

Article

Quantum Brain Dynamics and Holography

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Abstract: We describe non-equilibrium quantum brain dynamics (QBD) for the breakdown of symmetry and propose the possibility of hologram memory based on QBD. We begin with the Lagrangian density of QBD with water rotational dipole fields and photon fields in 3 + 1 dimensions, and derive time evolution equations of coherent fields. We show a solution for super-radiance derived from the Lagrangian of QBD and propose a scenario of holography by the interference of two incident super-radiant waves. We investigate the time evolution of coherent dipole fields and photon fields in the presence of quantum fluctuations in numerical simulations. We find that the breakdown of the rotational symmetry of dipoles occurs in inverted populations for incoherent dipoles. We show how the waveforms of holograms with interference patterns evolve over time in an inverted population for incoherent dipoles. The optical information of hologram memory can be transferred to the whole brain during information processing. The integration of holography and QBD will provide us with a prospective approach in memory formation.

Keywords: quantum brain dynamics; holography; breakdown of symmetry; super-radiance; memory



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1. Introduction

What is the physical mechanism of memory in a brain? To date, the physical mechanisms for storing memories in the human brain have not been fully ascertained. Learning and memory are commonly understood as mechanistically correlated with synaptic plasticity involving brain neurons, affecting cognitive activities through neuronal networks, and supported by so-called “long-term potentiation” (LTP). This is an experimental paradigm stating that brief repetitive pre-synaptic stimulation causes prolonged post-synaptic sensitivity, e.g., to glutamate. Glutamate receptor binding opens membrane calcium channels, and causes calcium ion fluxes into dendritic spines in neurons, shafts and cell bodies which then result in various downstream effects including the activation and phosphorylation of the calcium-calmodulin kinase II (CaMKII) holoenzyme. A computational model demonstrating how this mechanism can lead to the development of encoded phosphorylation patterns in neuronal microtubules has been proposed by Craddock et al. [1]. However, there has been no experimental validation of the model to date. Moreover, there could be different mechanisms involved in short- and long-term memory formation, each of which would rely on the different stability of the encoded state and its erasure. Several properties of memory storage in the human brain differ from the way that computer memory works [2]. The human brain memorizes information by sequences of patterns. For example, it recalls a melody or a song forward in sequences, and not backward in sequences. It recalls memorized patterns auto-associatively, that is, it is able to recall information from a given sample. The human brain memorizes patterns in an invariant form [3,4]. For example, the motions generated when drinking tea in a cup are different each time; however, we recognize them as a single motion. When we see our

friend's face, we can recognize their face every time even from various angles and distances, with shades, and with various facial expressions. Over visual cortical processing, we find that the invariant representations of objects with respect to translation, size, and view [5–7]. We process patterns in a hierarchical form, namely $V_1, V_2, \dots, V_5$, for visual processing. Furthermore, the brain's memory is robust against damages to parts of a brain and is diffused in a non-local storage manner. We represent the equipotentiality and the mass action principle. Equipotentiality refers to the property that memory is recalled by the other undamaged regions even when local regions in a brain are damaged [8]. The mass action principle represents whether each memory is lost depends on the severity of extensive lesions in a brain [9].

Quantum field theory is a powerful tool for describing a variety of physical phenomena in cosmology, elementary particle physics, nuclear physics, condensed matter physics, to name but a few areas of application. It can be also applied to biological systems, especially brain dynamics. Quantum brain dynamics (QBD) is a hypothesis formulated to describe the physical mechanism of memory in a brain [10,11]. It originates from the monumental work of Ricciardi and Umezawa in 1967 [12]. Non-local memory storage, the mechanism of recall, and stability of memory were discussed in detail in [13,14]. According to the QBD hypothesis, a brain is a mixed system of classical neurons and quantum degrees of freedom. In 1968, Fröhlich suggested that Bose–Einstein condensation might occur in biological systems at the cellular level and lead to quantum coherence with long-range correlations (a so-called Fröhlich condensation phenomenon). This was determined to be theoretically possible if the frequencies of oscillating molecular dipoles are within a narrow range around the resonant frequency and the coupling constants for their mutual interactions and their interactions with the heat bath and the energy pump are sufficiently large [15,16]. In 1976, Davydov and Kislukha proposed a theory of solitary waves propagating along the alpha-helical structures of DNA and protein chains, whose result was a stable localized propagating wave of coupled exciton–phonon interactions referred to as the Davydov soliton [17]. The Fröhlich condensation and the Davydov soliton emerged as static and dynamical properties, respectively, in a non-linear Schrödinger equation of an equivalent Hamiltonian [18]. In the 1980s, Del Giudice et al. investigated a quantum field theoretical approach to biological systems [19–22]. Specifically, they introduced the quantum field theory of water rotational degrees of freedom and photons [21]. By introducing the rotational degrees of freedom of quantum water electric dipole fields interacting with photon fields, the laser-like behaviors in quantum electrodynamics of water dipoles and photons were studied. In the 1990s, Jibu and Yasue proposed a set of concrete physical degrees of freedom in QBD, namely water electric dipole fields and photon fields [10,23–27]. Memory in QBD is envisaged as ordered patterns of dipoles aligned in the same direction owing to the breakdown of rotational symmetry. The vacua of these aligned dipoles are maintained by long-range correlations owing to the symmetry breakdown in the system of the Nambu–Goldstone quanta. Vitiello suggested that a huge capacity of memory in QBD is realized by the squeezed coherent states of Nambu–Goldstone bosons in open systems [28]. Meanwhile, Pribram proposed the holographic brain theory to describe non-local memory storage and perception [29,30]. Holography is a technique for recording and reconstructing three-dimensional (3D) images achieved by electromagnetic wave interference [31,32]. Holography has several properties of equipotentiality and mass action because the recorded information in a hologram is robust against the damaged of parts in a hologram, and the non-damaged parts can reconstruct the recorded information.

We aimed to describe non-equilibrium QBD in the presence of quantum fluctuations in $3 + 1$ dimensions and to show how ordered patterns of holograms evolve over time. We propose the integration of the QBD and holographic brain theory. We adopt Schrodinger-like equations for coherent dipole fields and Klein–Gordon equations for coherent electric fields in non-equilibrium QBD in $3 + 1$ dimensions. We consider water dipoles and photons around microtubules in the brain as a candidate of such a system which may generate super-radiance, inducing a flash of light by the cooperative spontaneous emission of radiation.

The interference of two super-radiant waves induces holographic memory with optical interference patterns. We find that ordered the patterns of aligned dipoles in memory are amplified from their initial patterns by quantum fluctuations on picosecond (ps) time scales. Incoherent dipoles in the first excited state of quantum fluctuations amplify the initial ordered pattern in memory printing. The properties in open systems are significant in memory storage because the flow of dipole fields from upstream to downstream induced by external incoherent photons exciting dipoles from the ground state to first excited states is necessary for the maintenance of aligned dipoles. The optical information of hologram memory might be transferred to a whole brain using parallel information processing. Holograms might propagate in a hierarchical manner in the brain's cortex area with optical information processing, and memory can be stored in an invariant form of memory. The auto-associative property of holograms can be useful in describing memory in a brain. If the sequences resulting from super-radiant emission are in the forward direction owing to sequences of neuron firings involving super-radiant emission, sequential properties can appear in the integration of QBD and holography. Holography and QBD will provide a promising approach regarding the study of brain memory.

This paper is organized as follows. In Section 2, we introduce the Lagrangian density in QBD in $3 + 1$ dimensions, and derive time evolution equations for coherent fields. In Section 3, we show a solution of super-radiance adopted in holography and introduce a scenario of hologram memory. In Section 4, we show numerical simulations of the breakdown of symmetry and for dynamical hologram memory. In Section 5, we discuss our results. In Section 6, we provide concluding remarks. We adopt the natural unit, where the light speed and the Planck constant divided by 2π are set to unity. We adopt the metric tensor $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ with space-time subscript $\mu, \nu = 0, 1, 2, 3$.

2. Lagrangian Density and Time Evolution Equations

In this section, we introduce the Lagrangian density in quantum brain dynamics (QBD) and show time evolution equations for coherent fields. The flowchart of the derivation in this section is depicted in Figure 1. We begin with the Lagrangian density for QBD in $3 + 1$ dimensions by referring [21,33]. Then, we derive a two-particle-irreducible effective action for the expectation values of quantum fields (coherent fields) and those of quantum fluctuations [34–36]. Finally, we derive the time evolution equations for coherent fields (Schrödinger-like eqs. and the Klein–Gordon eq.) and those for quantum fluctuations called the Kadanoff–Baym equations [37–39] by differentiating the effective action with expectation values. The derivations of terms with quantum fluctuations in the Klein–Gordon equation are given in the Appendix A.

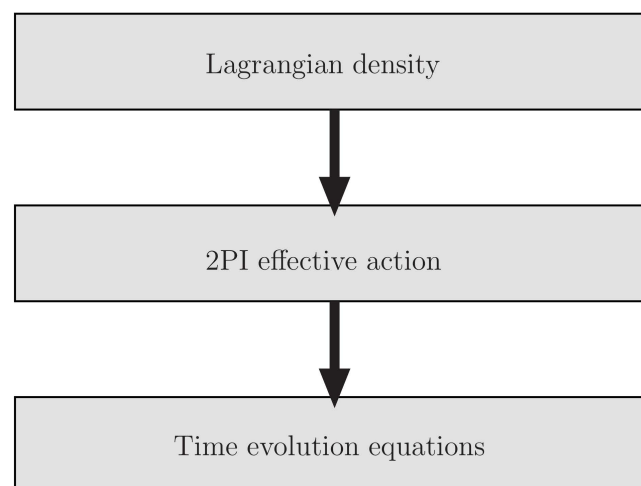


Figure 1. The flowchart of derivation.

We show the variables and constants in Table 1.

Table 1. Variables and constants.

ψ_α	dipole fields for first excited states $\alpha = 0, \pm 1$
ψ_s	dipole field for the ground state
$\bar{\psi}_\alpha$	expectation value of ψ_α
$\bar{\psi}_s$	expectation value of ψ_s
A^μ	background photon fields with $\mu = 0, 1, 2, 3$
a^μ	fluctuations of photon fields
$\delta\psi = \psi - \bar{\psi}$	fluctuations of dipole fields
I (constant)	moment of inertia
$2ed_e$ (constant)	dipole moment
N/V (constant)	number density of water dipoles
μ_i	dipole moment density with $i = 1, 2, 3$
$F^{\mu\nu}[A] = \partial^\mu A^\nu - \partial^\nu A^\mu$	electromagnetic tensor or field strength
$E_i = -F^{0i}$	electric field $i = 1, 2, 3$
Δ	4×4 matrix of Green's functions for quantum fluctuations of dipole fields
D	Green's functions for quantum fluctuations of photon fields a^i with $i = 1, 2, 3$
Δ_0^{-1}	inverse of Green's functions Δ without self-energy
D_0^{-1}	inverse of Green's functions D without self-energy
P_3	time-derivative of μ_3 multiplied by moment of inertia I
$Z_0 = \bar{\psi}_0 ^2 - \bar{\psi}_s ^2$	population difference between first excited state and the ground state
$\mathcal{Z} = Z_0/(N/V)$	population difference divided by N/V
$\mathcal{M}_3 = \mu_3/(ed_e N/V)$	dipole moment density in the x^3 direction divided by $ed_e N/V$
$\mathcal{P}_3 = P_3/(ed_e N/V)$	time derivative of $\mathcal{M}_3$ multiplied by I or P_3 divided by $ed_e N/V$

The Lagrangian density for QBD in $3 + 1$ dimensions [21,33] is given by

$$\begin{aligned}
 \mathcal{L}[\Psi^*(x, \theta, \varphi), \Psi(x, \theta, \varphi), A(x), a(x)] = & -\frac{1}{4}F^{\mu\nu}[A+a]F_{\mu\nu}[A+a] - \frac{(\partial^\mu a_\mu)^2}{2\alpha_1} \\
 & + \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta \left[i\Psi^* \frac{\partial}{\partial x^0} \Psi + \frac{1}{2m} \Psi^* \nabla_i^2 \Psi \right. \\
 & + \frac{1}{2I} \Psi^* \left(\frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \varphi^2} + \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) \right) \Psi \Big] \\
 & - 2ed_e \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta \Psi^* u^i \Psi F^{0i}[A+a], \quad (1)
 \end{aligned}$$

where the electric dipole field is denoted by $\Psi(x, \theta, \varphi)$ and its complex conjugate is denoted by $\Psi^*(x, \theta, \varphi)$ in polar coordinates denoted by (θ, φ) , the background photon fields are

denoted by A^μ in the background field method [40–43], and its quantum fluctuations are denoted by a^μ , the field strength is denoted by $F^{\mu\nu}[A] = \partial^\mu A^\nu - \partial^\nu A^\mu$, the gauge fixing parameter is denoted by $\alpha_1 = 1$, the mass of dipoles is denoted by m , the moment of inertia of dipoles I is denoted by $1/I = 4 \text{ meV}$ (the average of the moment of inertia in three spatial dimensions of water dipoles), the dipole moment is denoted by $2ed_e$ with elementary charge $e = 0.3$ and $d_e = 0.2$ and the orientation of dipoles is denoted by $u^i = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$.

