

Determination of the Dynamic Behavior of Thin-Walled Hollow-Box Sandwich Beams [†]

Hugo Miguel Silva ^{1,*}  and Jerzy Wojewoda ²

¹ proMetheus, Instituto Politécnico de Viana do Castelo, Rua Escola Industrial e Comercial Nun'Álvares, 34, 4900-347 Viana do Castelo, Portugal

² Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, 90-924 Lodz, Poland; jerzy.wojewoda@p.lodz.pl

* Correspondence: hugosilvasci@gmail.com

[†] Presented at the 4th International Electronic Conference on Applied Sciences, 27 October–10 November 2023; Available online: <https://asec2023.sciforum.net/>.

Abstract: Sandwich geometries, mainly panels and beams, are widely used in several transportation industries, namely aerospace, aeronautics, and automotive. They are known for some advantages in structural applications: high specific stiffness, low weight, and possibility of design optimization prior to manufacturing. This study aims to discover the dynamic behaviour of a model of novel sandwich type of beam simply supported-at-its-ends by use of finite element method. There are 12 geometries studied herein, with the same base configuration. The models were previously subjected to a design optimization routine. The dynamic behavior of the initial models in relation to their final versions is considered. The influence of the geometry on the characteristic frequencies is discussed, as well as its improvement in relation to the initial models. It is shown that the statically optimized models represent a significant improvement over the initial ones. In some cases, the improvement surpasses 20%. It can, therefore, be concluded that the design optimization approach, developed for static analysis, might be moderately effective in improving the modal behavior of the studied beams.

Keywords: sandwich beams; dynamic analysis; finite element method (FEM)



Citation: Silva, H.M.; Wojewoda, J. Determination of the Dynamic Behavior of Thin-Walled Hollow-Box Sandwich Beams. *Eng. Proc.* **2023**, *56*, 321. <https://doi.org/10.3390/ASEC2023-15885>

Academic Editor: Ana Paula Betencourt Martins Amaro

Published: 7 November 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The finite element method (FEM) functions by employing partial derivative equations to solve the discretization of the domain into many elements. In the context of dynamic analysis, the utilization of the finite element method (FEM) necessitates the use of a computer system that possesses enough computational capabilities [1]. The literature encompasses several studies that are relevant to the topic of the present work.

The authors of reference [2] extended the conventional Vlasov theory for thin-walled beams with open and closed cross sections by incorporating distortional displacement forces. The engineering relevance of the eigenvalues identified through dynamic analysis lies in their ability to mitigate the amplification of distortional eigenmodes. The authors of [3] offer a beam that resembles a thin-walled hollow tube, which is reinforced by ribs. The objective of the study is to examine the impact behavior of the beam in order to enhance energy absorption and reduce the first peak force. Subsequently, the process of shape optimization design is undertaken. The research investigates the characteristics of free vibration in single-cell thin-walled tubes with regular convex polygonal cross-sections. The authors also conducted a comprehensive analysis of the many modes of these beams.

In the study conducted by [4], a precise formulation of dynamic stiffness is presented using one-dimensional, higher-order theories. This formulation is subsequently employed to investigate the characteristics of free vibration in both solid and thin-walled structures. The objective of the study conducted by [5] was to expand the range of applications for the Generalized Beam Theory (GBT) formulation, which had been recently developed

for conducting elastic linear buckling analyses of thin-walled components. The paper in question aims to provide data on the dynamic behavior of internally reinforced thin-walled beams through a comparative analysis. Article [6] presents results of studies of a geometrically stiffened honeycomb core to increase the structural performance results in an upgraded honeycomb structure known as the stiffened honeycomb sandwich panel (stiffened HSP). The horizontal stiffened HSP has a lower natural frequency and a bigger buckling stress when compared to the standard and vertical stiffened HSPs [6].

