physical sciences forum

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Proceeding Paper

The Regular Black Hole by Gravitational Decoupling †

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† Presented at the 2nd Electronic Conference on Universe, 16 February–2 March 2023; Available online: https://ecu2023.sciforum.net/.

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Abstract: In this paper, we consider using the gravitational decoupling method to obtain a hairy regular black hole which corresponds to the Hayward model. We modify the hairy Schwarzschild solution to obtain the regular Kretschmann scalar. The energy momentum of a new model is considered, and we show that there is an energy exchange between its parts.

Keywords: gravitational decoupling; regular black hole; hairy black hole

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Citation: Vertogradov, V.; Misyura, M. The Regular Black Hole by Gravitational Decoupling. *Phys. Sci. Forum* **2023**, *7*, 27. https://doi.org/10.3390/ECU2023-14058

Academic Editor: Gonzalo Olmo

Published: 17 February 2023

1. Introduction

The gravitational collapse of massive stars can end in the formation of a black hole or naked singularity. According to the famous singularity theorem [1] the singularity must form during the gravitational collapse if the weak energy condition is held. However, singularities are generally regarded as indicating the breakdown of the theory, and one needs to modify the theory in this region by considering the quantum effects. Quantum field theory, applied to the stationary black holes, leads to the Hawking radiation [2], which requires the negative ingoing energy flux. This fact means that a black hole evaporates up to the singularity. For this reason, a regular (non-singular) black hole is considered [3]. Hayward has considered the formation and evaporation of regular black holes [4]. Another famous fact about black holes is the no-hair theorem, i.e., a black hole can have only three charges: mass, electric charge and angular momentum. However, it was shown that a black hole can have soft hair [5]. Recently, it was understood that one can obtain a hairy black hole by using the gravitational decoupling method [6,7]. In this paper, we use the gravitational decoupling method and Hayward's regular black hole model to obtain a hairy regular black hole. This paper is organized as follows: in Section 2 we briefly describe the gravitational decoupling. In Section 3 we introduce the hairy regular black hole. Section 4 is the conclusion, including the discussion of the formation and evaporating hairy regular vaidya black hole. The system of units $c = 8\pi G = 1$ will be used throughout the paper. We also use the signature $\{-,+,+,+\}$.

2. The Gravitational Decoupling

It is well known that obtaining the analytical solution of the Einstein equations is, in most cases, a really hard task. We know that we can obtain an analytical solution of the spherically symmetric spacetime in the case of the perfect fluid as the gravitational source. However, if we consider the more realistic case when the perfect fluid is coupled with other matter, then it is nearly impossible to obtain the analytical solution. In papers [8–10] it was shown, by means of the Minimal Geometric Deformation (MGD) [11,12], that we can decouple the gravitational sources. For example, one can write the energy-momentum tensor T_{ik} as:

$$T_{ik} = \tilde{T}_{ik} + \alpha \Theta_{ik} \,. \tag{1}$$

where $\tilde{T}_{ik}$ is the energy-momentum tensor of the perfect fluid and α is the coupling constant to the energy–momentum tensor Θ_{ik}. Moreover, there is only gravitational interaction between two sources, i.e.,

$$T^{ik}_{;k} = 0 \rightarrow \tilde{T}^{ik}_{;k} = \alpha \Theta^{ik}_{;k} = 0 \,. \tag{2}$$

This fact allows us to think about Θ_{ik} as a possible dark matter candidate. However, there can be an energy exchange between two sources:

$$\tilde{T}^{ik}_{;k} = -\alpha \Theta^{ik}_{;k} \,. \tag{3}$$

The Einstein tensor G_{ik}, which corresponds to the energy–momentum tensor (1), can be decoupled in the following way:

$$G_{ik} = \tilde{G}_{ik} + \hat{G}_{ik} \,. \tag{4}$$

where one has two separated Einstein equations to solve:

$$\begin{aligned} \tilde{G}_{ik} &= \tilde{T}_{ik} \\ \hat{G}_{ik} &= \alpha \Theta_{ik} \,. \end{aligned} \tag{5}$$