We adopt the two-energy-level approximation for angular momentum squared $\mathbf{I}^2 = -(\frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial}{\partial \theta}))$. Electric dipole fields are expanded by the dipole field for the ground state $\psi_s(x)$ and the dipole fields for the first excited states $\psi_\alpha(x)$ as

$$\Psi(x, \theta, \varphi) = \psi_s(x) Y_{00}(\theta, \varphi) + \sum_{\alpha=0, \pm 1} \psi_\alpha(x) Y_{1\alpha}(\theta, \varphi), \quad (2)$$

$$\Psi^*(x, \theta, \varphi) = \psi_s^*(x) Y_{00}^*(\theta, \varphi) + \sum_{\alpha=0, \pm 1} \psi_\alpha^*(x) Y_{1\alpha}^*(\theta, \varphi), \quad (3)$$

with the spherical harmonics for the ground state $Y_{00}(\theta, \varphi)$ (the eigenvalue $\mathbf{I}^2 = 0$) and the first excited states $Y_{1\alpha}(\theta, \varphi)$ with $\alpha = 0, \pm 1$ (eigenvalues $\mathbf{I}^2 = 2$) given by

$$Y_{00}(\theta, \varphi) = \frac{1}{\sqrt{4\pi}}, \quad (4)$$

$$Y_{1\pm 1}(\theta, \varphi) = \mp i \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\varphi}, \quad Y_{10}(\theta, \varphi) = i \sqrt{\frac{3}{4\pi}} \cos \theta. \quad (5)$$

We can then rewrite the fourth term in Equation (1) with the electric fields $E_i = -F^{0i}$ ($i = 1, 2, 3$) by

$$2ed_e \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta \Psi^* u^i \Psi E_i = \mu_i(x) E_i(x), \quad (6)$$

where μ_i ($i = 1, 2, 3$) are defined by

$$\mu_1 = \frac{2ed_e i}{\sqrt{6}} (\psi_s^* \psi_{-1} - \psi_s^* \psi_1 + \psi_1^* \psi_s - \psi_{-1}^* \psi_s), \quad (7)$$

$$\mu_2 = \frac{2ed_e}{\sqrt{6}} (\psi_s^* \psi_{-1} + \psi_s^* \psi_1 + \psi_1^* \psi_s + \psi_{-1}^* \psi_s), \quad (8)$$

$$\mu_3 = \frac{2ed_e i}{\sqrt{3}} (\psi_s^* \psi_0 - \psi_0^* \psi_s), \quad (9)$$

and we can then rewrite

$$\begin{aligned} \mu_i(x) E_i(x) &= \frac{2ed_e}{\sqrt{6}} \left[(iE_1 + E_2)(\psi_s^* \psi_{-1} + \psi_1^* \psi_s) + (-iE_1 + E_2)(\psi_s^* \psi_1 + \psi_{-1}^* \psi_s) \right. \\ &\quad \left. + \sqrt{2}iE_3(\psi_s^* \psi_0 - \psi_0^* \psi_s) \right] \\ &= \frac{2ed_e}{\sqrt{6}} \sum_{\alpha=0, \pm 1} \left[(-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3) \psi_s^* \psi_\alpha \right. \\ &\quad \left. + (i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3) \psi_\alpha^* \psi_s \right]. \end{aligned} \quad (10)$$

We then find that the Lagrangian density is rewritten by

$$\begin{aligned}
\mathcal{L}[\Psi^*(x, \theta, \varphi), \Psi(x, \theta, \varphi), A(x), a(x)] &\rightarrow \mathcal{L}[\psi^*(x), \psi(x), A, a] \\
&= -\frac{1}{4}F^{\mu\nu}[A+a]F_{\mu\nu}[A+a] - \frac{(\partial^\mu a_\mu)^2}{2\alpha_1} \\
&\quad + \psi_s^* \left[i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} \right] \psi_s + \sum_{\alpha=0, \pm 1} \psi_\alpha^* \left[i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I} \right] \psi_\alpha \\
&\quad + \frac{2ed_e}{\sqrt{6}} \sum_{\alpha=0, \pm 1} \left[\left(-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right) \psi_s^* \psi_\alpha \right. \\
&\quad \left. + \left(i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right) \psi_\alpha^* \psi_s \right]. \tag{11}
\end{aligned}$$

We then show a two-particle-irreducible (2PI) effective action [34,35] for the above Lagrangian density in the closed-time path (CTP) $\mathcal{C}$ formalism [44,45]. We set the gauge fixing $a^0 = 0$ in the path integral in CTP. The 2PI effective action in $\mathcal{C}$ with path 1 from $-\infty$ to ∞ and path 2 from ∞ to $-\infty$ is given by

$$\begin{aligned}
\Gamma_{2\text{PI}}[A, \bar{a}^i, \bar{\psi}, \bar{\psi}^*, \Delta, D] &= \int_{\mathcal{C}} d^4x \left[-\frac{1}{4}F^{\mu\nu}[A+\bar{a}]F_{\mu\nu}[A+\bar{a}] - \frac{(\partial^i \bar{a}_i)^2}{2} \right. \\
&\quad + \bar{\psi}_s^* \left(i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} \right) \bar{\psi}_s + \sum_{\alpha=0, \pm 1} \bar{\psi}_\alpha^* \left(i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I} \right) \bar{\psi}_\alpha \\
&\quad + \frac{2ed_e}{\sqrt{6}} \sum_{\alpha=0, \pm 1} \left[\left(-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right) \bar{\psi}_s^* \bar{\psi}_\alpha \right. \\
&\quad \left. + \left(i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right) \bar{\psi}_\alpha^* \bar{\psi}_s \right] \\
&\quad + i\text{Tr} \ln \Delta^{-1} + i\text{Tr} \Delta_0^{-1} \Delta \\
&\quad \left. + \frac{i}{2} \text{Tr} \ln D^{-1} + \frac{i}{2} \text{Tr} D_0^{-1} D + \frac{\Gamma_2[\Delta, D]}{2} \right], \tag{12}
\end{aligned}$$

where the bar represents the expectation value of coherent fields, $i\Delta_0^{-1}(x, y)$ and $iD_{0,ij}^{-1}(x, y)$ are defined by

$$\begin{aligned}
i\Delta_0^{-1}(x, y) &\equiv \frac{\delta^2 \int_w \mathcal{L}}{\delta \psi^*(x) \delta \psi(y)} \Big|_{\bar{a}=0} \\
&= \begin{bmatrix} i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} & \frac{2ed_e}{\sqrt{6}}(iE_1 + E_2) & \frac{2ed_e}{\sqrt{3}}iE_3 & \frac{2ed_e}{\sqrt{6}}(-iE_1 + E_2) \\ \frac{2ed_e}{\sqrt{6}}(-iE_1 + E_2) & i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I} & 0 & 0 \\ \frac{-2ed_e}{\sqrt{3}}iE_3 & 0 & i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I} & 0 \\ \frac{2ed_e}{\sqrt{6}}(iE_1 + E_2) & 0 & 0 & i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I} \end{bmatrix} \\
&\quad \times \delta_{\mathcal{C}}^4(x - y), \tag{13}
\end{aligned}$$

for $(s, -1, 0, 1)$ components and

$$\begin{aligned}
iD_{0,ij}^{-1}(x, y) &\equiv \frac{\delta^2 \int_w \mathcal{L}}{\delta a^i(x) \delta a^j(y)} \Big|_{\bar{a}=0} \\
&= -\delta_{ij} \partial_x^2 \delta_{\mathcal{C}}^{d+1}(x - y), \tag{14}
\end{aligned}$$

and the Green's functions $\Delta(x, y)$ for quantum fluctuations of incoherent dipoles and $D_{ij}(x, y)$ for quantum fluctuations of incoherent photons are given by

$$\Delta(x, y) = \begin{bmatrix} \Delta_{ss}(x, y) & \Delta_{s-1}(x, y) & \Delta_{s0}(x, y) & \Delta_{s1}(x, y) \\ \Delta_{-1s}(x, y) & \Delta_{-1-1}(x, y) & \Delta_{-10}(x, y) & \Delta_{-11}(x, y) \\ \Delta_{0s}(x, y) & \Delta_{0-1}(x, y) & \Delta_{00}(x, y) & \Delta_{01}(x, y) \\ \Delta_{1s}(x, y) & \Delta_{1-1}(x, y) & \Delta_{10}(x, y) & \Delta_{11}(x, y) \end{bmatrix}, \quad (15)$$

with $\Delta_{s-1}(x, y) = \langle T_C \delta\psi_s(x) \delta\psi_{-1}^*(y) \rangle$ with $\delta\psi_s = \psi_s - \bar{\psi}_s$ or the 2×2 matrix notation in $\mathcal{C}$

$$\begin{aligned} \Delta_{s-1}(x, y) &= \begin{bmatrix} \Delta_{s-1}^{11}(x, y) & \Delta_{s-1}^{12}(x, y) \\ \Delta_{s-1}^{21}(x, y) & \Delta_{s-1}^{22}(x, y) \end{bmatrix} \\ &= \begin{bmatrix} \langle T(\delta\psi_s(x) \delta\psi_{-1}^*(y)) \rangle & \langle \delta\psi_{-1}^*(y) \delta\psi_s(x) \rangle \\ \langle \delta\psi_s(x) \delta\psi_{-1}^*(y) \rangle & \langle \bar{T}(\delta\psi_s(x) \delta\psi_{-1}^*(y)) \rangle \end{bmatrix}, \end{aligned} \quad (16)$$

with time-order product T and anti-time-order product $\bar{T}$ and

$$D_{ij}(x, y) = \langle T_C a_i(x) a_j(y) \rangle. \quad (17)$$

Time evolution equations are derived by differentiating 2PI effective action by variables $\bar{a}^i$, $\bar{\psi}^{(*)}$, Δ and D as

$$\left. \frac{\delta \Gamma_{2PI}}{\delta \bar{a}^i} \right|_{\bar{a}=0} = \left. \frac{\delta \Gamma_{2PI}}{\delta A^i} \right|_{\bar{a}=0} = 0, \quad (18)$$

$$\left. \frac{\delta \Gamma_{2PI}}{\delta \bar{\psi}_{s,-1,0,1}^{(*)}} \right|_{\bar{a}=0} = 0. \quad (19)$$

$$\left. \frac{\delta \Gamma_{2PI}}{\delta \Delta} \right|_{\bar{a}=0} = 0, \quad (20)$$

$$\left. \frac{\delta \Gamma_{2PI}}{\delta D} \right|_{\bar{a}=0} = 0, \quad (21)$$

where Equations (20) and (21) are the Kadanoff–Baym equations, time evolution equations of quantum fluctuations, or Green's functions. Equation (18) is written by

$$\partial^\mu F_{\mu i}[A] = J_i, \quad (22)$$

with the current terms

$$J_1 = -\frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^0} \sum_{\alpha=0,\pm 1} [-i\alpha(\bar{\psi}_s^* \bar{\psi}_\alpha - \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} - \Delta_{s\alpha})], \quad (23)$$

$$J_2 = -\frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^0} \sum_{\alpha=0,\pm 1} [|\alpha|(\bar{\psi}_s^* \bar{\psi}_\alpha + \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} + \Delta_{s\alpha})], \quad (24)$$

$$J_3 = -\frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^0} \sum_{\alpha=0,\pm 1} \left[\sqrt{2}i(1 - |\alpha|)(\bar{\psi}_s^* \bar{\psi}_\alpha - \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} - \Delta_{s\alpha}) \right]. \quad (25)$$

When we introduce J_0 as

$$J_0 = -\frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^1} \sum_{\alpha=0,\pm 1} [-i\alpha(\bar{\psi}_s^* \bar{\psi}_\alpha - \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} - \Delta_{s\alpha})] \\ - \frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^2} \sum_{\alpha=0,\pm 1} [|\alpha|(\bar{\psi}_s^* \bar{\psi}_\alpha + \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} + \Delta_{s\alpha})] \\ - \frac{2ed_e}{\sqrt{6}} \frac{\partial}{\partial x^3} \sum_{\alpha=0,\pm 1} [\sqrt{2}i(1 - |\alpha|)(\bar{\psi}_s^* \bar{\psi}_\alpha - \bar{\psi}_\alpha^* \bar{\psi}_s + \Delta_{\alpha s} - \Delta_{s\alpha})]. \quad (26)$$

we can derive

$$\partial^\mu F_{\mu 0}[A] = J_0, \quad (27)$$

owing to the conservation law $\partial^0 J_0 = \partial_i J_i = \partial_i \partial^\mu F_{\mu i} = \partial^\nu \partial^\mu F_{\mu\nu} - \partial^i \partial^\mu F_{\mu i} = \partial^0 \partial^\mu F_{\mu 0}$ with the identity $\partial^\nu \partial^\mu F_{\mu\nu} = 0$ and integration with time x^0 . Time-independent terms interpreted as initial conditions are set to be zero. The Schrödinger-like equations (Equation (19)) are given by

$$\left(i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m}\right) \bar{\psi}_s + \frac{2ed_e i}{\sqrt{6}} \sum_{\alpha=0,\pm 1} [-\alpha(E_1 + i\alpha E_2) + \sqrt{2}(1 - |\alpha|)E_3] \bar{\psi}_\alpha = 0, \quad (28)$$

$$\left(i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{I}\right) \bar{\psi}_\alpha + \frac{2ed_e i}{\sqrt{6}} [\alpha(E_1 - i\alpha E_2) - \sqrt{2}(1 - |\alpha|)E_3] \bar{\psi}_s = 0, \quad (29)$$

and their complex conjugates. Using Equations (28) and (29), we can show the population conservation for coherent dipoles as

$$\frac{\partial}{\partial x^0} \left(\bar{\psi}_s^* \bar{\psi}_s + \sum_{\alpha=0,\pm 1} \bar{\psi}_\alpha^* \bar{\psi}_\alpha \right) \\ + \frac{1}{2im} \nabla_i \cdot \left[\bar{\psi}_s^* \nabla_i \bar{\psi}_s - \bar{\psi}_s \nabla_i \bar{\psi}_s^* + \sum_{\alpha=0,\pm 1} (\bar{\psi}_\alpha^* \nabla_i \bar{\psi}_\alpha - \bar{\psi}_\alpha \nabla_i \bar{\psi}_\alpha^*) \right] = 0. \quad (30)$$

We shall consider the case $E_1 = E_2 = 0$ and $\psi_{\pm 1}^{(*)} = 0$ to be spatially homogeneous in the x^3 direction. Using Equations (28) and (29) and assuming $\nabla_i^2/2m \ll 1/I = 4 \text{ meV}$ where the energy of the translational motion of water dipoles is smaller than that of rotational motion in fixed positions of water molecules, we can derive relations

$$\partial^0 \bar{\mu}_3 = \frac{P_3}{I}, \quad (31)$$

$$\partial^0 P_3 = -\frac{\bar{\mu}_3}{I} - \frac{8(ed_e)^2}{3} Z_0 E_3, \quad (32)$$

$$\partial^0 Z_0 = 2P_3 E_3. \quad (33)$$

for variables, the dipole moment density in the x^3 direction $\bar{\mu}_3 = \frac{2ed_e i}{\sqrt{3}} (\bar{\psi}_0 \bar{\psi}_s^* - \bar{\psi}_s \bar{\psi}_0^*)$, the population difference of coherent dipole fields $Z_0 = |\bar{\psi}_0|^2 - |\bar{\psi}_s|^2$, and the time derivative of the dipole moment density $P_3 = \frac{2ed_e}{\sqrt{3}} (\bar{\psi}_0 \bar{\psi}_s^* + \bar{\psi}_s \bar{\psi}_0^*)$. Differentiating Equation (22) for $i = 3$ by time x^0 , the time evolution equation of the coherent electric field $E_3 = F_{03} = \partial_0 A_3 - \partial_3 A_0$ with $A_0 = 0$ (the solution of Equation (27)) is written by

$$[(\partial^0)^2 - (\partial^1)^2 - (\partial^2)^2] E_3 = \frac{\bar{\mu}_3}{I^2} + \frac{8(ed_e)^2}{3I} Z_0 E_3 + \frac{8(ed_e)^2}{3I} E_3 \int \frac{d^4 p}{(2\pi)^4} (F_{00}(x, p) - F_{ss}(x, p)), \quad (34)$$

with $\int_p F_{00}(x, p) = \frac{1}{2}(\Delta_{00}^{21}(x, x) + \Delta_{00}^{12}(x, x))$ and $\int_p F_{ss}(x, p) = \frac{1}{2}(\Delta_{ss}^{21}(x, x) + \Delta_{ss}^{12}(x, x))$. The third term on right-hand side can be derived from the Kadanoff–Baym equations in Equation (20), as shown in Appendix A.

