{\Sigma_2} && \Sigma_{2\rho} = -P\dot{\Sigma}_{1\rho} + \frac{1}{2}\dot{P}\Sigma_{1\rho} && (41) \\
& [\delta_r, \delta_{\Sigma_1}] = \delta_{\Sigma_2} + \delta_\lambda && \Sigma_{2\rho} = \frac{1}{2}\left(j_-^i + j_+^i\right)_{\rho\sigma} r^i \Sigma_{1\sigma} \\
& && \lambda^I = (1+\alpha) V_A^{i\,I} r^i \delta_{\Sigma_1} x^A && (42) \\
& [\delta_{\tilde{r}}, \delta_{\Sigma_1}] = \delta_{\Sigma_2} && \Sigma_{2\rho} = -\frac{1}{2}(j_+^i)_{\rho\sigma}\Sigma_\sigma \tilde{r}^i && (43) \\
& [\delta_{\Sigma_1}, \delta_{\Sigma_2}] = \delta_P + \delta_r + \delta_{\tilde{r}} + \delta_\lambda && P = 2i\Sigma_{1\rho}\Sigma_{2\rho} \\
& && r^i = \frac{2i}{1+\alpha}(j_-^i)_{\rho\sigma}(\dot{\Sigma}_{1\rho}\Sigma_{2\sigma} - \Sigma_{1\rho}\dot{\Sigma}_{2\sigma}) \\
& && \tilde{r}^i = \left(\frac{2\alpha i}{1+\alpha} j_+^i - \frac{2i}{1+\alpha} j_-^i\right)_{\rho\sigma}(\dot{\Sigma}_{1\rho}\Sigma_{2\sigma} - \Sigma_{1\rho}\dot{\Sigma}_{2\sigma}) \\
& && \lambda^I = -2i\left(a^I \Sigma_{1\rho}\Sigma_{2\rho} + h^I \frac{d}{dt}(\Sigma_{1\rho}\Sigma_{2\rho})\right) && (44) \\
\mathrm{SC}: \quad & \mathcal{L}_\xi (J^i)^A{}_B = 0 && \partial_{[A} V_{B]}^{iI} = 0 && (45) \\
& \mathcal{L}_{\omega^i}(J^j)^A{}_B = \frac{1}{1+\alpha}\epsilon_{ijk}(J^k)^A{}_B + k_I^A V_C^{iI}(J^j)^C{}_B - (J^j)^A{}_C k_I^C V_B^{iI} && && (46) \\
\mathrm{GC}: \quad & \mathcal{L}_\xi C_{ABC} = -C_{ABC} && C_{ABC}\xi_\perp^C = 2k_{I[A}\partial_{B]} h^I && (47)
\end{aligned}$$

Here, $(j_\pm^i)_{\mu\nu} := \mp(\delta_{\mu i}\delta_{\nu 4} - \delta_{\mu 4}\delta_{\nu i}) - \epsilon_{i\mu\nu 4}$ denote the (anti-)selfdual 't Hooft symbols.

4. Conclusions and Discussion

We highlight some of the results of this investigation

- The set of constraints we obtained for the $D(2,1;\alpha)$ symmetry of the gauged superconformal sigma model turns out to be a deformed version of its ungauged counterpart. In particular, in the ungauged case, conformal invariance requires the one-form dual to the vector ξ to be exact [9,10], while in the gauged model it is sufficient that this holds for its projection $\xi_\perp$ orthogonal to the symmetry orbits. Therefore, the gauging procedure can be seen through the digression of vector $\xi_\perp$ from ξ, which is parametrized by the potentials $h^I(x)$. It turns out then that those models with nonvanishing h^I are only scale invariant before gauging. It is just through gauging (part of) their isometries that they can admit full conformal invariance via the existence of $\xi_\perp$, which satisfies (17).
- An application of, and motivation for, the work of [4] is provided by the Coulomb branch quiver mechanics describing the dynamics of D-brane systems in an AdS_2 scaling limit. As a special class, these systems exhibit $D(2,1;0)$ symmetry [6]. They are important due to their connection to (n)AdS_2/(n)CFT_1 and black hole physics.

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