By solving (5) separately, one obtains the solutions $\tilde{g}_{ik}$ and $\hat{g}_{ik}$, the combination of which gives the solution to the Einstein equation:

$$G_{ik} = T_{ik} \,. \tag{6}$$

By using this method, Ovale [6] has obtained the hairy Schwarzschild solution:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \,, \tag{7}$$

where $d\Omega$ is the metric on the unit sphere and the lapse function f is given by:

$$f = 1 - \frac{2M}{r} + \alpha e^{-\frac{r}{M}} \,. \tag{8}$$

This solution satisfies dominant and strong energy conditions at $r > 2M$ and has the singularity at $r = 0$. We will use (7) to obtain the hairy regular solution in the next section. The parameter M (8) is related to the Schwarzschild black hole mass $\mathbf{M}$ by following relation:

$$M = \mathbf{M} + \frac{\alpha L}{2} \,. \tag{9}$$

where the parameter L has the dimension of the length and is related to a new black hole charge. The impact of this charge on particle motion and thermodynamics has been studied in [13,14]. One should note that the Kretschmann scalar for the lapse function (8) is divergent at the limit $r \rightarrow 0$.

3. Hairy Regular Black Hole

A minimal model of a regular black hole suggested by Hayward [4] is:

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right) dt^2 + \left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right) dr^2 + r^2 d\Omega^2 \,. \tag{10}$$

where l is a convenient encoding of the central energy density $\frac{3}{l^2}$, which is assumed to be positive. The solution (10) behaves the same as the Schwarzschild solution at the limit $r \to \infty$, and at the limit $r \to 0$ it behaves as:

$$g_{00} \simeq 1 - \frac{r^2}{l^2}\,. \tag{11}$$

Under this limit, the Einstein tensor has the cosmological constant form:

$$G \sim \lambda g\,, \tag{12}$$

and

$$\lambda = \frac{3}{l^2}\,. \tag{13}$$

Now, we modify the hairy Schwarzschild solution (7) to obtain a regular hairy black hole. First, let us write the metric in the advanced Eddington–Finkelstein coordinates:

$$ds^2 = -f(r)dv^2 + 2dvdr + r^2 d\Omega^2\,. \tag{14}$$

Now, we assume that the lapse function has the form:

$$f(r) = 1 - \frac{2Mr^2}{r^3 + 2Ml^2} + \alpha e^{-\frac{r}{M}}\,. \tag{15}$$

The problem with this solution is that the Kretschmann scalar diverges at the limit $r \to 0$, indicating that there is a singularity at the centre of this black hole. Hence, one needs to modify the last term $\alpha e^{-\frac{r}{M}}$. One should find the modification which satisfies the following conditions:

- This term must be divergence free at the limit $r \to 0$;
- It must vanish at the spatial infinity.

The following modification satisfies these conditions:

$$ds^2 = -\left(1 - \frac{2M\,r^2}{2l^2M + r^3} + \alpha \exp\left(\frac{-2l^2M - r^3}{M\,r^2}\right)\right)dv^2 + 2dvdr + r^2 d\Omega^2\,. \tag{16}$$

The Kretschmann scalar has a horrific form, which we do not share here. However, it has a finite limit at $r \to 0$:

$$\lim_{r\to 0} K \equiv R_{iklm}R^{iklm} = \frac{24}{l^4}\,, \tag{17}$$

which coincides with Hayward model [4].

