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Borel and the Emergence of Probability on the Mathematical Scene in France †

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Abstract: In 1928, the Henri Poincaré Institute opened in Paris thanks to the efforts of the mathematician Emile Borel and the support of the Rockefeller Foundation. Teaching and research on the mathematics of chance were placed by Borel at the center of the institute's activity, a result imposed by the French mathematicians in the face of indifference and even hostility towards a discipline accused of a lack of seriousness. This historical account, based in large part on the results of Matthias Cléry's thesis, presents the way in which Borel became convinced of the importance of making up for the gap between France and other countries as regards the place of probability and statistics in the educational system, and elaborates the strategy that led to the creation of the IHP and how its voluntarist functioning enabled it to become in ten years one of the main world centers of reflection on this subject.

Keywords: Emile Borel; Institut Henri Poincaré; history of probability theory; Rockefeller Foundation

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This paper deals with the way in which, in the 1920s, probability was imposed on the French scientific scene under the decisive impulse of the mathematician Emile Borel.

Like all French mathematicians at the end of the 19th century after the harsh defeat of 1870 [1], Borel had been educated by teachers who had discovered with amazement the extent of German mathematics. For Borel's generation, Georg Cantor played a specific role among the German mathematicians with their treatment of sets and their studies of the transfinite, studies that were justly seen as something radically new. At first, Borel had been a fascinated follower of Cantor. In his PhD devoted to complex analysis, which he defended in 1894, he made a striking use of the Cantorian approach to prove the compactness of the closed unit interval.

However, at the turn of the 20th century, Borel's fascination with Cantor decreased and was gradually replaced by a kind of worry. From Borel's point of view, Cantor's theories were of course logically sound; however, he asked: was logic really the ultimate basis on which mathematics must rely? A good example of Borel's concern appears in this comment on mathematical activity:

> The mathematician who is absorbed in their dream is similar to the situation of the pupil for whom the francs of the problems are not real francs, used to buy objects; he lives in a world apart, built in their mind, having the feeling that this world often has nothing to do with the real world. One of the following two events usually occurs: either the mathematician builds an a priori real world, adequate to their world of ideas; it then leads to a metaphysical system that is not based on anything. Or, he establishes an absolute demarcation between their theoretical life and their practical life, and their science serves them nothing to understand the world; he accepts, almost without thinking, the beliefs of the environment in which he lives. [2]

Following a tradition he discovered in du Bois-Reymond, Borel became convinced of the necessity for the mathematician to base their work not on logic alone but on mathematical facts—such as models or representations—even if they are ideal models or analogies. In a way, Borel was close to Poincaré's famous opinion that mathematics is "the art of calling different things by the same name" ([3], p. 31) as long as the mathematician has a precise representation of at least one of these equivalent things. A specific problem under scrutiny in these years was Zermelo's axiom of choice. It gave rise to a famous correspondence exchanged between Borel and the French mathematicians Baire, Hadamard, and Lebesgue. Hadamard declared complete acceptance of Zermelo's axiom; however, Borel expressed his doubts. Hadamard observed that in this difference of approach, one could detect two opposed conceptions of mathematics: whether or not to admit that an object or a concept (such as "to order a set") can be used in mathematics without being able to construct or realize it effectively. For Hadamard, Borel's worry was not new: some had for instance opposed the general definition of a function, which they found meaningless in the absence of an analytic expression. However, what was at stake for Borel was nothing less than the unity of the mathematical community. As he would later suggest, mathematicians should accept to deal only with objects that satisfy the following criterion: a mathematical being A is well defined when any two mathematicians who mention A are sure to mention the same being, without any possible ambiguity. All the more when this being is the fundamental concept of mathematics: a number. For Borel, questions about the elements of the real line should be examined alongside the consideration of their constructibility.