The study [7] proposes a theoretical model of a stiffened plate with numerous dynamic vibration absorbers under various boundary restrictions. The model presented in [7] improves the equivalent mass solution efficiency by 90% when compared to FEM [7]. In reference [8], the vibration properties of sandwich panels with a sandwich core made of hierarchical composite honeycomb are presented. To offer an equivalent model (two-dimensional model), an orthotropic constitutive model of the hierarchical composite honeycomb sandwich core was used.

The natural frequencies and mode shapes of the sandwich panels were predicted using modal testing, two-dimensional (2D) and three-dimensional (3D) finite element models. The comparable model's prediction results agreed with the findings of the 3D finite element analysis and the experiment [8]. In [9], the homogenized beam-like model for the transverse dynamics of reticulated structures is presented in a finite element formulation. The study deals with elastic periodic lattice structures, whose unit cell is composed of connected beams or plates and repeats itself in a single direction. Examples of these structures include foams, crystals, honeycombs, and multistory skyscrapers. The motion is given by a sixth-order differential equation, and the examined model is a one-dimensional enriched form of the fourth-order Timoshenko beam equation. It is demonstrated that the homogenized beam finite element solution presented approaches the whole detailed finite element structural model and recovers the analytical results [9].

Industrial machinery with movable parts can be made smaller while still having the same mechanical performance to increase speed. Because thin-walled structures can be reinforced internally [10,11] and have high mass-unit effectiveness, hollow solid sections outperform bulk counterparts in engineering applications with the same outer section dimension and shape. Previous research on the mechanical behavior of beams similar to the one here has been conducted [12,13]. Similar beams were also subjected to design optimization procedures [14,15]. In addition, a beam comparable to the one under study has been produced and put through experimental testing [16].

The present study focuses on the modal analysis of internally reinforced beams that were designed and optimized for static loads. No study was found in which the geometries used in this study, or similar ones, were studied in modal analysis. Also, no studies were found that prove the effectiveness of design optimization, performed in static analysis, on the improvement of the modal behavior of internally reinforced thin-walled beams.

2. Numerical Procedure

2.1. The FEM Models

For the purpose of the analysis of dynamic behavior, 12 finite element method (FEM) models were designed in the ANSYS Mechanical APDL. These models represent different versions of the project of novel beams. They are composed of two sandwich panels on the top and on the bottom, and a reinforcement pattern on the sides, as shown in Figure 1: Such models were earlier discussed by the authors in other application aspects [12–14,16,17]. Simple hollow-box beams, named hollow-solid sections, and abbreviated HSS in the results were also studied, using the same conditions as on the sandwich beams. The variables for the studied beams are shown in Figure 1 (right).

In order to obtain an effective response to transversal beam deflection in terms of stiffness, 12 FEM models were built. These models were subjected to modal analyses, via the Block-Lanczos method, simply supported at their ends. The supports are shown in Figure 2. The mesh is a quadrilateral free mesh, using SHELL63 elements and with a mean

element size of 0.0025 [m]. This construction is based on the principle that such a type of beam needs a zone along which accessories pass, such as compressed air tubes and electric cables. The central zone of the beam was chosen because that zone contains the neutral axis. In the peripheral zone, there are two lateral zones, and two other zones: one at the top and the other at the bottom.

