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Proceeding Paper

String-Inspired Correction to R^2 Inflation †

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Abstract: We study the Starobinsky–Bel–Robinson inflationary model in the slow-roll regime. In the framework of higher-curvature corrections to inflationary parameters, we estimate the maximal possible value of the dimensionless positive coupling constant β coming from M-theory.

Keywords: Starobinsky's scalaron; Starobinsky–Bel–Robinson modified gravity; inflation

1. Introduction

The Starobinsky–Bel–Robinson (SBR)-modified gravity was proposed in [1,2] as an extension of the non-perturbative $(R+R^2)$ gravity [3] by the perturbative quantum correction inspired by M-theory. This quantum correction is given by the Bel–Robinson tensor squared [4,5].

The four-dimensional SBR action can be presented in the form:

$$S_{SBR} = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\left[R + \frac{R^2}{6m^2} + \frac{\beta}{32m^6}\left(\mathcal{G}^2 - P_4^2\right)\right], \tag{1}$$

where

$$\mathcal{G} = {}^*R_{\mu\nu\lambda\rho}{}^*R^{\mu\nu\lambda\rho} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \quad P_4 = {}^*R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}, \tag{2}$$

$${}^*R_{\mu\nu\lambda\rho} = \frac{1}{2}E_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\lambda\rho}, \quad E_{\mu\nu\lambda\rho} = \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho}, \tag{3}$$

β is a dimensionless positive coupling constant. The Starobinsky–Bel–Robinson modified gravity is the model with higher-curvature terms. In [2], we obtained the equations of motion in the Starobinsky–Bel–Robinson model. We considered solutions to the model, their series expansions, and a slow-roll regime, with inflationary parameters. Now, we consider, in detail, how the expressions to inflationary parameters with the influence of higher-curvature corrections were obtained.

2. Starobinsky–Bel–Robinson Modified Gravity in the Friedmann Universe

The application of the linearization procedure and variation principle leads to the system of equations of motions. In the spatially flat Friedmann Universe

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right),$$

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the P_4 term does not contribute to the equations of motion, which have the following form

$$\left(R_{\rho\nu} - \frac{g_{\rho\nu}}{2}R\right)\left(1 + \frac{R}{3m^2}\right) + \frac{R^2}{12m^2}g_{\rho\nu} - \frac{1}{3m^2}\left(\frac{\nabla_\rho\nabla_\nu + \nabla_\nu\nabla_\rho}{2} - g_{\rho\nu}\Box\right)R$$
$$+ \frac{\beta}{64m^6}\Big[\mathcal{G}^2 g_{\rho\nu} + 8\{(Rg_{\rho\nu} - 2R_{\rho\nu})\Box\mathcal{G} - R\nabla_\rho\nabla_\nu\mathcal{G} \tag{4}$$
$$+ 2(R^\alpha_\nu\nabla_\alpha\nabla_\rho\mathcal{G} + R^\alpha_\rho\nabla_\alpha\nabla_\nu\mathcal{G}) - 2(g_{\rho\nu}R_{\alpha\beta} + R_{\alpha\rho\nu\beta})\nabla^\beta\nabla^\alpha\mathcal{G}\}\Big] = 0\,.$$

We consider the $(0,0)$-component of the system (4)

$$3H^2\left(1 + \frac{R}{3m^2}\right) - \frac{R^2}{12m^2} + \frac{H\dot{R}}{m^2} - \frac{\beta}{64m^6}\left[\mathcal{G}^2 - 48H^3\dot{\mathcal{G}}\right] = 0 \tag{5}$$

and the trace equation

$$R + \frac{1}{m^2}(\ddot{R} + 3H\dot{R}) - \frac{\beta}{16m^6}\left[\mathcal{G}^2 - 12\left(H^2\ddot{\mathcal{G}} + 2H\dot{H}\dot{\mathcal{G}} + 3H^3\dot{\mathcal{G}}\right)\right] = 0, \tag{6}$$

where $H = \dot{a}/a$, the dot denotes the time derivative. Equation (6) can be obtained by the summing of the first derivative of Equation (5) multiplied to H^{-1} and Equation (5) multiplied to 4. Therefore, Equation (6) is a consequence of Equation (5).