Now, we should write down the Einstein tensor to make sure that (16) is the combination of two separated Einstein equations. The Einstein tensor components at $\alpha = 0$ are given by:

$$\begin{aligned}
\tilde{G}^0_0 &= -\frac{12l^2M^2}{(2l^2M + r^3)^2} \\
\tilde{G}^1_1 &= -\frac{12l^2M^2}{(2l^2M + r^3)^2} \\
\tilde{G}^2_2 &= -\frac{24l^2M^2(l^2M - r^3)}{(2l^2M + r^3)^3} \\
\tilde{G}^3_3 &= -\frac{24l^2M^2(l^2M - r^3)}{(2l^2M + r^3)^3}
\end{aligned} \tag{18}$$

Now, we should write down the Einstein tensor components for the metric:

$$ds^2 = -\left(1 + \alpha e^{-\frac{r^3+2l^2M}{Mr^2}}\right)dv^2 + 2dvdr + r^2 d\Omega^2\,, \tag{19}$$

these components are:

$$\begin{aligned}
\hat{G}^0_0 &= \frac{\left(4l^2M + M\,r^2 - r^3\right)\alpha \mathrm{e}^{\frac{-2l^2M-r^3}{r^2M}}}{r^4M} \\
\hat{G}^1_1 &= \frac{\left(4l^2M + M\,r^2 - r^3\right)\alpha \mathrm{e}^{\frac{-2l^2M-r^3}{r^2M}}}{r^4M} \\
\hat{G}^2_2 &= \frac{\left(16l^4M^2 - 4l^2M^2r^2 - 8l^2M\,r^3 - 2M\,r^5 + r^6\right)\alpha \mathrm{e}^{\frac{-2l^2M-r^3}{r^2M}}}{2r^6M^2} \\
\hat{G}^3_3 &= \frac{\left(16l^4M^2 - 4l^2M^2r^2 - 8l^2M\,r^3 - 2M\,r^5 + r^6\right)\alpha \mathrm{e}^{\frac{-2l^2M-r^3}{r^2M}}}{2r^6M^2}
\end{aligned} \tag{20}$$

If we consider the sum $\tilde{G}^i_k + \hat{G}^i_k$, then we obtain the Einstein tensor components for the metric (16):

$$\begin{aligned}
G^0_0 &= \frac{16\alpha\left(l^2M + \frac{r^3}{2}\right)^2\left(\left(l^2 + \frac{r^2}{4}\right)M - \frac{r^3}{4}\right)\mathrm{e}^{\frac{-2l^2M-r^3}{M\,r^2}} - 12l^2M^3r^4}{r^4M(2l^2M + r^3)^2} \\
G^1_1 &= \frac{16\alpha\left(l^2M + \frac{r^3}{2}\right)^2\left(\left(l^2 + \frac{r^2}{4}\right)M - \frac{r^3}{4}\right)\mathrm{e}^{\frac{-2l^2M-r^3}{M\,r^2}} - 12l^2M^3r^4}{r^4M(2l^2M + r^3)^2} \\
G^2_2 &= \frac{1}{2r^6(2l^2M + r^3)^3M^2}\left(128\alpha\left(l^2M + \frac{r^3}{2}\right)^3\left(\left(l^4 - \frac{1}{4}l^2r^2\right)M^2 + \right.\right. \\
&\quad \left.\left. + \left(-\frac{1}{2}l^2r^3 - \frac{1}{8}r^5\right)M + \frac{r^6}{16}\right)\mathrm{e}^{\frac{-2l^2M-r^3}{M\,r^2}} - 48l^4M^5r^6 + 48l^2M^4r^9\right) \\
G^3_3 &= \frac{1}{2r^6(2l^2M + r^3)^3M^2}\left(128\alpha\left(l^2M + \frac{r^3}{2}\right)^3\left(\left(l^4 - \frac{1}{4}l^2r^2\right)M^2 + \right.\right. \\
&\quad \left.\left. + \left(-\frac{1}{2}l^2r^3 - \frac{1}{8}r^5\right)M + \frac{r^6}{16}\right)\mathrm{e}^{\frac{-2l^2M-r^3}{M\,r^2}} - 48l^4M^5r^6 + 48l^2M^4r^9\right)
\end{aligned} \tag{21}$$