We shall adopt the scaling of variables $\mathcal{Z} \equiv Z_0/(N/V)$, $\mathcal{M}_3 \equiv \bar{\mu}_3/(ed_e N/V)$, $\mathcal{P}_3 \equiv P_3/(ed_e N/V)$, and $\mathcal{E}_3 \equiv ed_e I E_3$ by the number of the density of water dipoles $N/V = 3.3 \times 10^{28} \text{ m}^{-3}$ and the dipole moment divided by 2, that is ed_e with $e = 0.3$ and $d_e = 0.2$. Using these parameters, we can derive

$$\sqrt{\frac{8(ed_e)^2}{3} \cdot \frac{N}{V} \cdot I} \times \frac{1}{I} \simeq \frac{13}{I}, \quad (35)$$

which corresponds to the frequency of the collective oscillation of dipoles $\Omega = \frac{13}{I}$. We can then derive

$$\partial^0 \mathcal{M}_3 = \frac{\mathcal{P}_3}{I}, \quad (36)$$

$$\partial^0 \mathcal{P}_3 = -\frac{\mathcal{M}_3}{I} - \frac{8}{3I} \mathcal{Z} \mathcal{E}_3, \quad (37)$$

$$\partial^0 \mathcal{Z} = \frac{2}{I} \mathcal{P}_3 \mathcal{E}_3, \quad (38)$$

from Equations (31)–(33), and

$$\left[(\partial^0)^2 - (\partial^1)^2 - (\partial^2)^2 \right] \mathcal{E}_3 = \frac{3\Omega^2}{8} \mathcal{M}_3 + \Omega^2 \mathcal{Z} \mathcal{E}_3 + \Omega^2 f_2 \mathcal{E}_3, \quad (39)$$

from Equation (34) with the term of quantum fluctuations $f_2 \equiv \int \frac{d^4 p}{(2\pi)^4} (F_{00}(x, p) - F_{ss}(x, p)) / (N/V)$. The magnetic fields $\mathcal{B}_1$ and $\mathcal{B}_2$ with scaling of $ed_e I$ obey

$$\partial_0 \mathcal{B}_1 = -\partial_2 \mathcal{E}_3, \quad (40)$$

$$\partial_0 \mathcal{B}_2 = \partial_1 \mathcal{E}_3, \quad (41)$$

by the identity relation of field strength $F^{\mu\nu}$. The scaled total coherent population squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)$ is conserved

$$\partial_0 \left[\mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2) \right] = 0, \quad (42)$$

as shown from Equations (36)–(38) even in the presence of terms of quantum fluctuations. The scaled energy ϵ_{tot} is written by

$$\epsilon_{\text{tot}} = \frac{1}{2} \left[\mathcal{E}_3^2 + \mathcal{B}_1^2 + \mathcal{B}_2^2 \right] + \frac{3 \cdot 13^2}{16} \mathcal{Z}. \quad (43)$$

The integration of ϵ_{tot} with 3D spatial coordinates is shown to be conserved from Equations (22) and (33) when we neglect the terms of quantum fluctuations.

3. Scenario of Hologram Memory

In this section, we introduce a scenario of hologram memory.

3.1. Super-Radiance

We begin with time evolution equations for coherent dipoles and electric fields, and show a solution of super-radiant emission. We derive the super-radiance solution by refer-

ring to [46,47]. We consider the case $E_1 = E_2 = 0$ and $\psi_{\pm 1}^{(*)} = 0$ as spatially homogeneous in the x^2 and x^3 directions. Time evolution equations are given by

$$\left[(\partial^0)^2 - (\partial^1)^2 \right] E_3 = -\frac{2ed_e i}{\sqrt{3}} (\partial^0)^2 (\bar{\psi}_0 \bar{\psi}_s^* - \bar{\psi}_s \bar{\psi}_0^* + \Delta_{0s} - \Delta_{s0}), \quad (44)$$

$$\partial^0 Z_0 = +\frac{4ed_e}{\sqrt{3}} (\bar{\psi}_0 \bar{\psi}_s^* + \bar{\psi}_s \bar{\psi}_0^*) E_3, \quad (45)$$

$$\partial^0 (\bar{\psi}_0 \bar{\psi}_s^*) = -\frac{i}{I} \bar{\psi}_0 \bar{\psi}_s^* - \frac{2ed_e}{\sqrt{3}} Z_0 E_3, \quad (46)$$

with $E_i = \partial_0 A_i$ and, $Z_0 \equiv |\bar{\psi}_0|^2 - |\bar{\psi}_s|^2$. Equation (44) is derived by differentiating Equation (22) with $i = 3$ by time x^0 . Equations (45) and (46) are derived from the Schrödinger-like Equations (28) and (29).

We set $\omega = k_0 = \frac{1}{I}$ which is the energy difference in the two-energy-level approximation. Subsequently, we denote E_3 and $\bar{\psi}_0 \bar{\psi}_s^*$ by

$$E_3(x^0, x^1) = \frac{1}{2} \epsilon_3(x^0, x^1) e^{-i(\omega x^0 - k_0 x^1)} + (c.c.), \quad (47)$$

$$\bar{\psi}_0 \bar{\psi}_s^* = \frac{1}{2} R_0(x^0) e^{-i(\omega x^0 - k_0 x^1)}. \quad (48)$$

Here, the rotating-wave approximation where we neglect the non-resonant terms $\propto e^{\pm 2i\omega x^0}$ and the quantum fluctuations (Δ_{0s} and Δ_{s0}) is adopted. We shall neglect higher-order derivatives

$$|\partial^0 \epsilon_3| \ll |\omega \epsilon_3|, \quad |\partial^1 \epsilon_3| \ll |k_0 \epsilon_3|, \quad |\partial^0 R_0| \ll |\omega R_0|. \quad (49)$$

We can then derive the following Maxwell–Bloch equations from Equations (44)–(46),

$$\partial^0 \epsilon_3 + \partial^1 \epsilon_3 = -\frac{ed_e k_0}{\sqrt{3}} R_0, \quad (50)$$

$$\partial^0 Z_0 = \frac{ed_e}{\sqrt{3}} (\epsilon_3 R_0^* + \epsilon_3^* R_0), \quad (51)$$

$$\partial^0 R_0 = -\frac{2ed_e}{\sqrt{3}} Z_0 \epsilon_3. \quad (52)$$

We set $\epsilon_3 \rightarrow -\epsilon_3$, and assume that ϵ_3 and R_0 are real variables, then the above equations are rewritten by

$$\partial^0 \epsilon_3 + \partial^1 \epsilon_3 = \frac{ed_e k_0}{\sqrt{3}} R_0, \quad (53)$$

$$\partial^0 Z_0 = -\frac{2ed_e}{\sqrt{3}} \epsilon_3 R_0, \quad (54)$$

$$\partial^0 R_0 = +\frac{2ed_e}{\sqrt{3}} Z_0 \epsilon_3. \quad (55)$$

Equations (54) and (55) satisfy the dipole number conservation

$$\partial^0 (Z_0^2 + R_0^2) = \partial^0 \left[(|\bar{\psi}_0|^2 + |\bar{\psi}_s|^2)^2 \right] = 0, \quad (56)$$

and Equations (53) and (54) satisfy the energy conservation (by neglecting the derivative ∂^1 terms)

$$\partial^0 \left[\frac{1}{2} \epsilon_3^2 + \frac{1}{2} k_0 Z_0 + \frac{1}{2} k_0 R_0 \right] = 0, \quad (57)$$

with $B \equiv |\bar{\psi}_0|^2 + |\bar{\psi}_s|^2$.

Since we can derive $\partial^0(Z_0^2 + R_0^2) = \partial^0(|\bar{\psi}_1|^2 + |\bar{\psi}_s|^2)^2 = \partial^0 B^2 = 0$, we can write

$$Z_0(x^0) = B \cos \theta(x^0), \quad R_0(x^0) = B \sin \theta(x^0), \quad (58)$$

and we find

$$\partial^0 R_0(x^0) = Z_0 \partial^0 \theta(x^0) = \frac{2ed_e}{\sqrt{3}} Z_0 \epsilon_3(x^0), \quad \text{or,} \quad \partial^0 \theta(x^0) = \frac{2ed_e}{\sqrt{3}} \epsilon_3(x^0). \quad (59)$$

Then, the $\theta(x^0)$ is

$$\theta(x^0) = \theta_0 + \frac{2ed_e}{\sqrt{3}} \int_0^{x^0} dx'^0 \epsilon_3(x'^0). \quad (60)$$

The release of radiation in Equation (53) is given with the term $\frac{1}{L} \epsilon_3$ with the propagation length L . When we rewrite the equation as

$$\partial^0 \epsilon_3(x^0) + \frac{1}{L} \epsilon_3(x^0) = \frac{ed_e k_0}{\sqrt{3}} B \sin \theta(x^0), \quad (61)$$

we use Equation (59) and investigate the case $\kappa \equiv \frac{1}{L} \gg \partial^0$, we can derive

$$\partial^0 \theta(x^0) = \frac{2(ed_e)^2 k_0 B}{3\kappa} \sin \theta(x^0). \quad (62)$$

We then find the solution of $\theta(x^0)$,

$$\theta(x^0) = 2 \tan^{-1} \left[\exp \left(\frac{2(ed_e)^2 k_0 B x^0}{3\kappa} \right) \tan \frac{\theta_0}{2} \right]. \quad (63)$$

Due to the relation $\epsilon_3(x^0) = \frac{\sqrt{3} \partial^0 \theta(x^0)}{2ed_e}$, we arrive at

$$\epsilon_3(x^0) = \frac{\sqrt{3}}{2ed_e} \frac{1}{\tau_R} \left[\cosh \left(\frac{x^0 - \tau_0}{\tau_R} \right) \right]^{-1}, \quad (64)$$

with $\tau_R = \frac{3\kappa}{2(ed_e)^2 k_0 B}$ and $\tau_0 = -\tau_R \ln \left(\tan \frac{\theta_0}{2} \right)$. Because the conserved quantity B is the number density of dipoles ($B = \frac{N}{V}$) with volume V , we find $1/\tau_R \sim B = \frac{N}{V}$. Considering that the N dipoles with correlation among them decay within $1/N$ times faster than the decay of a single dipole system, the intensity of electric fields is instantly $\sim N^2$.

We adopt microtubules (MTs), the cytoskeletons inside cells with cylindrical structures composed of 13 proto-filaments, for super-radiant emission. When we set the length of MT $L = 1 \mu\text{m}$, the energy difference $k_0 = 4 \text{ meV}$, the elementary charge $e = 0.3$ with $d_e = 0.2$ for water dipoles (dipole moment $2ed_e$), and the number density of dipoles $B = N/V = 3.3 \times 10^{28} \text{ m}^{-3}$, we find the time scale of a flash light of super-radiance to be $\tau_R = 0.2 \text{ ps}$ and the strength to be $\frac{\sqrt{3}}{2ed_e \tau_R} = 480 \text{ MeV/m}$. Since the time scale is smaller than that of the thermal loss [25], the super-radiant photons propagate without thermal loss in a brain.

We make use of the MT structures to overcome the dephasing where the phases in each quantum state with a superposition in super-radiance are shifted differently in time evolution due to asymmetric interactions among dipoles. When the dipole–dipole interactions among neighboring dipoles are asymmetric and the phase shifts in time evolution of quantum states of many-body systems of dipoles occur, the super-radiance is weakened [48]. To overcome dephasing, the symmetric geometry of the system of dipoles is necessary. We can adopt a ring arrangement of neighboring dipoles as a candidate of

symmetric geometry. The MTs characterized by their cylindrical structures will provide ring arrangement of water dipoles inside MTs, allowing the system to overcome dephasing.

We estimate the Fresnel number of MTs. In Figure 2, we show the super-radiant emission from an MT with the inner diameter of MT $2w = 15$ nm and a typical longitudinal length of an MT $L = 1$ μ m. The wavelength of super-radiant light is $\lambda = 310$ μ m since the energy difference of dipoles between the ground state and the first excited states is $1/I = 4$ meV. Then, the Fresnel number $\mathcal{F} = \frac{\pi w^2}{L\lambda}$ is in the order of 10^{-6} . Since $\mathcal{F} \ll 1$, the diffraction occurs in the propagation of super-radiant light outside MTs. The super-radiant lights might propagate in a wide range in a brain due to diffraction:

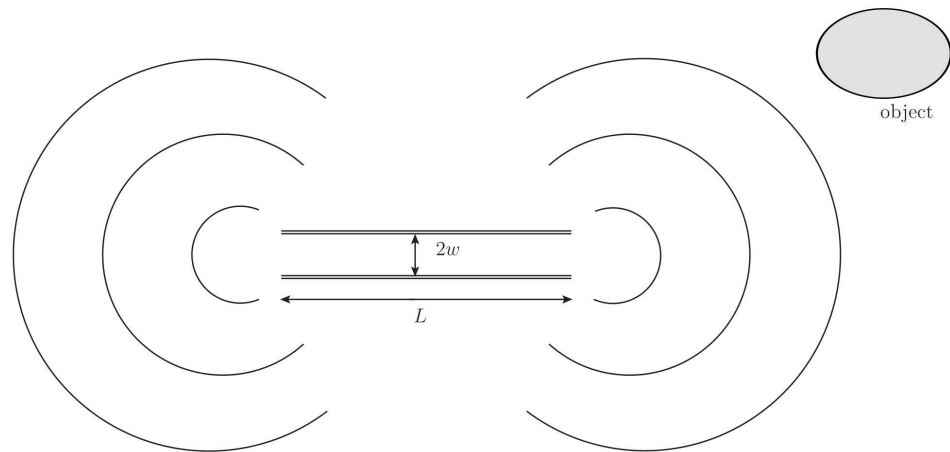


Figure 2. Super-radiant emission from a microtubule.

3.2. Hologram Memory with the Interference of Reference and Object Waves

Super-radiant light from one side of an MT might be reflected by objects and then the light will interfere with the super-radiant light from the other side of an MT.

In Figure 3, we show the interference of the reference and object waves. The polarization in the electric fields for the reference and object waves is set to be in the x^3 direction. When we set the angle of the incident beam of the reference wave with momenta $(k^1, k^2, 0)$ and the vertical line of x^1x^3 plane as θ_1 and the angle of the incident beam of the object wave with momenta $(p^1, p^2, 0)$ and the vertical line of x^1x^3 plane as θ_2 , the angle of amplification in the interference of two waves is $\frac{\theta_1 + \theta_2}{2}$ (the average of two angles). We set the angle $\frac{\theta_1 + \theta_2}{2}$ as 0, the interference pattern for the amplification of waves is in parallel to x^2 direction as shown in Figure 3. We print hologram memory by creating interference patterns. We investigate how interference patterns evolve over time in the next section.