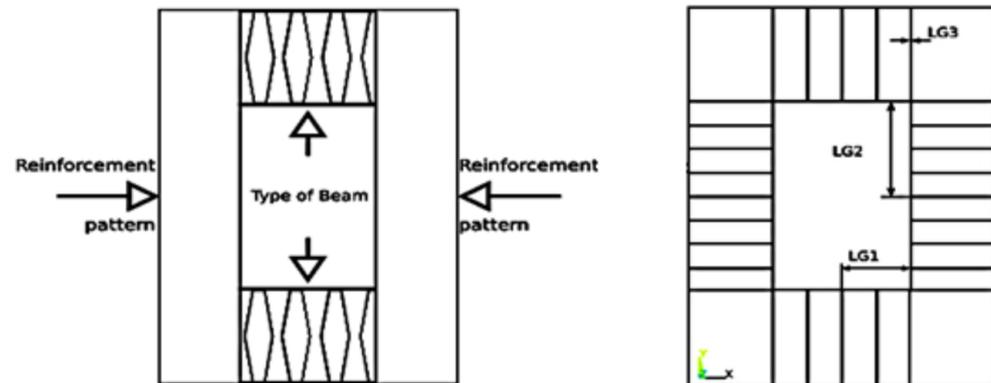


Figure 1. Configuration of the beam (left) [12–14,16,17] and geometric variables of the FEM model used on the design optimization (right) [14,17].

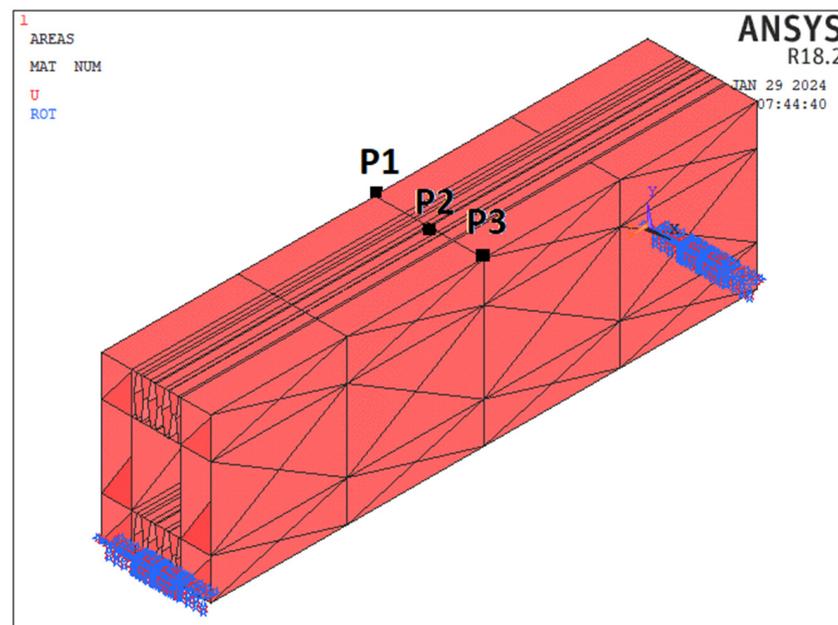


Figure 2. Type of support applied to all FEM models, and points used to calculate displacements, adapted from [12–14,16,17].

The ANSYS input file contains instructions to collect the displacements on the nodes attached to the keypoints indicated in Figure 2 (one by each keypoint), which are situated at the edges and at the center, in order to gather the displacements on the same points in each iteration. These keypoints were selected because, even when the variable values change during optimization, their coordinates remain unchanged. Since the ribs at these sites provide considerable reinforcement, it is anticipated that the local deformation will not be significant for the thicknesses under consideration. The 12 geometries studied are shown in Figure 3.

In the present study, the following material properties, typical of steel, were considered: Young's modulus (E) of 210 GPa; density (ρ) of 7890 kg/m³; Poisson's ratio (ν) of 0.29 [-]. The modal analyses were performed in case of the model supported at its ends.