Thus, we have shown that the model is the combination of two matter fields. Now, we should find out if there is an energy exchange between these sources. For this purpose, one needs to consider the energy-momentum tensor at the limit $\alpha \to 0$ and consider only $\tilde{T}^{ik}_{;k}$ because $T^{ik}_{;k} = 0$ in virtue of the Einstein equation. The energy–momentum tensor components of the pure Hayward model are given by:

$$\begin{aligned}
\tilde{T}_{00} &= \frac{12M^2l^2\left(2l^2M - 2M\,r^2 + r^3\right)}{(2l^2M + r^3)^3} \\
\tilde{T}_{01} &= -\frac{12M^2l^2}{(2l^2M + r^3)^2} \\
\tilde{T}_{22} &= -\frac{24r^2M^2l^2\left(l^2M - r^3\right)}{(2l^2M + r^3)^3} \\
\tilde{T}_{33} &= -\frac{24r^2M^2l^2\left(l^2M - r^3\right)\sin(\theta)^2}{(2l^2M + r^3)^3}
\end{aligned} \tag{22}$$

If there is an energy exchange between two sources $\tilde{T}_{ik}$ and Θ_{ik}, then the following condition is held:

$$\tilde{T}^{ik}_{;k} = -\alpha\Theta^{ik}_{;k}\,. \tag{23}$$

The covariant derivative is taken with regard to the metric (16). Considering the covariant derivative of (22) one obtains:

$$\tilde{T}^{ik}_{;k} = -\frac{12M\,l^2 e^{\frac{-2l^2M-r^3}{M\,r^2}}\alpha\left(8l^4M^2+4l^2M^2r^2+2l^2M\,r^3-4M\,r^5-r^6\right)}{\left(2l^2M+r^3\right)^3r^3}\delta^i_0\,. \tag{24}$$

Additionally, one can see that there is an energy exchange between two sources. However, this energy exchange is negligible because the minimal geometrical deformation assumes $\alpha \ll 1$ and, according to the Hayward model, l is the Planck scale. Moreover, the expression (24) is regular at the limit $r \to 0$:

$$\lim_{r\to 0}\tilde{T}^{ik}_{;k} = 0\,. \tag{25}$$

As we pointed out above, there is an energy exchange between these sources, and this exchange is negligible. In particle physics, there are several scenarios in which a small energy exchange between dark and ordinary matters is allowed. For this reason, we can still think about Θ_{ik} as a possible dark matter candidate. However, a deeper investigation should be conducted in this direction, and we leave this question to be answered in future research.

4. Conclusions

In this paper, we have modified the hairy Schwarzschild solution to obtain the model of a hairy regular black hole. This model is regular in the centre of a black hole, and the Kretschmann scalar is the same in the limit $r \to 0$ as in the Hayward model. We have shown that there is an energy exchange between two sources and the energy–momentum tensor T_{ik} is conserved, but its parts $\tilde{T}_{ik}$ and Θ_{ik} are not conserved separately. The main questions about the formation and evaporation of this black hole will be answered in future research. However, we can consider the hairy Vaidya solution for this purpose, in which gravitational decoupling has been considered, as described in [15].

Author Contributions: Conceptualization V.V. and M.M.; methodology, V.V. and M.M.; software, V.V. and M.M.; validation, V.V. and M.M.; formal analysis, V.V. and M.M.; investigation, V.V. and M.M.; resources, V.V. and M.M.; data curation, V.V. and M.M.; writing V.V. and M.M.; writing—review and editing, V.V. and M.M.; visualization, V.V. and M.M.; supervision, V.V. and M.M.; project administration, V.V. and M.M.; funding acquisition, V.V. and M.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Russian Science Foundation grant number 22-22-00112.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors gratefully acknowledge financial support from the RSF grant 22-22-00112. This work was performed for SAO RAS state assignment "Conducting Fundamental Science Research".

Conflicts of Interest: The authors declare no conflict of interest.

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