To their surprise, in a paper written by the Swedish Anders Wiman [4], Borel discovered the following problem: if a real number is chosen at random between 0 and 1, how is the distribution of the terms in its decomposition in continued fractions determined?Wiman made use of the sigma-additivity of the lengths of subsets of the real line to perform the calculations of the distribution function of the terms, though he certainly ignored any concept of the measure of sets. Countable additivity was for Wiman merely a straight extension of finite additivity; however, Borel was familiar to an argument he had introduced as early as in his PhD as a central tool for analysis, opening the way to Lebesgue's revolutionary theory of integration. Wiman's paper about continued fraction expansions became proof for Borel that the measure of sets was adequate to tackle probabilistic questions and, furthermore, that a probabilistic approach may be an efficient way to describe real numbers. With probability, Borel would find the intuitive framework he was looking for in order to overtake Cantor's logical approach.

If this evolution may seem trivial today, it was far from obvious for a French mathematician of Borel's stature to become interested in probability at the beginning of the 20th century. It is true that since 1850, thanks to the efforts of Poisson, the worthy heir of his master Laplace, the tutelary figure of the French 19th century probabilistic scene, there existed at the Sorbonne a Chair of Probability Calculus and Mathematical Physics; however, this very title emphasizes that mathematical physics had been added to compensate for the not-very-serious reputation of probabilities, inherited from the ramblings of the same Poisson on judicial probabilities. Furthermore, it was indeed mathematical physics that constituted the courses offered over six decades. Joseph Bertrand (1822–1900), on the chair of physics and mathematics at the Collège de France, did propose a certain renewal of the teaching of probability by publishing in 1889 an interesting treatise on the subject; however, he always clearly considered it to be rather minor on the mathematical side. Moreover, Bertrand's skepticism about the interpretation of continuous probabilities is well known [5], a skepticism that he illustrated with famous paradoxes about the random drawing of a string on a circle.

Borel could, it is true, recommend an illustrious predecessor in Henri Poincaré (1854–1912), whose chair at the Sorbonne was their first university post in 1886. Poincaré seriously considered the probabilistic question, though not immediately: during the first years, the courses he gave were only devoted to questions of physics [6]. It was during the publication of their course on thermodynamics and a polemic with the English physicist

Peter Tait (1831–1901), who reproached them for ignoring everything about Maxwell's statistical mechanics, that Poincaré began to question, not without resistance, his strictly deterministic vision of physics inherited from Newton, Laplace, and Hamilton. Poincaré being Poincaré, it was first of all himself that he wanted to convince that a serious mathematician could reasonably use probabilities for physical models and, deciding to take the bull by the horns, he decided to teach probability in 1893 in an attempt to give it a presentable face. His course, which introduced fundamental innovations such as the method of arbitrary functions, was published in 1896 [7] when he left this chair for the chair of celestial mechanics (a second edition of the book, considerably expanded, was published a few months before his death in 1912). It is clear, however, that Poincaré always remained on the threshold of randomness, keeping in mind an ideal of a deterministic horizon for physics. He was replaced by the physicist Joseph Boussinesq (1842–1929). As one can see, a French mathematician of the beginning of the 20th century had no business wasting their time in a discipline that was, at best, a mere handmaiden of physics, and at worst a series of purely recreational problems, and it therefore took Borel real scientific courage to take this path, which he did with all the usual power of his hyperactivity.