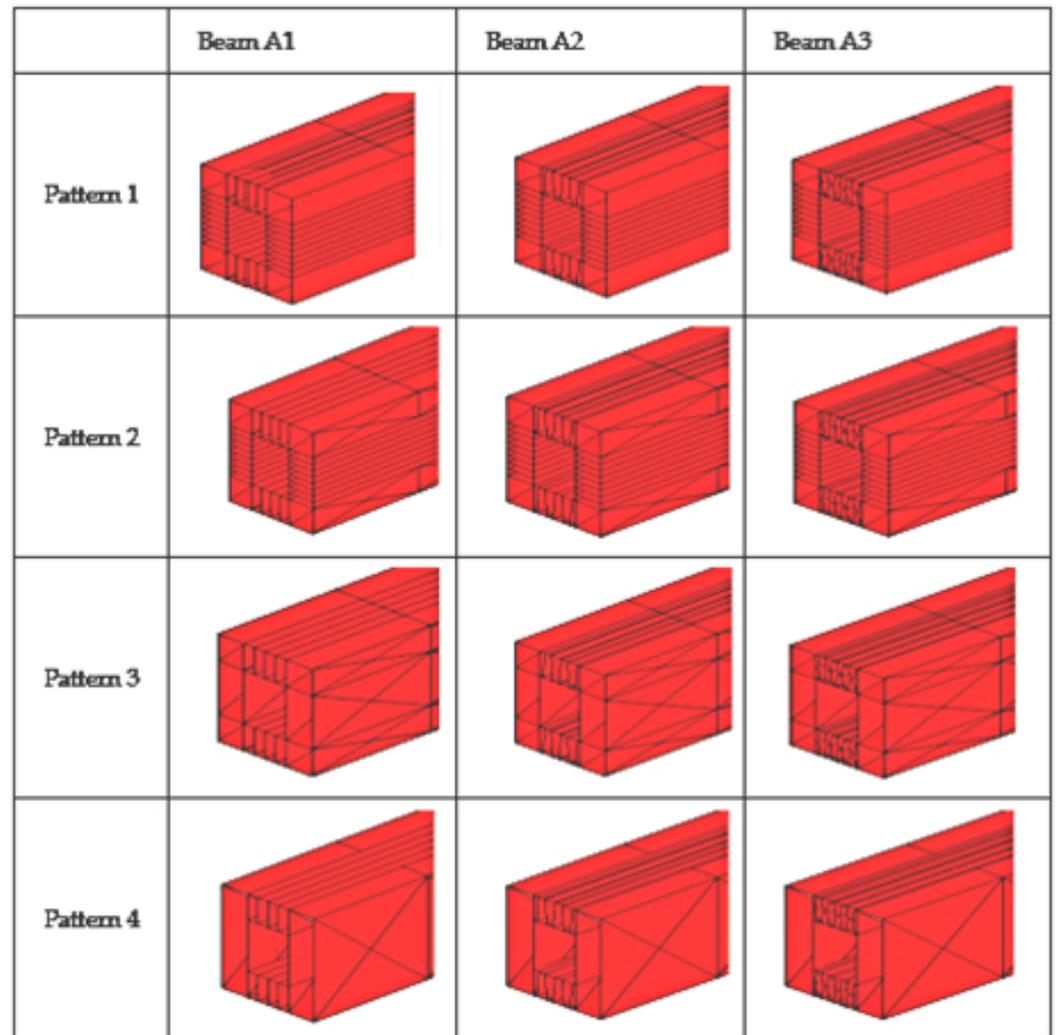


Figure 3. Geometries of the FEM models [14,17].

2.2. Design Optimization

The models were optimized with respect to their total mass and nodal displacements in the z direction, which were measured at three different places. The interaction between ANSYS 2020 R2 and the MATLAB 2019 R1 optimization programming code is seen in Figure 4. ANSYS and the MATLAB application collaborate in this process. In [14,15,17], the authors state that ANSYS computes the FEM models, while MATLAB manages the optimization by means of a programming code. The optimization process was built according to the scheme shown in Figure 4.