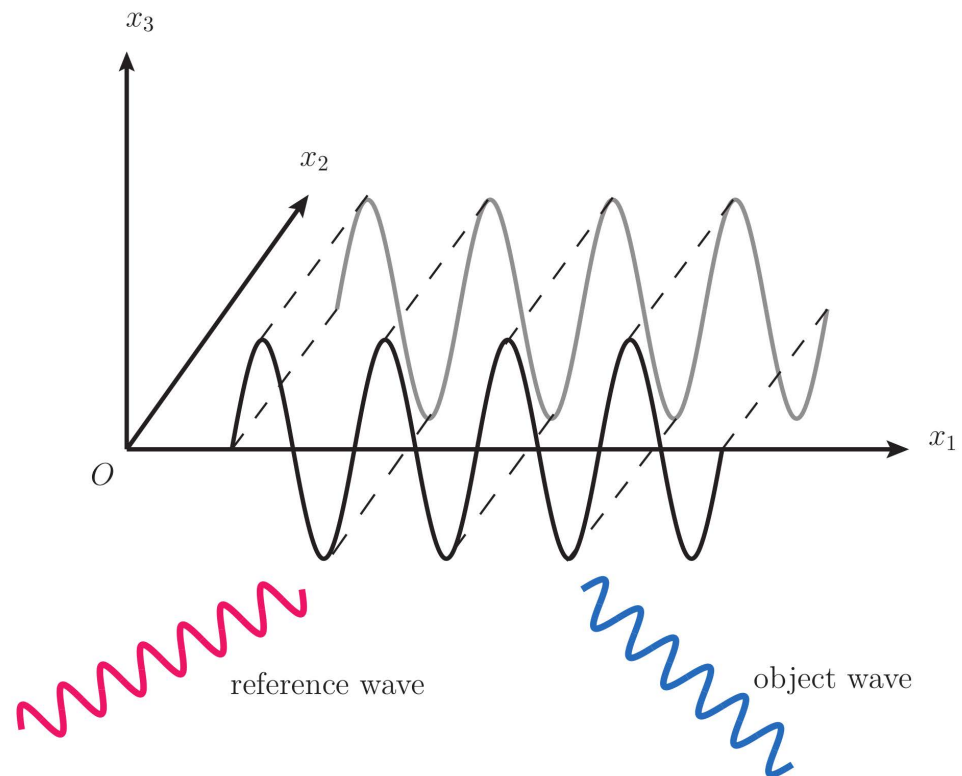


Figure 3. Interference of reference and object waves.

The memory recall mechanism from a hologram is envisaged as follows. The time evolution of electric field E_3 is given by

$$\left[(\partial^0)^2 - (\partial^1)^2 - (\partial^2)^2 \right] E_3 - \Omega^2 \mathcal{Z} E_3 = \Omega^2 \tilde{E}_3 + (\text{other terms}), \quad (65)$$

with collective mode $\Omega = \frac{13}{T}$ as in Equation (34) with the external field. Here, $\tilde{E}_3$ is an external electric field. When the external electric field is $\tilde{E}_3 = \mathcal{A} e^{-i(\sqrt{(k^1)^2 + (k^2)^2} x^0 - k^1 x^1 - k^2 x^2)}$, the special solution of the above equation is

$$E_3 = \frac{\mathcal{A}}{(-\mathcal{Z})} e^{-i(\sqrt{(k^1)^2 + (k^2)^2} x^0 - k^1 x^1 - k^2 x^2)} + O(\partial \mathcal{Z}). \quad (66)$$

Using the conservation law of the scaled total coherent population squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2) = 1$, we find $-\mathcal{Z} = \sqrt{1 - \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)} \sim 1 - \frac{3}{8}\mathcal{M}_3^2$. When $\mathcal{M}_3$ is proportional to the electric field E'_3 in memory printing ($\mathcal{M}_3 = \eta E'_3$) with the proportionality constant η , we find

$$E_3 \simeq \left(1 + \frac{3}{8} \eta^2 E_3'^2 \right) \mathcal{A} e^{-i(\sqrt{(k^1)^2 + (k^2)^2} x^0 - k^1 x^1 - k^2 x^2)}. \quad (67)$$

In $x^1 x^3$ plane, when we write $E_3'^2 = |r(x^1, x^3) + o(x^1, x^3)|^2 = |r|^2 + |o|^2 + r o^* + r^* o$ for the object wave $o(x^1, x^3)$ and the reference wave $r(x^1, x^3)$ and $\mathcal{A} e^{-i(\sqrt{(k^1)^2 + (k^2)^2} x^0 - k^1 x^1 - k^2 x^2)} = r$, we find that

$$E_3 \simeq \left(r + \frac{3}{8} \eta^2 (|r|^2 r + |o|^2 r + o |r|^2 + r^2 o^*) \right). \quad (68)$$

Term $o|r|^2$ corresponds to the reconstructed image in memory recall.

4. Numerical Simulations for Dynamical Holograms

In this section, we show how ordered patterns evolve over time in the presence of evolving quantum fluctuations. We adopt the time evolution equations for the coherent fields in Section 2 with dynamically evolving quantum fluctuations. We adopt the fourth-order Runge–Kutta method to study the time evolution of our numerical simulations.

4.1. Time Evolution towards a Breakdown of Rotational Symmetry in a Spatially Homogeneous Case

In this section, we investigate the dynamics of coherent fields in the spatially homogeneous system. Time evolution equations are Equations (36)–(38) and (44). The time step a_t is set to be $a_t \frac{1}{I} = 0.006$. We assume that the population difference of incoherent dipoles $f_2 \equiv \int \frac{d^4 p}{(2\pi)^4} (F_{00}(x, p) - F_{ss}(x, p)) / (N/V)$ in the term for quantum fluctuations in Equation (34) is dependent on time x^0 (open systems) and evolves in

$$\partial_0 f_2 = -\gamma f_2 \quad (69)$$

with initial $f_2(x^0 = 0) = 0.06$ and $\gamma I = 0.05$ (Case I), initial $f_2(x^0 = 0) = 0.05$ and $\gamma I = 0.05$ (Case II),

$$\partial_0 f_2 = -\gamma(f_2 + 0.001) \quad (70)$$

with $f_2(x^0 = 0) = 0.06$ and $\gamma I = 0.05$ (Case III), and

$$(\partial_0)^2 f_2 = -\omega'^2(f_2 + 0.001) \quad (71)$$

with $f_2(x^0 = 0) = 0.005$ and $\omega' I = 0.03$ (Case IV). Initial values are set to be $\mathcal{M}_3 = 0$, $\mathcal{P}_3 = 0$, $\mathcal{Z} = -1$, $\mathcal{E}_3 = 0.001$ and $\partial_0 \mathcal{E}_3 = 0$.

In Figure 4, we show the time evolution of electric fields E_3 with f_2 in Cases I and II. In Cases I and II, f_2 is positive at any time point and decays exponentially. At early times $x^0 < 5$ ps, the electric field E_3 is fluctuating around zero in Cases I and II. In Case I, the E_3 is monotonically increasing at intermediate times $5 \text{ ps} \leq x^0 < 30 \text{ ps}$. At around $x^0 \sim 35 \text{ ps}$, the increase becomes steeper. At later times $x^0 \geq 40 \text{ ps}$, the E_3 is still monotonically increasing. In Case II, at intermediate times $10 \text{ ps} \leq x^0 < 60 \text{ ps}$, the E_3 gradually increases. The increase becomes steeper at around $x^0 \sim 65 \text{ ps}$.

In Figure 5, we show the time evolution of electric fields E_3 with f_2 in Cases III and IV. In Case III, f_2 starts from a positive value, flips the sign at $x^0 = 13.7 \text{ ps}$ and converges towards -0.001 . We find that the electric field E_3 monotonically increases in Case III when f_2 is positive. When f_2 becomes negative, the increase in electric field E_3 in Case III becomes moderate, and E_3 starts oscillating around zero in the later times $x^0 > 20 \text{ ps}$. In Case IV, the f_2 oscillates around -0.001 . When f_2 is positive at $25 \text{ ps} \leq x^0 < 40 \text{ ps}$, we find that E_3 starts increasing. When f_2 becomes negative at $45 \text{ ps} \leq x^0 < 60 \text{ ps}$, E_3 starts oscillating. When f_2 is positive at $x^0 \geq 60 \text{ ps}$, E_3 starts increasing. In Case IV, the electric field periodically repeats the increase and oscillation with the flips of the sign of f_2 .

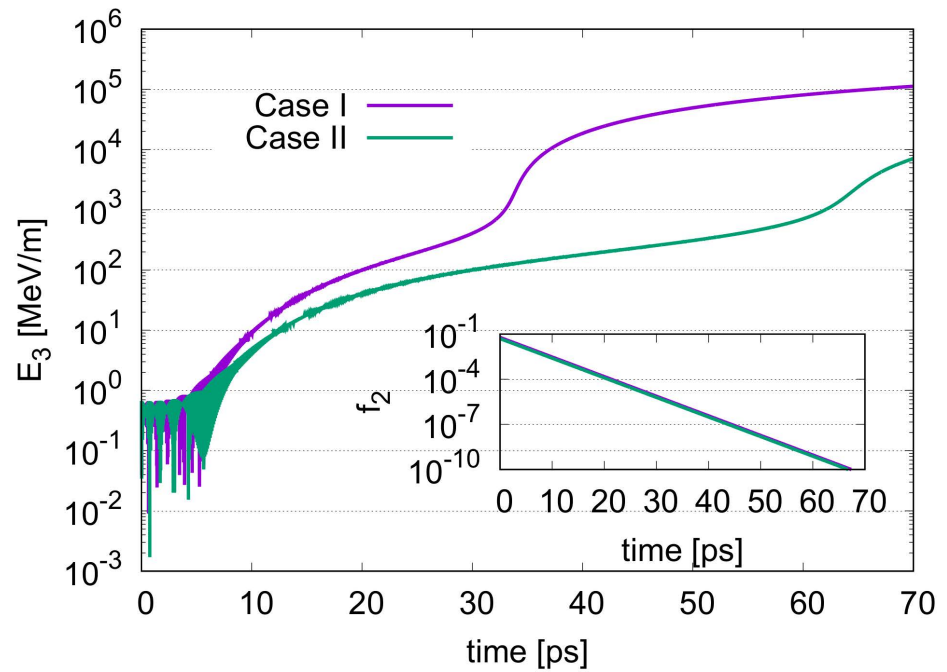


Figure 4. Time-evolution of the electric field E_3 with population difference f_2 in Cases I and II.

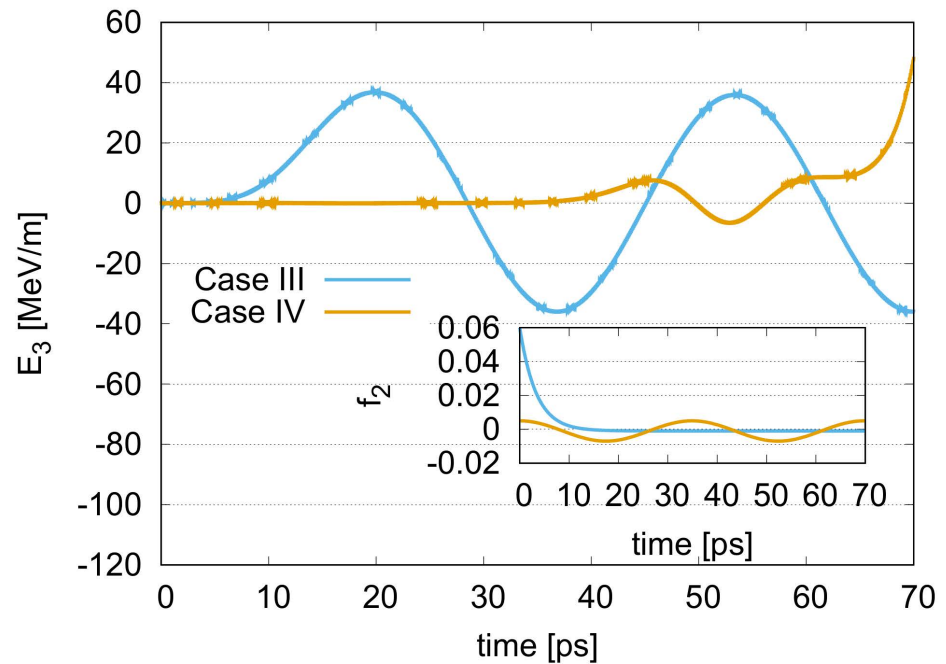


Figure 5. Time-evolution of the electric field E_3 with population difference f_2 in Cases III and IV.

In Figure 6, we show the time evolution of the dipole moment density $\bar{\mu}_3$ divided by its maximum value $\mu_{3,\max} = \frac{2}{\sqrt{3}}ed_eN/V$ derived from the conservation of the scaled total coherent population squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2) = 1$. At early times $x^0 < 10$ ps, the $\bar{\mu}_3$ is approximately zero since E_3 is fluctuating close to zero in Cases I and II. In Case I, at intermediate times $10 \text{ ps} \leq x^0 < 35$ ps, the $\bar{\mu}_3$ gradually increases and tends to converge towards its maximum value. Then, $\bar{\mu}_3$ saturates at $x^0 \geq 35$ ps. We find that the increase becomes steeper at approximately $x^0 \sim 35$ ps in Figure 4 when the saturation of $\bar{\mu}_3$ occurs.

Because energy is supplied from incoherent dipoles when the population difference f_2 is positive, the energy is transferred to the coherent electric field E_3 and coherent dipole fields. When the dipole moment density $\bar{\mu}_3$ becomes saturated, the energy is solely transferred to the electric field E_3 . Consequently, the increase in electric field becomes steeper. In Case II, at intermediate times $10 \text{ ps} \leq x^0 < 65 \text{ ps}$, the $\bar{\mu}_3$ gradually increases towards its maximum value in Figure 6. The $\bar{\mu}_3$ saturates at later times $x^0 \geq 65$ when the increase in electric field becomes steeper in Figure 4. When electric fields gradually increase, dipoles tend to be aligned in the same direction as electric fields. In Cases III and IV, we find that the time evolution of $\bar{\mu}_3$ in Figure 6 is similar to that of the electric field E_3 in Figure 5. When the electric field is positive, the $\bar{\mu}_3$ becomes positive. The $\bar{\mu}_3$ increases at $x^0 < 10 \text{ ps}$ in Case III in a similar manner to $\bar{\mu}_3$ in Case I. However, it begins oscillating at $x^0 \geq 10 \text{ ps}$ because the population difference f_2 becomes negative at $x^0 = 13.7 \text{ ps}$ in Case III.