In 1905, a few months after discovering Wiman's article, Borel published his first probabilistic article [8] entitled *On some questions of probabilities*. There, his goal was precisely to show that the measure of sets and the very young integral of Lebesgue made it possible to provide a precise mathematical formulation for some probabilistic questions that were previously intractable. This was the case, for example, said Borel, if one sought to assess the probability of obtaining a rational number by drawing a real number at random between 0 and 1. In their article, Borel first wanted to follow in the footsteps of Henri Poincaré, who was then the dominant figure of the French probabilistic scene. Poincaré defended a conventionalist approach, and he had in particular asserted in his famous essay 'Science and Hypothesis' [9] (p. 243) that if one is to undertake any calculation of probability, for this calculation to simply have a meaning, it was necessary to admit, as a starting point, a hypothesis or a convention that always involves some arbitrariness. However, Borel asserts with authority that the most convenient convention—at least in the case where the set of possible values of the various variables involved in the problem is bounded—consists of considering the value of the probability as proportional to the area defined by the variables (Borel, 1905). This assertion seems to illustrate that for Borel the arbitrariness of the convention, since it relies on our intuitive perception of a geometric measure (length, surface, volume...), to which Lebesgue's integral precisely gives an indisputable mathematical meaning, is not really arbitrary. This shift from geometry to probabilities allowed Borel to promote a new probabilistic approach to design the filling of the real line. The real line was a kind of large urn from which one can draw a number, a thought experiment of which we have an immediate intuition, and the calculus of probabilities based on the measurement of sets allows us to quantify the results of the experiment and its specific events (such as drawing a rational number). It is a mathematical explanation of the obscure instinct—as Poincaré expressed it—placed at the heart of the scientific approach to phenomena without which we cannot do [9] (p. 216).

In 1909, in an article [10] for the journal *Rendiconti del Circolo matematico di Palermo*, Borel's ideas reached their deepest development with his application of denumerable probability to decimal and continued fractional expansions of real numbers. In one of the most influential probabilistic works of the 20th century, Borel introduces the notion of almost sure convergence and an initial version of the strong law of large numbers. He used these to provide a proof for the existence of an object, such as a normal number, by showing that its probability of existence equals one. The strangeness of this kind of proof of existence, when contrasted with the classical methods, helps explain why the strong law of large numbers and denumerable probability "caught the mathematicians by surprise". This sort of semi-intuitionism, "half-axiomatic, and half-constructivist" as Brouwer would call it, became one of Borel's trademarks, but was sometimes received with a certain amount of skepticism.

There was another reason for Borel's interest in probability after 1905. That same year, he and his wife founded the journal *Revue du Mois*, which became one of the major publications for intellectual debates in France. Borel asked numerous members of their huge network to contribute to their journal [11,12]. In the typical atmosphere of the "radical-socialist" Third republic, Borel's journal defended the idea that the scientific enlightenment of citizens was a basis to improve social conditions. Borel himself used the journal to present an evolving conception of the presence of probability in everyday life. From their perspective, the calculus of probability and statistics provides citizens with tools allowing for the measurement of risks; therefore, he believed they were the most useful part of mathematics. The application of probability to social mathematics goes against the most antisocial aspects of a poorly conceived individualism, which, as Borel would write, is generally only a "stupid egoism". Such calculations of risks constantly bring to the foreground the man as a citizen belonging to a society and acting within it. It follows, then, that the study and practice of these different techniques, over and beyond scientific goals of analysis and prediction, have the virtue of limiting the "excesses of the individualistic mentality". Instead, they promote the values of social solidarity.

In 1914, Borel gathered the papers he wrote for the *Revue du Mois* to form his volume "le Hasard" [13]. One perceives in this book Borel's philosophy of "practical values" in science. Mathematics is at the heart of both the sciences and everyday life. This is an interpretation that is not wholly utilitarian, no more than it is axiomatic. The object is to find a middle path between an absolute faith in the results of mathematics, consisting of an application without any judgment, and a skepticism consisting in positing a radical rupture between mathematical equations and problems of everyday life. For Borel, the coefficient of probability was a clear answer to many questions corresponding to an absolutely tangible reality. He commented ironically that people who complain that "they prefer certainty" would probably also prefer that 2 plus 2 equals 5. As a commentator would summarize later, "probabilities appear to be the only possible path to the future in a world that is no longer endowed with the sharp edges of certainty, but instead presents itself as the fuzzy realm of approximations" [14].

Despite all this activity, Borel's support for a probabilistic approach to the world may have remained limited to the inner academic circle if an occasion had not provided a large-scale experience of the use of numbers, namely the outbreak of the Great War in 1914 [15,16]. Borel was deeply involved in the conflict at various levels. In 1915, he became the head of the *Commission des inventions intéressant la defense nationale* and, in 1917, he was secretary of Painlevé's government. In each of these positions, he realized that a sound statistical treatment of the enormous collection of data provided to the government was urgently needed for a modern approach to governance. At the end of the war, Borel became convinced to enter a political career and use his influence to implement his ideas for developing statistics and probability in France.