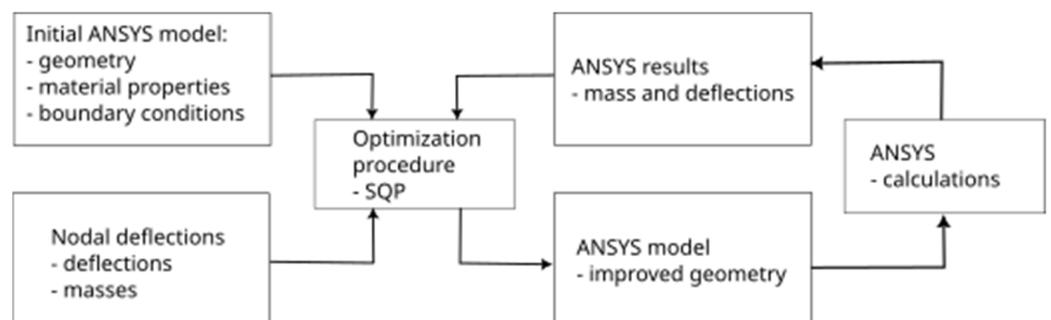


Figure 4. Functional flow chart of the optimization methodology [14,17], adapted from [18].

Despite the variance in the geometric variables, these spots were selected at locations where all coordinates remain constant. By doing this, the findings are not directly impacted when the design factors are changed. The approach used in this work’s MATLAB-based Finite Element Model Updating tool was first created in [18] for structural dynamic analysis. Additionally, in [19,20], it was modified for structural static analysis. In this work, Equation (1) served as the goal function that the MATLAB code used to optimize the models, as in [14,17].

$$O(m, \delta) = W_1 \frac{\sum_{j=1}^n M_j}{\sum_{j=1}^n M_j^i} + W_2 \frac{\sum_{j=1}^n |\delta_j|}{\sum_{j=1}^n |\delta_j^i|} \tag{1}$$

where the following variables are defined:

δ_j is the nodal deflection obtained in each nodal point, and in each iteration, δ_j^i is the nodal deflection obtained in each nodal point in the initial model.

M_j is the element mass obtained in each nodal point and in each iteration, and M_j^i is the element mass obtained in each nodal point in the initial model.

2.3. Improvement of the Models

Each beam model in its initial state was parametrized in ANSYS APDL and has the same values as the variables LG1, LG2, and LG3. These variables are shown in Figure 1 (right). Their initial values are LG1 = 45, LG2 = 75, and LG3 = 2 [mm]. The outer section dimensions are kept, by principle, unaltered. The models had been statically optimized earlier, in [14,17], and as such, the values of the design variables changed during the optimization routine. Their final values are shown in Table 1.

Table 1. Final variable and objective function values obtained on the optimized models.

Bending	A1	A2	A3		A1	A2	A3
	Pattern 1				Pattern 3		
LG1f	4.86	1.80	1.80	LG1f	4.50	2.17	1.80
LG2f	7.73	9.26	8.32	LG2f	7.51	9.02	9.52
LG3f	3.74	2.79	2.63	LG3f	3.61	2.76	2.58
Final objective	0.98	0.83	0.79	Final objective	0.97	0.86	0.80
	Pattern 2				Pattern 4		
LG1f	1.80	2.17	1.80	LG1f	1.80	2.17	1.80
LG2f	10.15	7.61	11.90	LG2f	8.05	7.70	8.36
LG3f	2.79	2.99	2.65	LG1f	2.75	3.55	2.76
Final objective	0.87	0.89	0.81	Final objective	0.80	0.85	0.77

The analyzed models were previously subjected to bending and torsion modelled as uncoupled loads, in [12–14,16,17].

3. Results and Discussion

Mode shapes for model A1 and Pattern 4 are shown in Figure 5 as an example. In Figure 5, both the displaced and initial mode shapes are presented.