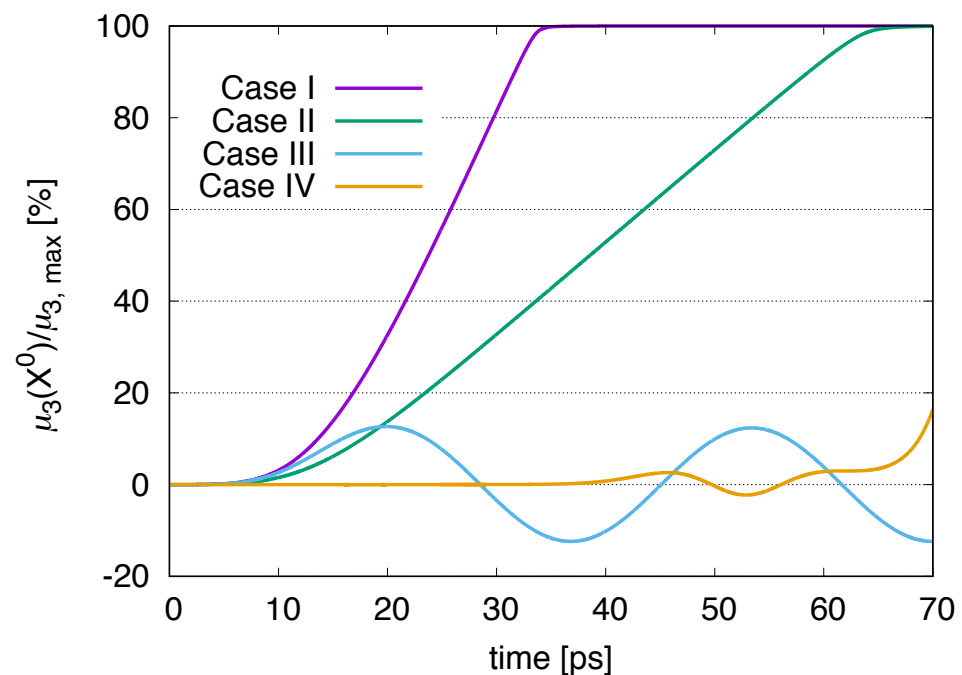


Figure 6. Time-evolution of dipole moment density $\bar{\mu}_3$ divided by its maximum value $\mu_{3,\max}$.

In Figure 7, we show the time evolution of the population difference of coherent dipole fields $\mathcal{Z} = Z_0/(N/V)$. In all cases, $\mathcal{Z}$ starts from -1 in its time evolution. In Case I, $\mathcal{Z}$ gradually increases and converges towards zero when the dipole moment density becomes saturated to its maximum value in Figure 6. In Case II, the increase in $\mathcal{Z}$ is moderate compared with that in Case I; however, $\mathcal{Z}$ also tends to converge towards zero. In Cases III and IV, the $\mathcal{Z}$ is near -1 in its time evolution; however, due to the conservation of the total coherent population, $\mathcal{Z} \geq -1$ is squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2) = 1$. The deviations of the total coherent population squared from 1 are $10^{-7}\%$ in Case I, and less than $10^{-7}\%$ in Cases II, III, and IV.

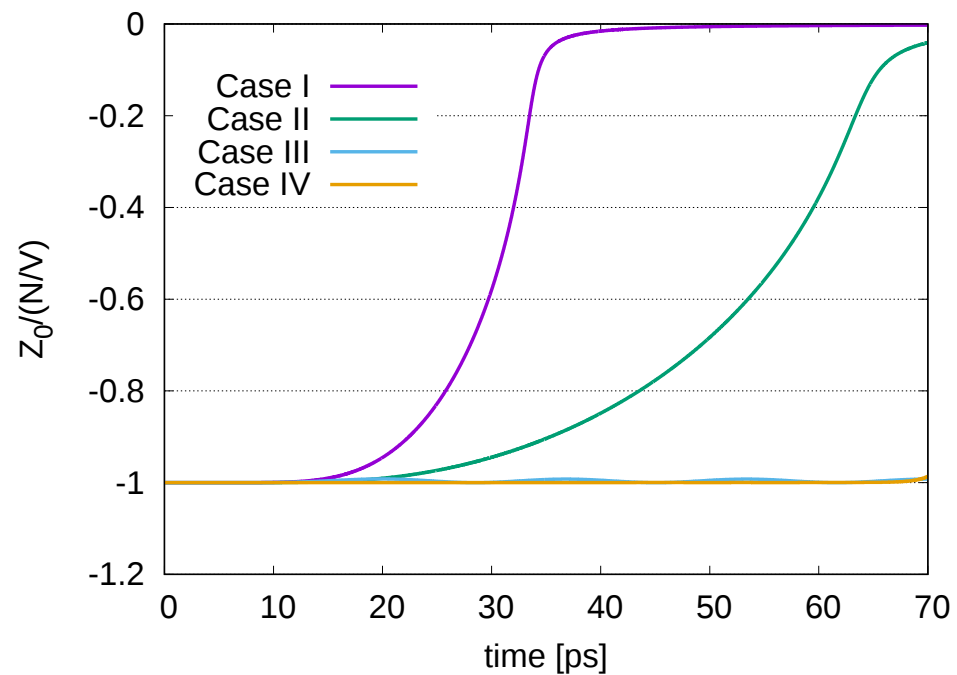


Figure 7. Time-evolution of population difference of coherent dipole fields $Z = Z_0/(N/V)$.

In Figure 8, we show the population difference of coherent dipole fields $-Z = -Z_0/(N/V)$ for the electric field E_3 . Here, $-1/Z$ corresponds to the transmission in amplitude holography shown in Section 3.2. When E_3 fluctuates at approximately zero, the population difference $-Z$ is dependent on the processes of time evolution in Cases I, II, III, and IV. However, when E_3 gradually increases, the population difference $-Z$ is independent of the processes. We find similar relations between $-Z$ and E_3 for Cases I and II for $\ln E_3 \geq 2$. We also find the linear relation between $-\ln(-Z)$ and $\ln E_3$ for $\ln E_3 \geq 6$.

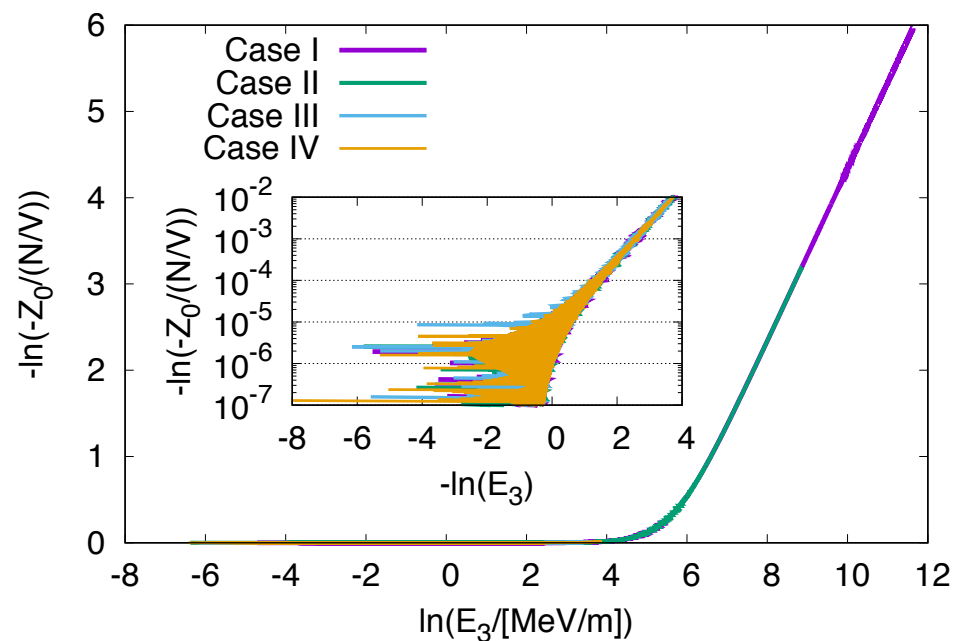


Figure 8. Population difference of coherent dipole fields $-Z$ for the electric field E_3 in Cases I, II, III, and IV.

4.2. Evolution of Spatially Inhomogeneous Hologram Memory

In this section, we show the time evolution of spatially inhomogeneous hologram memory. We set the initial condition of inhomogeneous electric fields by the interference of two super-radiant waves in Figure 3. We prepare the lattice $2N_1 \times 2N_2$ with $N_1 = N_2 = 64$ in the x^1x^2 -plane, the lattice spacing is $a_s \frac{1}{T} = 0.15$ on space $-N_1a_s, -(N_1 - 1)a_s, \dots, 0, \dots, (N_1 - 1)a_s, N_1a_s$, with time step $a_t/a_s = 0.04$. We assume periodic boundary conditions for x^1 and x^2 . We prepare the initial condition of the electric field $\mathcal{E}_3$ by

$$\mathcal{E}_3 = 0.001 \sin\left(\frac{\pi n_1}{N_1}\right), \quad (72)$$

with initial distributions set to $\mathcal{M}_3 = 0$, $\mathcal{P}_3 = 0$, $\mathcal{Z} = -1$ and $\partial_0 \mathcal{E}_3 = 0$ in any spatial points. This condition corresponds to the case in which interference patterns are parallel to the x^2 direction. The $\mathcal{E}_3$ takes its maximum value at $n_1 = 32$. Time evolution equations are Equations (36)–(38) and (44). We investigate the case in which f_2 satisfies

$$\frac{\partial f_2}{\partial x^0} = -0.009 f_2, \quad (73)$$

with initial condition $f_2(x^0 = 0) = 0.005$.

In Figure 9, we show the time evolution of the electric field at point ($x^1 = 32a_s, x^2 = 0$) where $\mathcal{E}_3$ attains its maximum value at the initial time. At early times $x^0 < 10$ ps, the E_3 fluctuates around zero, but gradually increases at intermediate times $10 \text{ ps} \leq x^0 < 50$ ps. The E_3 takes its maximum value at approximately $x^0 = 50$ ps where $f_2 = 3.3 \times 10^{-4} > 0$. At later times $x^0 > 50$ ps, the E_3 starts oscillating in time evolution. The amplitude in oscillation does not seem to change at later times. The reason why E_3 starts oscillating even in the positive population difference of incoherent dipoles $f_2 = \int_p (F_{00} - F_{ss}) / (N/V)$ is the Laplacian term in the time evolution Equation (44). In the previous section, the threshold of f_2 between amplification and oscillation in E_3 is found to be zero. With the existence of the Laplacian in Equation (44), the sum of the term with f_2 and the term with the Laplacian is

$$\Omega^2 f_2 E_3 \rightarrow \Omega^2 f_2 E_3 + [(\partial^1)^2 + (\partial^2)^2] E_3, \quad (74)$$

in the spatially inhomogeneous case. The Laplacian is determined by spatial frequency in E_3 . Owing to the Laplacian term, the threshold of f_2 between amplification and oscillation in E_3 is shifted to a positive value. When we increase the spatial frequency in initial E_3 , the threshold becomes larger and the E_3 starts oscillating at earlier times for damping f_2 in time evolution. The times when E_3 starts oscillating are determined by spatial frequency in E_3 and a population difference f_2 .

In Figure 10, the distributions of electric field E_3 at times $x^0 = 50$ ps, $x^0 = 300$ ps and $x^0 = 450$ ps are depicted. We find that the electric field has a maximum amplitude at approximately $x^0 = 50$ ps. The zero points in E_3 do not change in time evolution. At $x^0 = 450$ ps, the E_3 flips signs compared with E_3 at $x^0 = 50$ ps and 300 ps. However, the shapes of the sine curve do not change in time evolution. The amplitude in the sine curve increases at $x^0 < 50$ ps and the E_3 starts oscillating at $x^0 > 50$ ps with the shapes of sine curves maintained.

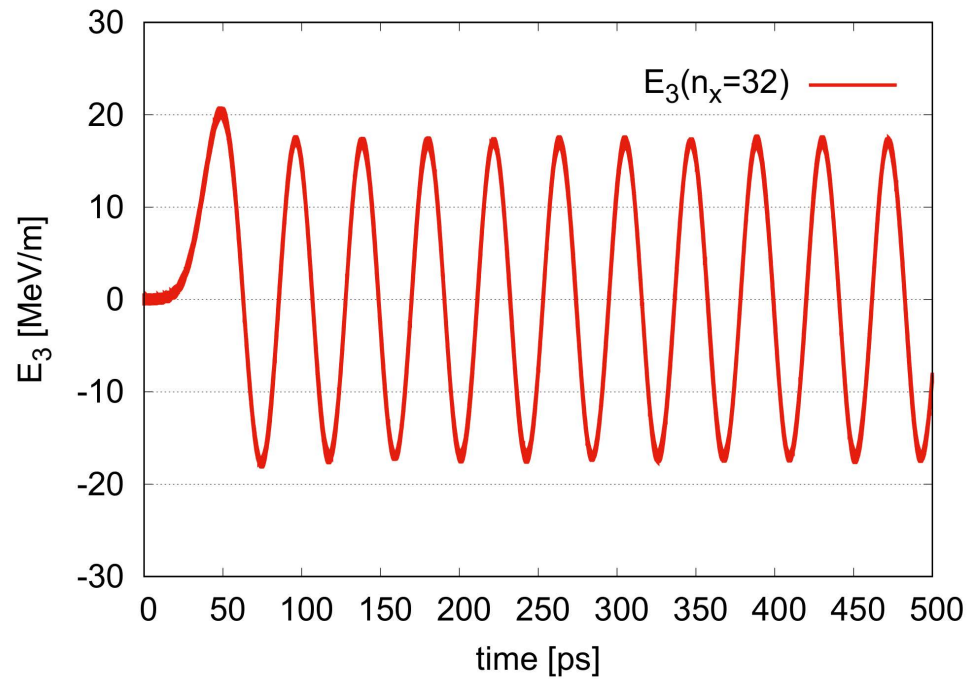


Figure 9. Time-evolution of electric field $E_3(x^0, x^1 = 32a_s, x^2 = 0)$.