Borel's main institutional efforts were first directed toward statistics. Due to the old Napoleonic conception of educating administrators as jurists, he was extremely conscious of France's backwardness. In 1922, Borel accepted to be the first mathematician to become president of the Société de Statistique de Paris, where he began a campaign to emphasize the importance of a probabilistic approach to statistics [17]. During the same year, he helped found the ISUP, where he soon asked his former student Georges Darmois to organize the teaching of mathematical statistics [18]. In 1928, Darmois wrote the first French textbook on probabilistic statistics [19] in which he began to realize a transfer of statistical technology from foreign countries, especially from England with Pearson's biometrical research.

Borel's involvement also concerned probability theory. His first step was to accept the Chair of Probability Calculus and Mathematical Physics at the Sorbonne in 1920. Under Borel's direction and for the first time, the chair clearly took a mathematical turn to the detriment of physics [18]. Probability theory became the main (and soon the only) topic of its syllabus. Moreover, in 1924, Borel launched a great project, a treatise collecting the probabilistic knowledge of the time, thereby proving that he was well aware of the exponential development of probability theory in those years. However, the treatise, which appeared in fascicles until 1939, was in many ways rather obsolete because Borel was skeptical of the use of too-sophisticated mathematics in probability [20]. This negative skepticism did not fail to attract the acidic criticism of the new generation of probabilists, such as Paul Lévy. One must, however, honestly observe that Borel always remained strategically open toward modern probabilities and facilitated the publication of their works by more advanced colleagues (such as Paul Lévy [21]).

After 6 years on the Chair of Probability Calculus and Mathematical Physics, an opportunity was presented to Borel that could give a decisive impulse to their project of making probability calculus a dynamical domain of mathematical research and teaching. It was an opportunity he grasped with his usual energy. The American mathematician George Birkhoff (1884–1944) stayed for several months in Europe between 1925 and 1926 but chose to make Paris his temporary residence of departure for his trips [22]. Birkhoff was sent by Augustus Trowbridge, head of the International Education Board (IEB) office in Paris, to enquire about the development of European mathematics. The IEB, directed by Wickliffe Rose, was created by the Rockefeller Foundation for financing traveling grants for young scientists and supporting the creation or the maintenance of scientific institutions. An admirer of Poincaré, Birkhoff used his stay to connect with French mathematicians. During an informal dinner, Birkhoff and Borel discussed the need to develop interactions between mathematicians and physicists in Paris to strengthen both groups. They suggested creating an institute, which Borel wanted to be located on the rue Pierre Curie, close to the Institut du Radium (directed by Marie Curie), the Laboratory of physical chemistry (directed by Jean Perrin), and the École normale supérieure.

Borel, supported by Birkhoff, started discussions with Trowbridge in may 1926 in order to obtain financial support from the IEB to create an institute of mathematics and mathematical physics, as well as to construct a modest building for the mathematics department. He also asked for three or four new chairs in mathematical physics and applied mathematics. Trowbridge's reactions show that his concerns were less connected to the scientific aspect than to its financial sustainability. Above all, Trowbridge was troubled by the little number of first-rate scientists involved in the project, illustrating Rose's goal for the IEB: to make the peak higher. The project that Borel presented to the assembly of the Faculty of Science and to the University of Paris in June 1926 was twofold: the edification of a building to become the mathematical center of the university and the creation of an institute to develop mathematics and mathematical physics.