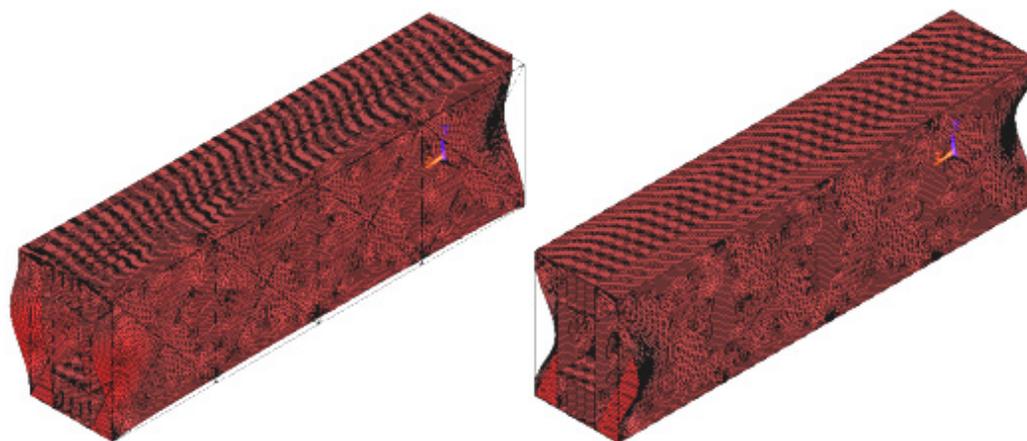


Figure 5. Mode shapes for the mode 1 at 246.145 Hz (left) and mode 2 at 330.343 Hz (right).

3.1. Mesh Convergence Analysis

To select a mesh size that originates accurate results, a mesh convergence analysis was performed on the model beam 3 pattern 3. This geometry is shown in Figure 2 (left).

As this model is the most complex, it is expected that if the mesh convergence yields accurate results for this geometry, the other models would originate accurate results, as well. Table 2 presents the mesh convergence study, showing the mean element size E_{size} , the frequency of the 1st mode ω_1 , and the error obtained by the application of Equation (2).

Table 2. Mesh convergence study.

Error [%]	ω_1	Esize [mm]
	145.88	40
1.28	144.01	20
1.39	142.01	10
0.32	141.55	5
0.04	141.61	2.5

The error E shown in Table 2 was calculated by using Equation (2):

$$E[\%] = \frac{|\omega_i - \omega_{i+1}|}{\omega_1} * 100 \tag{2}$$

Because the element size of 2.5 mm originates the most accurate results, in comparison to other element sizes, with a value of 0.04%, that element size was used in the simulations.

3.2. Analysis of the Frequencies

In order to study the dynamic response of the beams, modal analyses were performed. The modal extraction method was Block-Lanczos, with a frequency range between 0 and 20,000 Hz. The first 20 modes were expanded, and their eigenvalues collected. The results are shown in Figures 6–11. The results regarding the dynamic behavior presented in Figures 6 and 7, were compared by means of an improvement factor:

$$I_f = \frac{\omega_f - \omega_i}{\omega_i} * 100\% \tag{3}$$

where I_f is the improvement of the final models in relation to the initial models, in terms of characteristic frequencies, ω_i is the frequency of the initial models, and ω_f is the frequency of the final models.