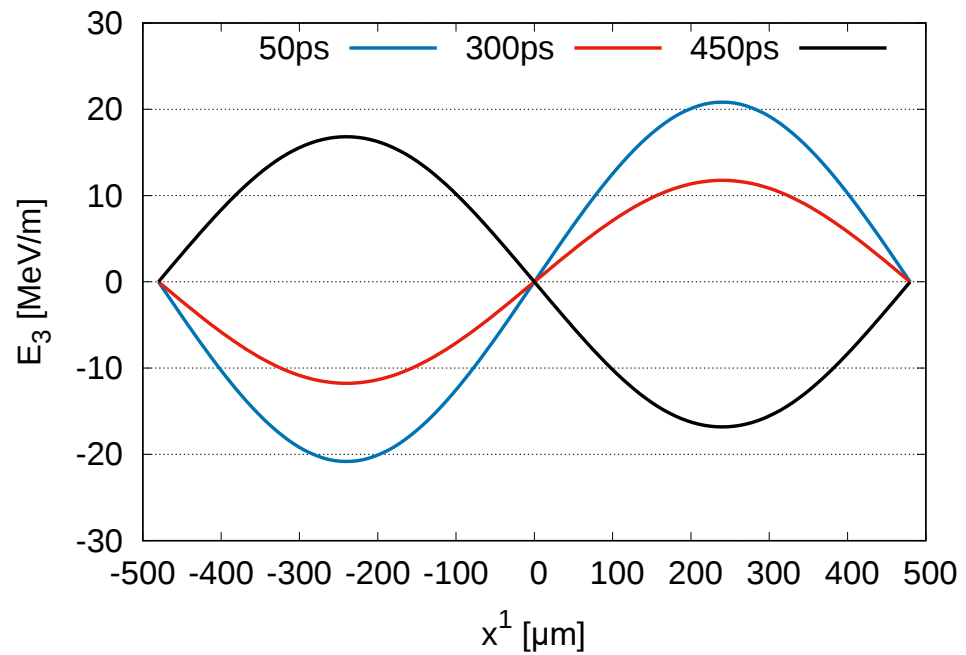


Figure 10. Distribution of the electric field E_3 at $x^0 = 50$ ps, $x^0 = 300$ ps and $x^0 = 450$ ps.

In Figure 11, we show the distribution of the dipole moment density at times $x^0 = 50$ ps, $x^0 = 300$ ps, and $x^0 = 450$ ps. The amplitude takes its maximum value at approximately $x^0 = 50$ ps in a similar manner to that in E_3 . The zero points in $\bar{\mu}_3$ ($x^1 = -480 \mu\text{m}, 0 \mu\text{m}, 480 \mu\text{m}$) are the same as those in E_3 although the dipole moment density $\bar{\mu}_3$ is set to be zero at any spatial point at the initial time. At $x^0 = 450$ ps, the $\bar{\mu}_3$ flip signs is compared with that at $x^0 = 50$ ps and 300 ps in a similar manner to E_3 at $x^0 = 450$ ps. The shapes of the sine curve are maintained for $\bar{\mu}_3$ in time evolution. Since both E_3 and $\bar{\mu}_3$ have shapes of sine curves, we find that the values of $\bar{\mu}_3$ become proportional to those of E_3 .

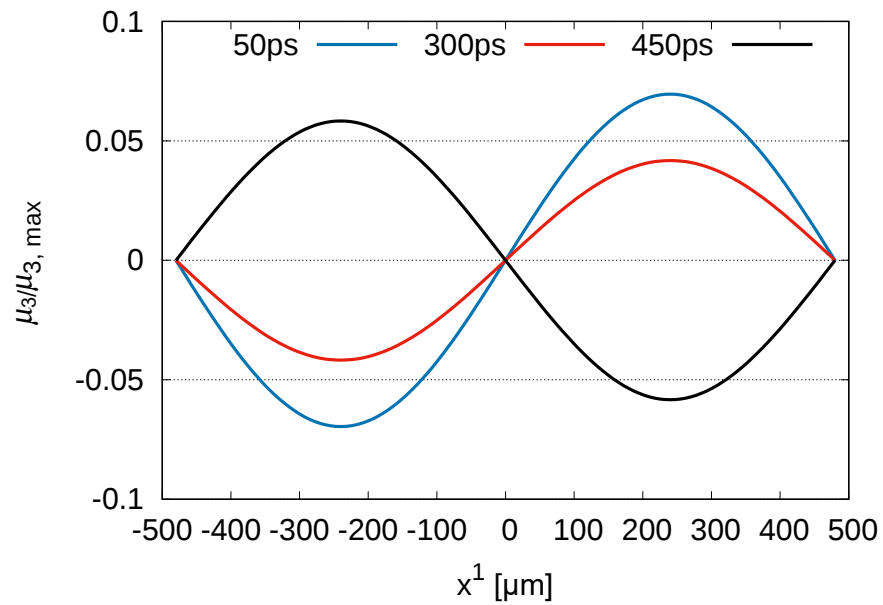


Figure 11. Distribution of the dipole moment density $\bar{\mu}_3$ divided by its maximum value $\bar{\mu}_{3,\max} = \frac{2}{\sqrt{3}}ed_eN/V$ at $x^0 = 50$ ps, $x^0 = 300$ ps and $x^0 = 450$ ps.

In Figure 12, we show the distribution of the population difference of coherent dipoles $\mathcal{Z} = Z_0/(N/V)$ where $-1/Z_0$ corresponds to transmission in hologram. At approximately $x^0 = 50$ ps, the $\mathcal{Z} = Z_0/(N/V)$ takes maximum values at $x^1 = \pm 32a_s = \pm 240 \mu\text{m}$. The $\mathcal{Z}$ at $x^1 = \pm 32a_s = \pm 240 \mu\text{m}$ at $x^0 = 300$ ps is less than that at $x^0 = 450$ ps. We find that $\mathcal{Z}$ is dependent on the absolute values of the dipole moment density $\bar{\mu}_3$ in Figure 11. The $\mathcal{Z}$ is related with $\bar{\mu}_3$ due to the conservation law of the number density of coherent dipoles. The scaled total coherent population squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)$ is conserved in each spatial point. The deviations of $\mathcal{N}$ from 1 are less than $10^{-7}\%$ at any points in the time evolution $x^0 \leq 500.0$ ps. Since $\mathcal{N}$ is conserved, we find the relation $\mathcal{Z} = -\sqrt{1 - \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)}$ in any points and times.

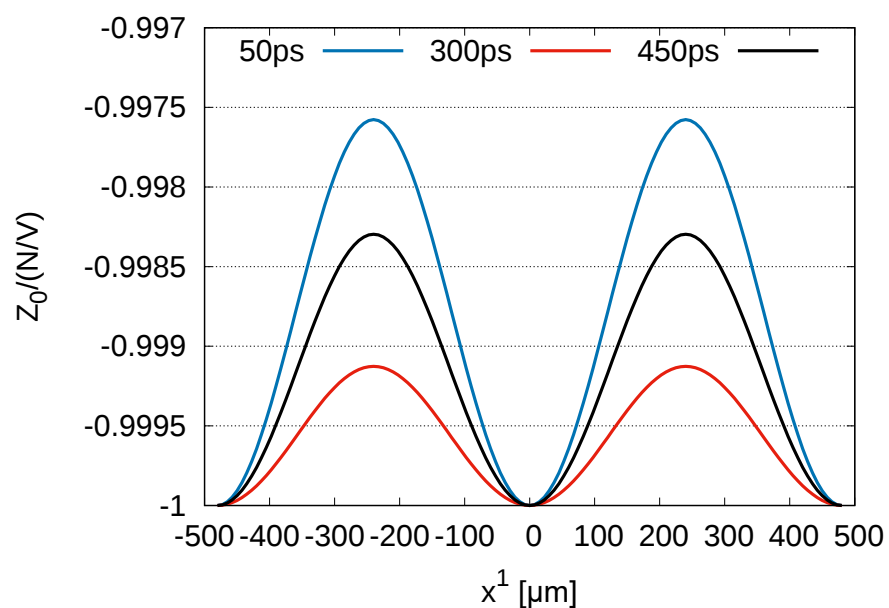


Figure 12. Distribution of the population difference of coherent dipoles $Z_0/(N/V)$ at $x^0 = 50$ ps, $x^0 = 300$ ps and $x^0 = 450$ ps.

We shall then investigate how interference patterns are amplified in fixed f_2 . We set $f_2 = 0.06$ at any points in space and time. The initial conditions are set to be

$$\mathcal{E}_3 = 0.001 \sin\left(\frac{\pi n_1}{N_1}\right) + 0.001 \sin\left(\frac{\pi(n_1 + n_2)}{N_1}\right), \quad (75)$$

$\partial_0 \mathcal{E}_3 = 0$, $\mathcal{Z} = -1$, $\mathcal{M}_3 = 0$ and $\mathcal{P}_3 = 0$. The conditions correspond to the case where two interference patterns were recorded.

In Figure 13, we show the distribution of electric fields E_3 at $x^0 = 0.0$ ps, $x^0 = 5.0$ ps, $x^0 = 6.0$ ps, and $x^0 = 7.0$ ps. We find that the waveforms of the electric fields do not seem to change in the course of time evolution, although maximum values in $|E_3|$ monotonically increase in time evolution. The spatial positions for both the maximum and minimum electric field values do not seem to change in the course of time evolution. The contrasts in E_3 are amplified with the maintained waveforms.

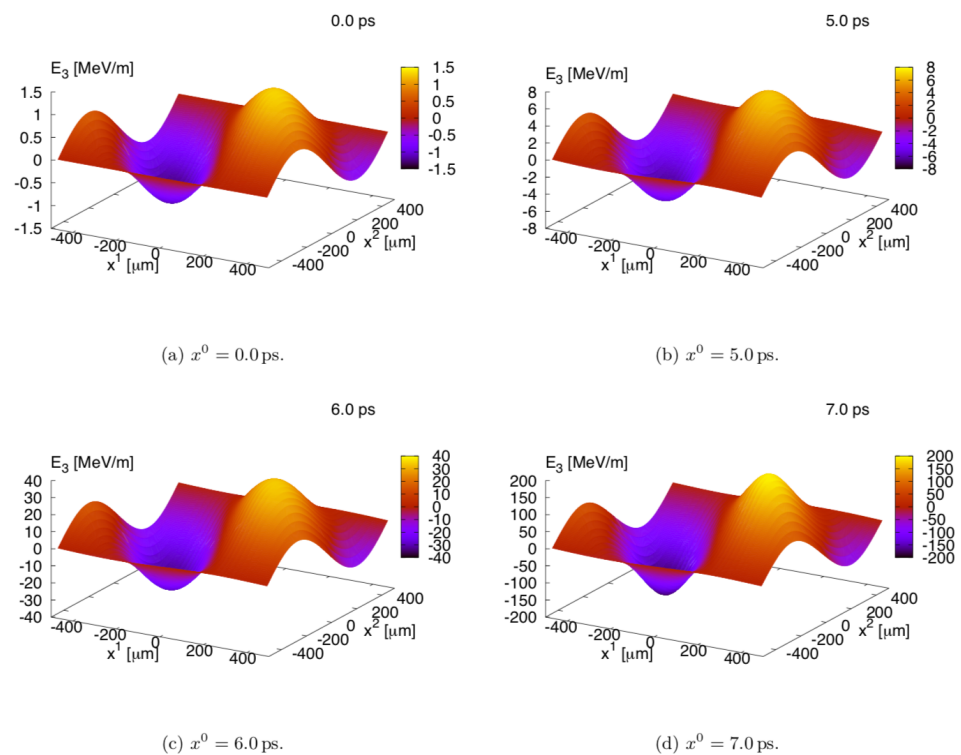


Figure 13. Distribution of electric fields E_3 at (a) $x^0 = 0.0$ ps; (b) $x^0 = 5.0$ ps; (c) $x^0 = 6.0$ ps; and (d) $x^0 = 7.0$ ps.

In Figure 14, the distribution of the dipole moment density $\bar{\mu}_3$ divided by its maximum value at time $x^0 = 7.0$ ps is represented. We find that waveforms of $\bar{\mu}_3$ are similar to those of E_3 in Figure 13. The spatial positions for both the maximum and minimum values of $\bar{\mu}_3$ correspond to those of E_3 . The $\bar{\mu}_3$ seems to be proportional to E_3 in the course of time evolution.

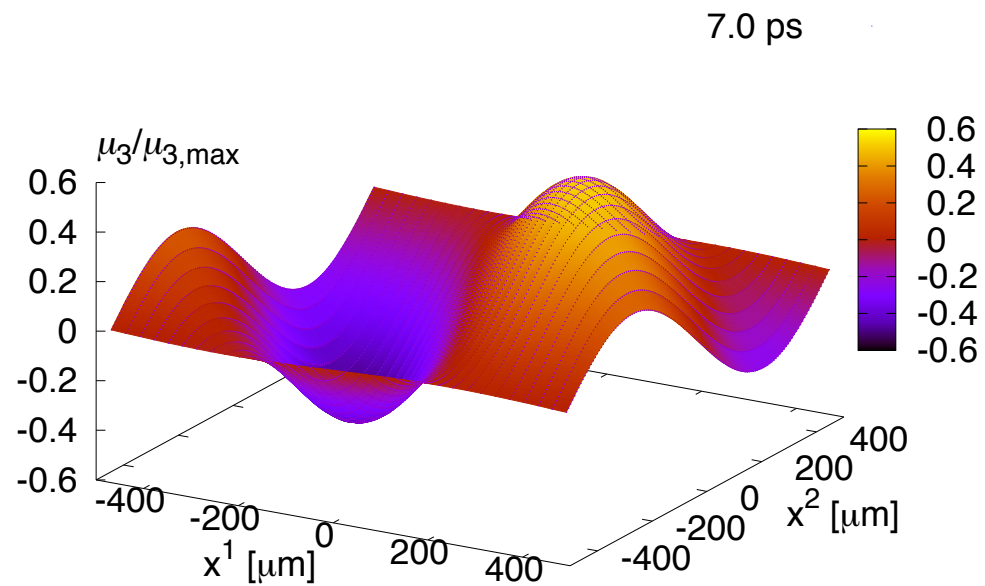


Figure 14. Distribution of the dipole moment density $\bar{\mu}_3$ divided by the maximum value $\mu_3 = \frac{2}{\sqrt{3}}ed_eN/V$ at $x^0 = 7.0$ ps.

In Figure 15, we show the distribution of $\mathcal{Z} = Z_0/(N/V)$. We find two peaks in Z_0 . The positions of peaks correspond to the maximum and minimum points of $\bar{\mu}_3$ in Figure 14 proportional to E_3 in Figure 13. We can check the conservation of the scaled total coherent population squared $\mathcal{N}^2 \equiv \mathcal{Z}^2 + \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)$. The deviations of $\mathcal{N}$ from 1 are less than $10^{-7}\%$ at any points on the lattice for $x^0 \leq 7.0$ ps. Because $\mathcal{N}$ is conserved, we find that the relation $\mathcal{Z} = -\sqrt{1 - \frac{3}{4}(\mathcal{M}_3^2 + \mathcal{P}_3^2)}$ with $\mathcal{M}_3 \propto \mathcal{E}_3$ or $\bar{\mu}_3 \propto E_3$.