Between June and December 1926, the project evolved towards the elaboration of an international scientific institution. Borel proposed to complete their chair with a new chair of physical theory. In addition, Borel proposed to organize lectures given by French or foreign scientists. This new project clearly met the implicit and explicit conditions of the IEB, which gave its agreement. On the 17th of November 1928, the new building of the Institut Henri Poincaré was inaugurated with a ceremony gathering scientists, both French and foreign, as well as politicians. In his speech, Borel emphasized probability calculus, both as a French science and a heritage from Poincaré. The Institut Henri Poincaré soon became a powerful institutional tool for scientific policy lead by four scientists: Charles Maurain, Émile Borel (who acted as director), Jean Perrin, and Paul Langevin. The quartet of close friends met when they were students at the École normale supérieure and shared common views on the need for the development of physics in Paris [15].

The institute officially started its two activities in November 1928. A board was in charge of choosing four or five members to compose a committee, which was in turn each year in charge of choosing the list of scientists to be invited. Between 1928 and 1939, this committee invited 85 scientists, among whom 76 gave a talk [18].

As far as probability calculus is concerned, the appointment of Maurice Fréchet and the lectures given by Darmois at the IHP in 1929, which was soon followed by his appointment at the institute, allowed Borel to be supported at the board and the committee by two younger mathematicians dedicated to probability calculus and mathematical statistics They defined a program aimed at developing topics in which each of them were involved as well as transforming the IHP into a first-rank institution for probability theory. They also trained young mathematicians in order to initiate a long-term development in probability and mathematical statistics. The talks and lectures by foreign scientists enabled a cultural transfer that allowed the emergence of specific lines of research fueled by works developed in other contexts.

Training in probability calculus underwent many changes after Borel became the owner of their chair in 1920. The title of the degree it delivered now included probability calculus because Borel changed the syllabus. The opening of the IHP gave Borel the opportunity to secure changes in two directions: the degree of probability calculus became a more specialized degree and the lectures became closer to current research topics. In 1930, three options were created for the degree: theoretical complements, statistics, and mathematical physics. Until 1937, Fréchet and Darmois, introduced contemporary research topics, notably Markov processes and statistical methods. However, in 1937, the syllabus became more strictly defined to prepare theoretical complements and statistics options; however, that same year, Borel instituted a seminar on the model of Hadamard's seminar in the Collège de France, whereby contemporary probabilistic publications were discussed with students and professional mathematicians. Note also that students were moreover explicitly called to participate in the talks of the IHP, which had to be in French.

Fréchet and Darmois' activity improved the situation of probability calculus as a legitimate field on the mathematical scene. They used a presentation of probability based on an analytic approach. For instance, Fréchet developed in his teaching of 1929 and 1930 a long presentation of Fredholm's integral equations and its application to the study of Markov chains. This way of presenting probability calculus contributed to inserting probability calculus in a mathematical curriculum and therefore contributed to make probability calculus a legitimate topic for students. With Borel's presence, the Chair of Probability Calculus and Mathematical Physics and the degree it provided to students were definitely acknowledged by the Faculty of Science as belonging to the mathematical field.

The program of the lectures on probability and statistics, defined by Borel, Fréchet, and Darmois, aimed at two intertwined goals: developing local research with international research and making the IHP an international scene for the mathematics of randomness.

Their first strategy was to invite probabilists of international renown. George Pólya was the first foreign speaker on probability calculus. This invitation was strongly symbolic: Pólya gained recognition at an international scale in the 1920s for developments in analysis for the central limit theorem and, as a Hungarian, he was seen as an intermediary between European probabilists from the West and the East.

Their second strategy was to connect the institute with already-existing foreign probabilistic communities. For instance, the attention given to Italian probabilists (who gave four series of talks in the 1930s) points to the fact that probability calculus and mathematical statistics were strongly institutionalized in Italy, especially with publications such as *Metron*, which was founded in 1920 with an explicit international vocation.