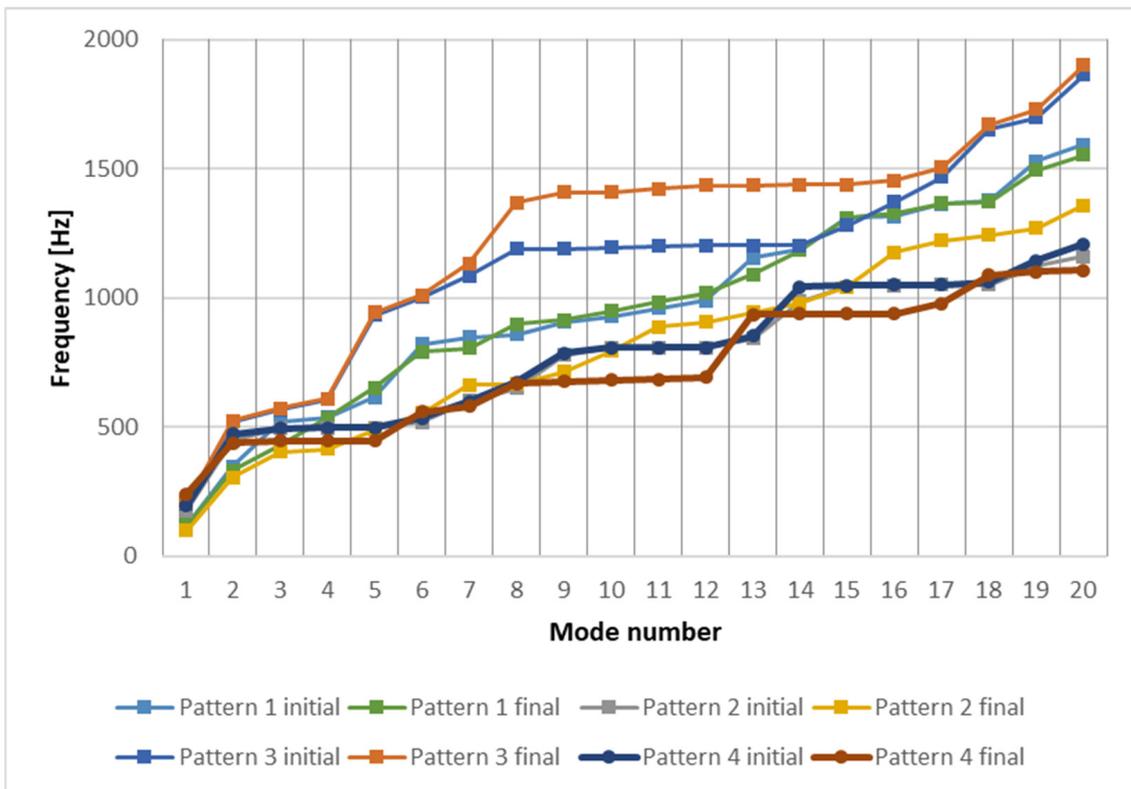


Figure 6. Frequency vs. mode number for beam A1.