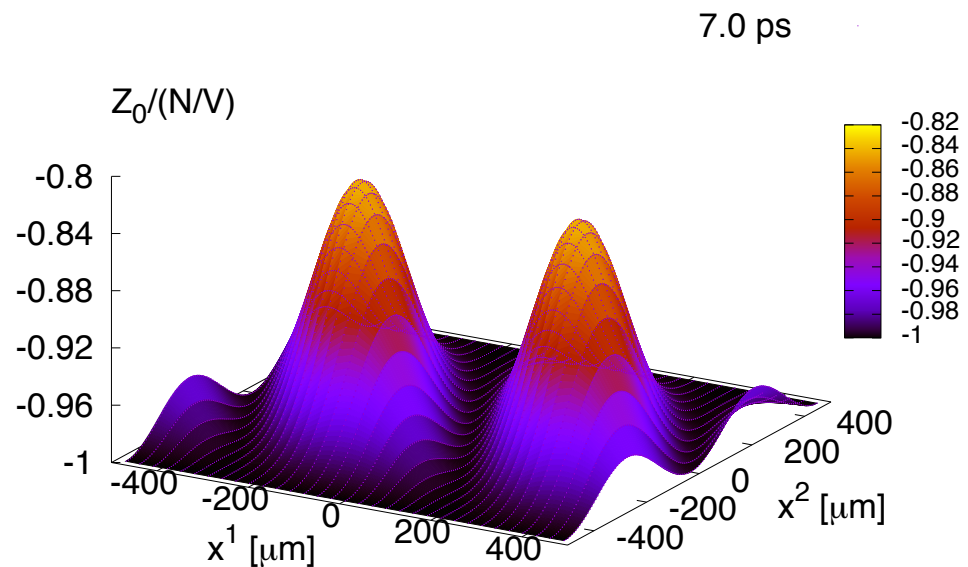


Figure 15. Distribution of population difference $Z_0/(N/V)$ at $x^0 = 7.0$ ps.

5. Discussion

In this paper, we showed the time evolution towards the breakdown of symmetry and time evolution of holograms with interference patterns. This was achieved based on the time evolution equations for coherent fields, namely the Schrödinger-like equations for coherent dipole fields and the Klein–Gordon equations for coherent electric fields, in the presence of evolving quantum fluctuations derived from the Lagrangian density in QBD in $3 + 1$ dimensions. We expanded quantum water dipole fields in terms of

spherical harmonics and adopted a two-energy level approximation for the water dipole field in ground state ψ_s , that in the first excited states is ψ_α with $\alpha = 0, \pm 1$ and their complex conjugates. Beginning with time evolution equations, we derived a solution for super-radiance, which is a cooperative spontaneous emission of radiation. The solution involves coherent waves from microtubules characterized by their cylindrical structures and abundantly present in all neuronal cells. The super-radiant waves propagate in a wide range by diffraction due to the small Fresnel number $\mathcal{F} \simeq 10^{-6}$ for the structures of microtubules. The super-radiance solution offers a new method to achieve holographic memory with the interference patterns of reference waves and object waves reflected by objects involving the information of external stimuli. The transmission of electric fields $\propto -1/Z_0$, with the population difference of coherent dipoles between the first excited states and the ground state $Z_0 = |\bar{\psi}_0|^2 - |\bar{\psi}_s|^2$ with bar representing expectation values, is expressed in terms proportional to the squared electric fields with interference patterns in recording the information in holograms. The recorded information is reconstructed by incident reference waves whose incident angle is equal to the angle in the recording. In simulations of the time evolution of interference patterns, the wavefronts of electric fields seem to be maintained and the contrasts of electric fields are amplified. The distribution of the dipole fields has a similar form to that of electric fields in Figures 13 and 14. The transmission $\propto -1/Z_0 \propto 1 + \frac{3}{8}\mathcal{M}_3^2 = 1 + \frac{3}{8}\eta^2 E_3'^2$ with a scaled value $\mathcal{M}_3 = \bar{\mu}_3/(ed_e N/V)$ for the dipole moment density and proportionality constant η contains terms dependent on the distribution of electric fields $|E_3'|^2$ in recording information. Considering the fact that the wavefronts E_3' are maintained with contrasts amplified and the transmission contains terms proportional to $|E_3'|^2$, the strength of the reconstructed images in recalling memory increases over the time evolution of holograms.

We initially began with simulations of time evolution of coherent fields in spatially homogeneous systems. We find that the breakdown of rotational symmetry emerges in time evolution with terms for quantum fluctuations derived in Appendix A. In our simulations, the positive values of the population difference of incoherent dipoles, namely the inverted population of incoherent dipoles where the population of incoherent dipoles in the first excited state is larger than that in the ground state in two-energy systems expressed by $f_2 = \int_p (F_{00} - F_{ss})/(N/V) > 0$ with $F_{00, \text{ or } ss}$ which represent populations of incoherent dipoles for the first excited state 00 and the ground state ss , play a significant role in the breakdown of rotational symmetry. Dipole fields can be energy sources for coherent electric fields. We show leading-order processes of interactions in the coupling expansion in Figure 16 where Figure 16a represents processes between electric fields and incoherent dipoles, and Figure 16b represents the process of incoherent dipoles in the ground state absorbing incoherent photons and excited by the first excited states (the inverse process is also possible). By adopting open systems, we can consider the flow of incoherent photons and exciting dipoles in the ground state to the first excited states as in Figure 16b. Incoherent dipoles in the first excited states amplify electric fields in an inverted population as shown in Figure 16a. Meanwhile, electric fields oscillate in the normal population $f_2 < 0$. The $f_2 = 0$ represents the threshold determining the amplification or oscillation of electric fields in spatially homogeneous systems. We can also discuss the values of electric fields in Figure 4 for which the dipole moment density $\bar{\mu}_3$ becomes saturated in Figure 6. Water dipoles are found to be all aligned approximately for $E_3 > 1000$ MeV/m in our approach based on quantum field theory (QFT). In the preceding work [49], the critical value of the external electric field inducing aligned water dipoles was determined as 800 MeV/m. We can observe that our approach provides a value of electric fields for the alignment of water dipoles similar to the value in the preceding work even if we adopt several approximations, namely an isotropic moment of inertia, a two-energy level approximation, rigid dipoles without the stretching of O–H bonds, and situations neglecting hydrogen bonding among water molecules.

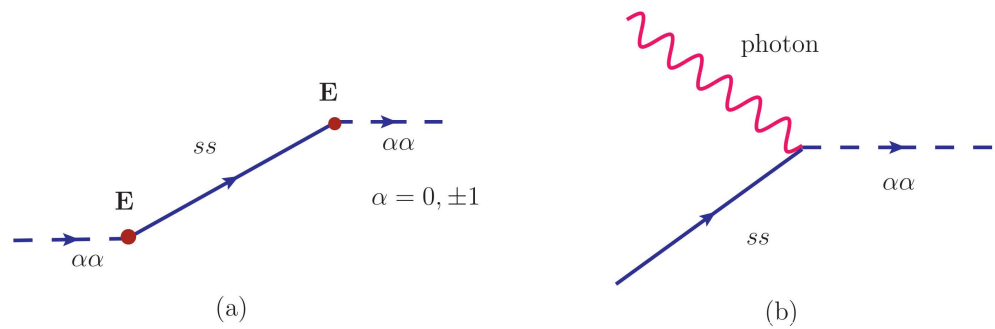


Figure 16. Leading-order processes of interactions in the coupling expansion of ed_e . (a) Interaction between electric fields and incoherent dipoles in the ground states and the first excited states; and (b) Interaction between incoherent photons and incoherent dipoles.

Linear dependence between the logarithm of transmission $-1/Z_0$ and the logarithm of electric fields E_3 is observed in Figure 7. The transmission is dependent on processes of E_3 when E_3 is fluctuating at approximately zero. As E_3 increases exponentially, we find the relation between transmission and E_3 independent of processes. Even if electric fields for interference patterns are small at the initial time, they are amplified due to incoherent dipoles in the inverted population. Subsequently, transmission which is linearly dependent on amplified electric fields involving initial interference patterns is achieved in the course of time evolution. We can adopt this linear dependence for hologram memory formation.

Why is the waveform in the hologram shown in Figure 13 maintained? We can explain the reason by referring to the structure of the Klein–Gordon Equation (34). We find that the equation has the following terms

$$\begin{aligned}\Omega^2 f_2 E_3 + [(\partial^1)^2 + (\partial^2)^2] E_3 &= \Omega^2 f_2 E_3 - (\Theta^2 + \delta\Theta^2) E_3 \\ &= [\Omega^2 f_2 - (\Theta^2 + \delta\Theta^2)] E_3,\end{aligned}\quad (76)$$

with a collective mode $\Omega = \frac{13}{T}$. We assume that the population difference of incoherent dipoles $f_2 = \int_p (F_{00} - F_{ss}) / (N/V)$ is positive and sufficiently large compared with the absolute value of the curvature or spatial frequency squared of electric fields Θ^2 emerging from the Laplacian operating on the electric fields. The $\delta\Theta^2$ represents the deviation of curvature from Θ^2 in E_3 . The larger the bracket in the above equation $\Omega^2 f_2 - (\Theta^2 + \delta\Theta^2)$ is, the more rapidly electric fields E_3 increase in the course of time evolution. When the deviation $\delta\Theta^2 > 0$ becomes large for the parts of interference patterns with $E_3 > 0$ or their absolute value of the curvature becomes large, the $\Omega^2 f_2 - (\Theta^2 + \delta\Theta^2)$ becomes smaller and the increase in parts in E_3 becomes moderate compared with the other parts in E_3 . Similarly for the deviation $\delta\Theta^2 < 0$, the parts in E_3 increase more rapidly than the other parts. As a result, the waveform in the hologram is maintained.

In this work, we adopted a two-dimensional flat surface with periodic boundary conditions. There are various structures in morphology, that is torus, cylindrical or spherical structures with curvatures representing dendrite, cell body and axion in a brain. In these structures, we can investigate the Laplacian with derivatives in perpendicular directions to electric fields in the Klein–Gordon equation. When electric fields are in the direction perpendicular to the surfaces of cylindrical structures, the derivatives of the Laplacian is in the direction parallel to the surfaces. Then, for the zero-points of electric fields $E_r = \sin(n\theta)$ with integer n and the variable of angle θ , the Laplacian $\frac{1}{r^2} \frac{\partial^2 E_r}{\partial \theta^2}$ is zero, so that electric fields in these zero-points remain zero. Except in those zero points, electric fields can be amplified in a similar way to a flat two-dimensional surface. The dipole moment density can be aligned in the same directions as electric fields in cylindrical structures. We can also discuss the various morphology of microtubules. When the diameter of microtubules

becomes larger, the number of correlated water dipoles increases inside microtubules, but the amplitude of super-radiance does not change in Equation (64) since it depends on the number density of water dipoles. The critical factor is the length L of microtubules. The larger the length is, the smaller τ_R is, so that the amplitude of super-radiance will increase since that is proportional to the length. The length of microtubules affects the initial electric fields in the interference patterns of holography. In addition, the number of microtubules becomes larger, the diverse angles and sources of super-radiant emission will be achieved, and the capacity of hologram memory can increase.

We consider memory printing and information processing induced by external stimuli in QBD and holography. The candidate of initial external objects might be phosphorylated tubulins [1] and ionic bio-plasma around microtubules, for example, in V1 for visual information. External objects in V1 are irradiated by super-radiant waves. Then, the interference patterns of reference and object waves are produced as holographic patterns. Once these optical patterns of holography are produced, they propagate in a brain with the other super-radiant waves from neurons in a higher visual cortex with parallel information processing. The strength and angle will be determined by the distances and geometry in the propagation of the visual cortex, which is the strength and angle of super-radiance that depend on the distance of propagation and the direction of information processing in the cortex.

In conventional neuroscience, we consider long-term potentiation (LTP) as a result of neurotransmitter-receptors as the mechanism of long-term memory. Our approach can connect the LTP and holographic approach with super-radiant waves. We shall consider the firing of a single neuron and subsequent super-radiant emissions. There can be various angles of super-radiant emissions as reference waves inducing the reconstruction of multiple holographic images. In considering the firing of two neurons interconnected by LTP with synchrony, the common reconstructed image can emerge in both printing and recalling, and then the amplitude of that image is twice that of the other images. Similarly, due to the firing of N multiple neurons with synchrony and subsequent super-radiant emissions, the amplitude of the common image is N times larger than that of the other images regarded as noises. As a result of LTP and subsequent super-radiant emission, the synchrony can induce the common holographic image in printing and recalling whose amplitude might be proportional to the number of neurons with synchrony.

We adopt the QFT approach involving the field variables of space–time coordinates. The QFT approach can be regarded as the generalization of multi-oscillator systems. In [50–53], the sets of oscillators with a mutual relationship with temporal synchrony for binding and robustness in conscious perception against inherent noise are reported. Similarly, we can consider the mass-spring system of oscillators with an infinite number of point-masses and spring among (neighboring) them in the continuum limit. This physical system will be represented by field variables defined in each space–time in QFT including strings, cloths, or boxes. The QFT with an infinite number of degrees of freedom can provide the Bose–Einstein condensation where massless Nambu–Goldstone bosons are condensed in the zero-energy state resulting in the binding of quantum degrees of freedom in the physical system where they behave as a single entity. The QFT generalized from oscillators can provide the binding of quantum degrees of freedom and synchrony in the physical system. The QFT might provide an answer to the binding problem in neuroscience.

Our approach can provide relations with a split brain study suggested in [54,55]. Among the consequences of cutting the corpus callosum, the breakdown of functional integration between the left and right hemispheres is reported. We assume that the brain is a mixed system of classical neurons and quantum degrees of freedom. Connections among neurons play a role in synchrony. In the split brain, although memory is not lost in each hemisphere due to the diffused non-local nature of memory, the synchrony among neurons is lost. Due to the loss of connections, the simultaneous neuron firing and synchrony of subsequent super-radiant emissions will disappear.

We can adopt parallel information processing in the hologram memory of QBD, where the processing is optically achieved in each point of the hologram through propagation in space. The optical information of a hologram memory might be transferred with the filtering of the hologram or processing in propagation through a hierarchy in the neocortex. (The digital–analog conversion with filtering is proposed in [56].) Hologram memory might propagate in the hierarchy and can be stored in an invariant form of memory. In storing information in the hologram, we do not necessarily adopt sine functions for transmission. We can also adopt step-function-like storage, where coherent domains with dipoles are all aligned and incoherent domains with dipoles whose directions are random are distributed as spatially inhomogeneous patterns after optical information processing. The difference between sine functions and step functions is one of efficiency. This denotes the percentage of incident reference beam used to reconstruct the object image. The holograms for long-term memory might be stored as step functions with coherent domains and incoherent domains and be diffused in a whole brain to achieve the equipotentiality and the mass action. When we consider step-function-like storage, we can then adopt the vacua emerging in the breakdown of rotational symmetry. The vacua are long-range correlations maintained by massless Nambu–Goldstone bosons, indicating robustness against disturbance. We find several stable examples of the breakdown of symmetry in our daily life. For example, magnets are maintained by magnons in the breakdown of symmetry and crystals are maintained by phonons in room temperature. We just simply propose to adopt the macroscopic order emerging in the breakdown of symmetry in quantum field theory (QFT) in QBD and holography. The QFT, the fundamental theory of the nature describes both macroscopic matter (including macroscopic order in the breakdown of symmetry) in classical mechanics and microscopic degrees of freedom in quantum mechanics. The QFT approach is different from other quantum mechanical approaches, such as Penrose–Hameroff theory [57]. The quantum mechanics cannot be applied to macroscopic matter. The robustness against damage in a brain is achieved in hologram memory. The whole image can be recreated from undamaged parts in hologram memory even if parts of the hologram are damaged. We can represent sequential characters of hologram memory. When patterns of incident super-radiant emission are sequential, this may be related to neuron firing and subsequent super-radiant beams in sequential properties—we recall memory as sequential patterns. We can also achieve the auto-associative character of memory in a brain by adopting hologram memory in QBD, where the entire memory is recreated from fragments of memory. When two object images are recorded on the same hologram and the reflected light by one of the object images extends to the hologram, the other object image is recreated by the light.