Their third strategy was to give room to trendy topics. Despite their (strong) skepticism towards any attempt at axiomatization of probability calculus, Borel, Fréchet, and Darmois organized talks on foundations—at that time the object of an important debate at an international scale. The heart of this debate was the first attempt of axiomatization, the theory of "kollektivs", designed by Richard von Mises in 1919, which he presented in 1930 at the IHP. The talk of Cantelli, in 1933, is a first response based on the identification of specific modes of convergence in probability theory. A second response came from Bruno de Finetti in 1935, this time based on a different approach to probability. If von Mises designed their theory on a frequentist approach of probability, de Finetti proposed to develop their theory on a strong subjectivist conception of probability. The last talk on this topic was by Hans Reichenbach, based on a logical approach. Incidentally, the invitation of Reichenbach shows Borel's attention toward exiled German mathematicians fleeing Nazi Germany.

Making the IHP an international scene also served the agenda of local actors. Fréchet started being interested in Markov chains in the 1920s through correspondence with the Czech mathematician Bohuslav Hostinský; however, he only really started working on this topic in 1928 [23]. Hostinský gave two series of talks on Markov chains at the IHP in 1930 and 1937. Observe that in the 1930s, Soviet mathematicians were the main specialists in research on Markov processes; however, the Stalinist regime made it almost impossible for them to come to Paris. Hostinský's role (together with the small active group of students he trained in Brno) was also partly that of a go-between. In his talk of 1937, he gave an original synthesis of research, including various Soviet results [24].

The talks on statistics also show how Georges Darmois's interest in statistics evolved. Following Borel, Darmois was firstly interested in Scandinavian statistics. Johann Steffensen, a Danish statistician, gave a talk in 1931, which was followed by Alf Guldberg, a Norwegian statistician, in 1932 and 1934 [25]. After 1936, with the talk of Jerzy Neyman followed by the one of Ronald Fisher in 1938, interest clearly shifted towards British statistics, more specifically towards the theory of estimation and the application of statistics to genetics [18]. These talks followed an international evolution, whereby British statistics took a decisive lead. All those talks served Darmois's agenda, which was to make probability the main basis for statistics, contrary to the state of mind of many French statisticians, who were wary, if not opposed, to this approach. Incidentally, Darmois, supported by Borel, made statistics a legitimate topic of research in mathematics.

The talks of the IHP were designed to give visibility to contemporary research and to be accessible to students. In 1930, they were completed with the *Annales de l'IHP* in order to publish the text of the talks given at the institute.

The combination of guest talks, lectures, and the *Annales*, made the institute a remarkable tool for the practice of a transfer. This unprecedented collective organization around probability theory, inside the building housing the mathematics department of the Sorbonne, favored an increasing number of candidates to choose degrees in probability calculus and mathematical physics. If a large majority of students chose the statistics option, the students of the École normale supérieure preferred the option "theoretical complements". This led to an unprecedented increase in the number of PhD candidates. If Bachelier had been the only candidate to defend a PhD in probability calculus between 1900 and 1918, and Robert Deltheil (in 1920) and Francis Perrin (in 1928) the only ones in the following decade, six candidates defended a PhD thesis in probability or statistics between 1929 and 1941 [18,26]. Some of them had passed the probability calculus degree: Jean Ville and Michel Loève in 1931, Daniel Dugué in 1932, and Wolfgang Dœblin in 1934. Fréchet and Darmois were particularly eager to attract young "normaliens", as Ville and Dugué were, because it was an important step in the process of making probability calculus and mathematical statistics legitimate topics of mathematical study and research. Moreover, they did not hesitate to encourage students from the Ecole Normale that had not formerly followed lectures in probability or statistics to embark on such a project: Robert Fortet started a PhD in probability calculus under Fréchet's advice and Gustave Malécot under Darmois' advice.

Nevertheless, this rather small group of PhD students promoted a dynamic collective life. They regularly met in the library in the building of the IHP and eventually organized a seminar on probability calculus. This seminar was organized by Ville, Dœblin, and Fortet, and they invited Paul Lévy as the first speaker. This student seminar became the basis of the more official seminar organized by Borel from November 1937 [18,27].