We rewrite Equation (5) in terms of the Hubble function $H(t)$ and its time derivatives

$$2\left(m^4 + 3\beta\,H^4\right)H\ddot{H} - \left(m^4 - 9\beta\,H^4\right)\dot{H}^2 + 6\left(m^4 + 3\beta\,H^4\right)H^2\dot{H} - 3\beta\,H^8 + m^6H^2 = 0\,. \tag{7}$$

using the relations

$$\dot{R} = 24\,\dot{H}\,H + 6\,\ddot{H}, \quad \dot{\mathcal{G}} = 48\,H\left(H^2 + \dot{H}\right)\dot{H} + 24\,H^2(2\,H\dot{H} + \ddot{H}). \tag{8}$$

We apply the slow-roll approximation $|\ddot{H}| \ll |H\dot{H}|$ and $|\dot{H}| \ll H^2$ to Equation (7):

$$6\left(m^4 + 3\beta H^4\right)\dot{H} - 3\beta H^6 + m^6 = 0 \tag{9}$$

and obtain the solution up to first-order corrections

$$H(t) \approx \frac{m^2(t_0 - t)}{6} - \beta\left(\frac{m}{6}\right)^6(t_0 - t)^5\left[\frac{m^2}{14}(t_0 - t)^2 + \frac{18}{5}\right] + \mathcal{O}(\beta^2)\,. \tag{10}$$

3. Inflationary Parameters

In [6], the approach for studying slow-roll inflation in the models with higher-curvature terms has been proposed. In the Friedmann Universe, the equations of motions can be presented, such as:

$$F(H^2) = 2\alpha l^2(\psi(\psi - H^2) - H\dot{\psi}), \quad 12\psi = R \tag{11}$$
$$\dot{H}F'(H^2) = -\alpha l^2(\ddot{\psi} - H\dot{\psi} + 2\dot{H}\psi) \tag{12}$$

where α, l are the model constants, $\alpha l^2 = 2/m^2$. Equation (12) is the direct time differential of Equation (11). The function $F(H^2)$ has the following polynomial structure

$$F(H^2) = H^2 + l^{-2}\sum_{n=3}^{\infty}(-1)^n\lambda_n(l\cdot H)^{2n} \tag{13}$$

In the case of the slow-roll regime, the inflationary parameters can be determined by the function $F(H^2)$.

Equation (5) is equivalent to (11) with

$$F(H^2) = H^2 - \frac{\beta}{192m^6}\left(48H^3\dot{\mathcal{G}} - \mathcal{G}^2\right). \tag{14}$$

In the slow-roll regime, the expression (14) can be simplified

$$48H^3\dot{\mathcal{G}} - \mathcal{G}^2 \approx 24^2 H^6(6\dot{H} - H^2).$$

With help of solution (10), we obtain $\beta\dot{H} \approx -\beta m^2/6$ and rewrite Equation (5) such as:

$$R\left(\frac{R}{12} - H^2\right) - H\dot{R} = 3\,m^2 F(H^2), \quad F(H^2) = \left(H^2 - \frac{3\,\beta\,H^8}{m^6} + \frac{18\,\beta\,H^6\dot{H}}{m^6}\right) . \tag{15}$$

To apply a comparative analysis, we rewrite the function $F(H^2)$ in the form

$$F(H^2) = H^2 - \lambda_3\left(l^2\right)^2\left(H^2\right)^3 + \lambda_4\left(l^2\right)^3\left(H^2\right)^4 \tag{16}$$

where

$$\lambda_3 = \frac{3\beta\alpha^2}{4}, \quad \lambda_4 = -\frac{3\beta\alpha^2}{8} \tag{17}$$

In [6], the expressions for the spectral index n_s and the tensor-to-scalar ratio r were obtained using coefficients λ_3 and λ_4 as follows

$$n_s = 1 - \frac{2}{N} - \frac{32\lambda_3 N}{27\alpha^2} + \frac{4\lambda_4 N^2}{3\alpha^3}, \quad r = \frac{12}{N^2} - \frac{16\beta}{9\alpha^2} + \frac{16\lambda_4 N}{3\alpha^3}. \tag{18}$$

Thus, Equation (17) leads to the inflationary parameters:

$$n_s = 1 - \frac{2}{N} - \frac{8\beta N}{9} - \frac{\beta N^2}{2}, \quad r = \frac{12}{N^2} - \frac{16}{3}\beta - 2\beta N \tag{19}$$

The numerical estimation of the spectral index [2] leads to the following interval for value of β:

$$0 \leq \beta \leq 3.9 \cdot 10^{-6} \tag{20}$$

If β belongs to the interval (20), then values of the tensor-to-scalar ratio and the amplitude of scalar perturbations do not contradict modern observations [7,8].

4. Conclusions

The SBR-modified gravity was suggested as part of the gravitational low-energy effective action in four space–time dimensions, originating from non-perturbative superstring theory (or M-theory) in higher space–time dimensions, and in the presence of the mass terms for dilaton and axion when their kinetic terms are ignored [1,2]. In [2], we study different solutions of the model using series representations. The attractor solutions are most interesting when studying the slow-roll inflationary scenarios. In [2], we analyzed the inflationary scenario in the Starobinsky–Bel–Robinson modified gravity. Here, we wrote just some details applied to obtain formulas of inflationary parameters in the frameworks of gravity models with higher-curvature terms [6].

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