6. Conclusions and Perspectives

We proposed the integration of quantum brain dynamics (QBD) and holography. Based on the Lagrangian density of QBD, we derived the time evolution equations for coherent dipole fields and coherent electric fields. We adopted a solution of super-radiance to achieve interference patterns for hologram memory. We discovered that the breakdown of rotational symmetry occurs in an inverted population of incoherent dipoles in spatially homogeneous systems. We investigated how interference patterns in holographic memory evolve over time with the existence of the flow of dipole fields in the first excited state (quantum fluctuations) excited by incoherent photons. Ordered patterns in a hologram were amplified for memory printing in the course of time evolution in the inverted population of the incoherent dipoles. Holography in QBD can describe several properties of memory in a brain, which are different from computer memory. The integration of QBD and holography may provide a promising approach to investigate the fundamental and applied aspects of the human brain and the mechanism of memory creation.

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Appendix A. Quantum Fluctuations

We derive the term of quantum fluctuations in the Klein–Gordon Equation (34), namely the third term on the right-hand side. In order to derive the term, we need to calculate the term,

$$-\frac{2ed_e}{\sqrt{6}} \frac{\partial^2}{\partial(x^0)^2} \sum_{\alpha=0,\pm 1} \left[\sqrt{2}i(1 - |\alpha|)(+\Delta_{\alpha s} - \Delta_{s\alpha}) \right],$$

which is time-derivative in Equation (22) with J_3 in Equation (25).

We begin with the Kadanoff–Baym equations for incoherent dipoles in the ground state and the first excited states. The Kadanoff–Baym equations for incoherent dipoles are given by $\left. \frac{\delta \Gamma_{2PI}}{\delta \Delta} \right|_{\bar{a}=0} = 0$ in Equation (20). The equations are written as

$$i\Delta_0^{-1} - i\Sigma = i\Delta^{-1}, \quad (\text{A1})$$

where $i\Delta_0^{-1}$ is given in Equation (13), Δ is given in Equation (15), and self-energy $i\Sigma = -\frac{1}{2} \frac{\delta \Gamma_2}{\delta \Delta}$. Here, the self-energy matrix Σ is written by $\Sigma = \text{diag}(\Sigma_{ss}, \Sigma_{-1-1}, \Sigma_{00}, \Sigma_{11})$ in the leading order in the coupling expansion.

Multiply Δ from the right in Equation (A1) and take the (α, s) component; we can then derive

$$i(\Delta_{0,\alpha\alpha}^{-1} - \Sigma_{\alpha\alpha})\Delta_{\alpha s} + \frac{2ed_e}{\sqrt{6}} \left[i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right] \Delta_{ss} = 0, \quad (\text{A2})$$

with $i\Delta_{0,\alpha\alpha}(x, y) = \left(i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - \frac{1}{i} \right) \delta_C(x - y)$ in the closed-time contour $\mathcal{C}$. We shall introduce the auxiliary Green's function $\Delta_{g,\alpha\alpha}$ satisfying

$$i\Delta_{g,\alpha\alpha}^{-1} = i(\Delta_{0,\alpha\alpha}^{-1} - \Sigma_{\alpha\alpha}). \quad (\text{A3})$$

Using Equation (A3) in Equation (A2), we can derive the relation

$$\Delta_{\alpha s}(x, y) = -\frac{1}{i} \frac{2ed_e}{\sqrt{6}} \int_{\mathcal{C}} dw \Delta_{g,\alpha\alpha}(x, w) \left[i\alpha(E_1(w) - i\alpha E_2(w)) - \sqrt{2}i(1 - |\alpha|)E_3(w) \right] \Delta_{ss}(w, y). \quad (\text{A4})$$

For $\alpha = 0$, we find

$$\Delta_{0s} = \frac{2ed_e}{\sqrt{3}} \Delta_{g,00} E_3 \Delta_{ss}. \quad (\text{A5})$$

Then, multiply Δ from the right in Equation (A1) and take the (s, s) component; we can then derive

$$i(\Delta_{0,ss}^{-1} - \Sigma_{ss})\Delta_{ss} + \frac{2ed_e}{\sqrt{6}} \sum_{\alpha=0,\pm 1} \left[-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right] \Delta_{\alpha s} = i\delta_C, \quad (\text{A6})$$

where $i\Delta_{0,ss}(x, y) = \left(i\frac{\partial}{\partial x^0} + \frac{\nabla^2}{2m} \right) \delta_C(x - y)$. Using Equation (A4), we arrive at

$$\begin{aligned} i(\Delta_{0,ss}^{-1} - \Sigma_{ss})\Delta_{ss} + i\frac{2(ed_e)^2}{3} \sum_{\alpha} \left[-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right] \Delta_{g,\alpha\alpha} \\ \times \left[i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right] \Delta_{ss} = i\delta_C. \end{aligned} \quad (\text{A7})$$

Then multiply Δ from the left in Equation (A1) and take the (s, α) component; we can then derive

$$\Delta_{ss} \frac{2ed_e}{\sqrt{6}} \left[-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right] + \Delta_{s\alpha} i(\Delta_{0,\alpha\alpha}^{-1} - \Sigma_{\alpha\alpha}) = 0. \quad (\text{A8})$$

Using the relation (A3), the above equation is transformed into

$$\Delta_{s\alpha}(x, y) = -\frac{1}{i} \frac{2ed_e}{\sqrt{6}} \int_C dw \Delta_{ss}(x, w) \left[-i\alpha(E_1(w) + i\alpha E_2(w)) + \sqrt{2}i(1 - |\alpha|)E_3(w) \right] \Delta_{g,\alpha\alpha}(w, y). \quad (\text{A9})$$

For $\alpha = 0$, we find

$$\Delta_{s0} = -\frac{2ed_e}{\sqrt{3}} \Delta_{ss} E_3 \Delta_{g,00}. \quad (\text{A10})$$

Multiply Δ from the left in Equation (A1), and take the (s, s) component; we can then derive

$$\Delta_{ss} i(\Delta_{0,ss}^{-1} - \Sigma_{ss}) + \sum_{\alpha} \frac{2ed_e}{\sqrt{6}} \Delta_{s\alpha} \left[i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right] = i\delta_C. \quad (\text{A11})$$

Using Equation (A9), the above equation is rewritten by

$$\begin{aligned} \Delta_{ss} i(\Delta_{0,ss}^{-1} - \Sigma_{ss}) + i\frac{2(ed_e)^2}{3} \Delta_{ss} \left[-i\alpha(E_1 + i\alpha E_2) + \sqrt{2}i(1 - |\alpha|)E_3 \right] \\ \times \Delta_{g,\alpha\alpha} \left[i\alpha(E_1 - i\alpha E_2) - \sqrt{2}i(1 - |\alpha|)E_3 \right] = i\delta_C. \end{aligned} \quad (\text{A12})$$

Multiply Δ from the right in Equation (A1), take the (α, α) component with $\alpha = 0$, and use the relation (A9); we can then derive

$$i\Delta_{g,00}^{-1} \Delta_{00} + i\frac{4(ed_e)^2}{3} E_3 \Delta_{ss} E_3 \Delta_{g,00} = i\delta_C. \quad (\text{A13})$$

From this relation, we arrive at

$$\Delta_{00} = \Delta_{g,00} - \frac{4(ed_e)^2}{3} \Delta_{g,00} E_3 \Delta_{ss} E_3 \Delta_{g,00}. \quad (\text{A14})$$

We then calculate the time derivatives of $\Delta_{s0}(x, x)$ and $\Delta_{0s}(x, x)$. We shall neglect self-energy terms which are a next-to-leading order contribution in the Klein–Gordon Equation (34). We consider the case of $E_1 = E_2 = 0$. The relation (A3) is then rewritten as

$$\left(i\frac{\partial}{\partial x^0} + \frac{\nabla_x^2}{2m} - \frac{1}{I}\right)\Delta_{g,\alpha\alpha}(x, w) = i\delta_C(x - w) \quad (\text{A15})$$

$$\left(-i\frac{\partial}{\partial x^0} + \frac{\nabla_x^2}{2m} - \frac{1}{I}\right)\Delta_{g,\alpha\alpha}(w, x) = i\delta_C(w - x). \quad (\text{A16})$$

The Equation (A7) is rewritten as

$$\left(i\frac{\partial}{\partial x^0} + \frac{\nabla_x^2}{2m}\right)\Delta_{ss}(x, w) + i\frac{4(ed_e)^2}{3}E_3(x) \int_C dy \Delta_{g,00}(x, y) E_3(y) \Delta_{ss}(y, w) = i\delta_C(x - w). \quad (\text{A17})$$

Similarly, the relation (A12) is rewritten as

$$\left(-i\frac{\partial}{\partial x^0} + \frac{\nabla_x^2}{2m}\right)\Delta_{ss}(w, x) + i\frac{4(ed_e)^2}{3} \int_C dy \Delta_{ss}(w, y) E_3(y) \Delta_{g,00}(y, x) E_3(x) = i\delta_C(w - x). \quad (\text{A18})$$

Using Equations (A15)–(A18), we can derive

$$\begin{aligned} \frac{\partial}{\partial x^0}\Delta_{0s}(x, x) &= \frac{2ed_e}{\sqrt{6}} \left[\left(\frac{1}{I}\Delta_{g,00} + i\delta_C \right) (-\sqrt{2}iE_3)\Delta_{ss} \right. \\ &\quad \left. + \Delta_{g,00}(-\sqrt{2}iE_3)i\frac{4(ed_e)^2}{3}\Delta_{ss}E_3\Delta_{g,00}E_3 - \Delta_{g,00}(-\sqrt{2}iE_3)i\delta_C \right], \end{aligned} \quad (\text{A19})$$

and

$$\begin{aligned} \frac{\partial}{\partial x^0}\Delta_{s0}(x, x) &= \frac{2ed_e}{\sqrt{6}} \left[i\delta_C\sqrt{2}iE_3\Delta_{g,00} - i\frac{4(ed_e)^2}{3}E_3\Delta_{g,00}E_3\Delta_{ss}\sqrt{2}iE_3\Delta_{g,00} \right. \\ &\quad \left. + \Delta_{ss}\sqrt{2}iE_3\left(-\frac{1}{I}\Delta_{g,00} - i\delta_C\right) \right], \end{aligned} \quad (\text{A20})$$

where we neglect the terms with $\frac{\nabla^2}{2m}$ which are higher-order contributions in the gradient expansion. Using relations (A19) and (A20), we find

$$\begin{aligned} \frac{\partial}{\partial x^0}(\Delta_{0s} - \Delta_{s0}) &= \frac{2ed_e}{\sqrt{6}} \frac{1}{I} \left(\Delta_{g,00}(-\sqrt{2}iE_3)\Delta_{ss} + \Delta_{ss}\sqrt{2}iE_3\Delta_{g,00} \right) \\ &= \frac{1}{iI}(\Delta_{0s} + \Delta_{s0}), \end{aligned} \quad (\text{A21})$$

and

$$\begin{aligned} \frac{\partial^2}{\partial (x^0)^2}(\Delta_{0s} - \Delta_{s0}) &= \frac{1}{iI} \frac{2ed_e}{\sqrt{6}} \left[2\sqrt{2}\Delta_{ss}E_3 - 2\sqrt{2}\Delta_{g,00}E_3 + \frac{1}{I}\Delta_{g,00}(-\sqrt{2}iE_3)\Delta_{ss} \right. \\ &\quad \left. - \frac{1}{I}\Delta_{ss}\sqrt{2}iE_3\Delta_{g,00} + \frac{8\sqrt{2}(ed_e)^2}{3}\Delta_{g,00}E_3\Delta_{ss}E_3\Delta_{g,00}E_3 \right]. \end{aligned} \quad (\text{A22})$$

We shall use the relation (A14), and then we find

$$\begin{aligned} \frac{\partial^2}{\partial(x^0)^2}(\Delta_{0s} - \Delta_{s0}) &= \frac{1}{iI} \frac{2ed_e}{\sqrt{6}} \left[2\sqrt{2}(\Delta_{ss} - \Delta_{00})E_3 + \frac{1}{I}\Delta_{g,00}(-\sqrt{2}iE_3)\Delta_{ss} \right. \\ &\quad \left. - \frac{1}{I}\Delta_{ss}\sqrt{2}iE_3\Delta_{g,00} \right] \\ &= -\frac{1}{I^2}(\Delta_{0s} - \Delta_{s0}) + \frac{4ed_e}{\sqrt{3}} \frac{1}{iI}(\Delta_{ss} - \Delta_{00}). \end{aligned} \quad (A23)$$

Finally, taking the 0th order terms in the gradient expansion and taking the statistical part of Green's functions, we arrive at

$$\begin{aligned} -\frac{2ed_e i}{\sqrt{3}} \frac{\partial^2}{\partial(x^0)^2}(\Delta_{0s} - \Delta_{s0}) &= +\frac{8(ed_e)^2}{3I}E_3 \int_p (F_{00}(x, p) - F_{ss}(x, p)) \\ &\quad + \frac{8(ed_e)^2}{3I^2}E_3 \int_p (\text{Re}\Delta_{g,00,R}F_{ss} + \Delta_{g,00,F}\text{Re}\Delta_{ss,R}), \end{aligned} \quad (A24)$$

where we introduced the retarded Green's functions $\Delta_{g,00,R} = i(\Delta_{g,00}^{11} - \Delta_{g,00}^{12})$ and $\Delta_{ss,R} = i(\Delta_{ss}^{11} - \Delta_{ss}^{12})$ and statistical functions $F_{ss} = \frac{\Delta_{ss}^{12} + \Delta_{ss}^{21}}{2}$ and $\Delta_{g,00,F} = \frac{\Delta_{g,00}^{12} + \Delta_{g,00}^{21}}{2}$ with their Fourier-transformed functions. In this paper, we discuss the first term on the right-hand side in the above equation by assuming that the second term is incorporated into the first term.

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