Moreover, the PhD students had the opportunity to meet international experts at the institute. As already mentioned, Darmois became particularly interested in British statistics in the mid-1930s. Daniel Dugué and Gustave Malécot were introduced to Ronald Fisher: Dugué, thanks to a Rockefeller fellowship, stayed at the Rothamsted Experimental Station to work under Fisher's guidance in 1937-1938 and Malécot was in the audience of Fisher's talk in 1938.

Those six PhD dissertations present research along the lines defined by Fréchet, Darmois, and Borel. Moreover, although Paul Lévy was outside the Faculty of Science, as he was professor at the École polytechnique, he was involved with both the talks of the IHP and the work of some PhD students. Lévy gave two talks, in 1929 and 1935, and, as already mentioned, gave the first talk at a student seminar in February 1937. He was also probably part of the audience of Borel's seminar, as Borel mentioned the presence of probabilists. However, only Borel, Fréchet, or Darmois were mentioned in those dissertations or were members of the jury, as Lévy had no university position.

Let us make a few comments on the six PhDs defended at the IHP. As mentioned earlier, Fréchet taught his own research area, using Fredholm's integral equations in order to study the asymptotic law for Markov processes with continuous-space states. This kind of probabilistic translation of analytical concepts and results is clearly visible in Fortet's dissertation. Fortet started their work in 1934 and developed a study of Markov chains with a denumerable space of states by means of Riesz's theory of operators. In 1935, Wolfgang Dœblin started working on Markov chains with a new approach through the study of trajectories, an approach partly related to their exchanges with Bohuslav Hostinský [28] and with Soviet mathematicians that Fréchet met during his trip to Moscow in 1935 [29].

Fréchet was interested in Markov chains partly as a model allowing to go beyond the hypothesis of independence in asymptotic theorems. Michel Loève's PhD, which he started in 1936, is explicitly an attempt to obtain a law of large numbers and central limit theorem without the independence assumption using conditional expectation. This work, defended in 1941, presents an original synthesis of the work of Paul Lévy on sums of random variables with Soviet works by Glivenko, Kolmogorov, and Bernstein.

As already mentioned, Daniel Dugué and Gustave Malécot defended a PhD in statistics under the guidance of Georges Darmois. Dugué started working in 1935 on the theory of estimators and defended in 1937 a PhD in which statistical estimation is based on probabilistic concepts. That same year, Malécot started to work on another side of Fisher's work—genetics—and presented a stochastic model of heredity.

Jean Ville's PhD was a most original work. Ville started working on his PhD in 1934 after a sojourn in Vienna where he made contact with Menger's circle of logical empiricism [30]. On this occasion, Ville discovered von Mises' theory of collectives, which he proposed to criticize. Fréchet accepted, reluctantly, Ville's PhD topic, which he considered as more philosophical than mathematical. Ville commented later that Borel supported the project but asked them to develop the mathematical side. The PhD was defended in 1939. To develop their argument, Ville invented the concept of martingale based on their reflections on the embryo of game theory that Borel had begun to set up in the 1920s [31] (observe that besides that, Ville provided the first proof of the 1926 von Neumann's minmax theorem by means of convex analysis [32]).

Now, some words of conclusion. As we have seen, during the 1930s, the IHP had become an impressive institutional tool for producing new probabilistic knowledge and training young mathematicians. If the works of the first generation were connected to the research of the masters Fréchet, Darmois, and Borel, the PhD they produced also opened new directions based on their contacts with international research. Those students were particularly involved in the collective organization and dynamics of research. Borel's utopia at the beginning of the 1920s finally came to fruition in the second half of the 1930s.

The outbreak of World War II suddenly stopped this promising development. The period of the occupation interrupted international communications. Although the building still housed the department of mathematics, the invited talks of the IHP stopped. Moreover the first generation was dispersed, in France or abroad. Wolfgang Dœblin tragically died in June 1940. Borel retired in 1941 and Fréchet became the new owner of the chair, while Fortet was a substitute to Darmois, who was in exile, first in London and then in Algiers. The situation of the institute after the war was therefore entirely different and belongs to another historical period.

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