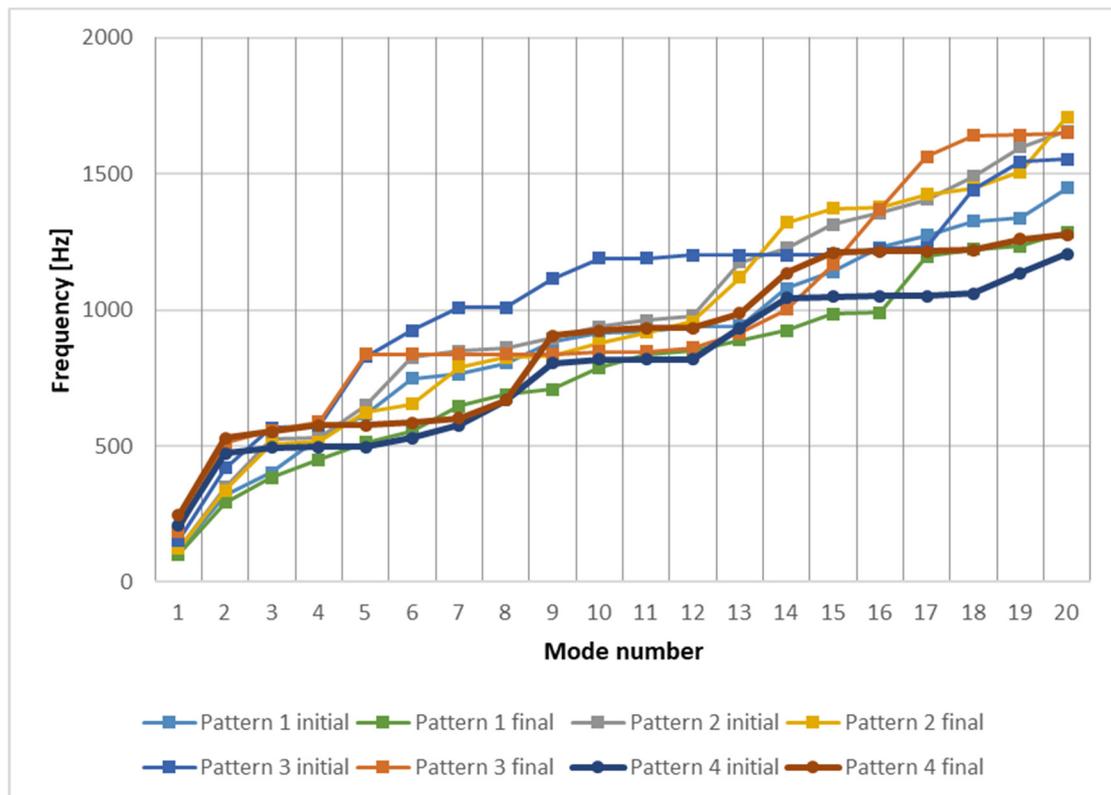


Figure 7. Frequency vs. mode number for beam A2.

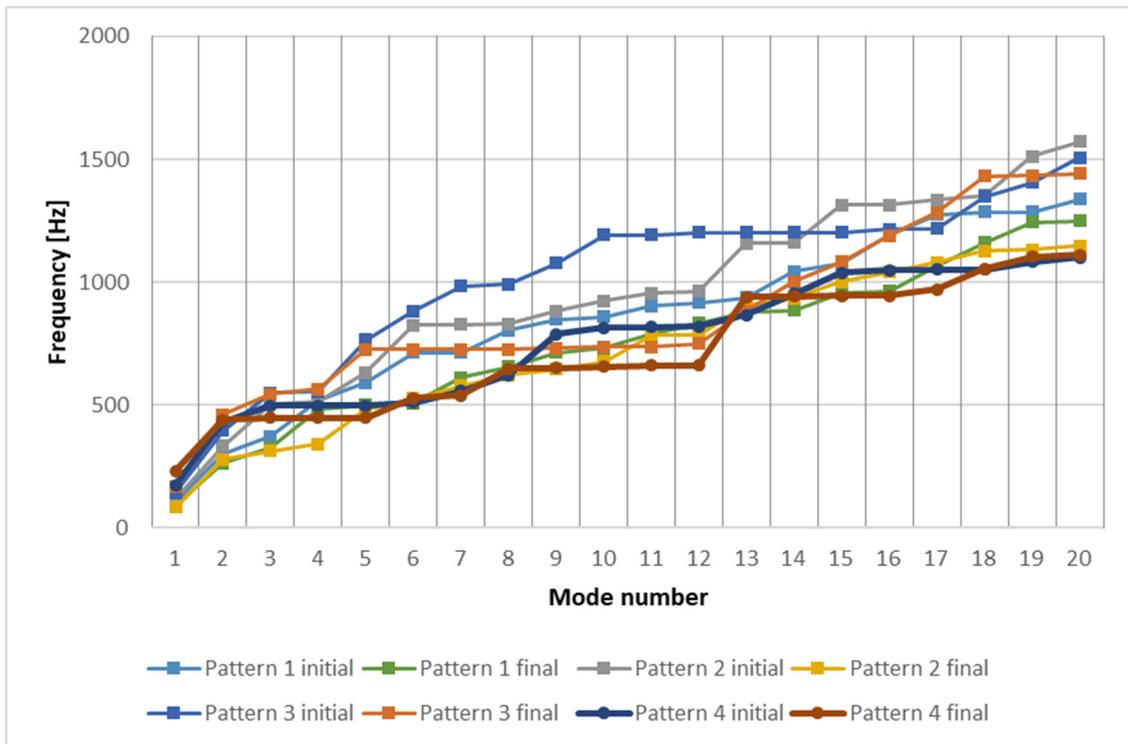


Figure 8. Frequency vs. mode number for beam A3.

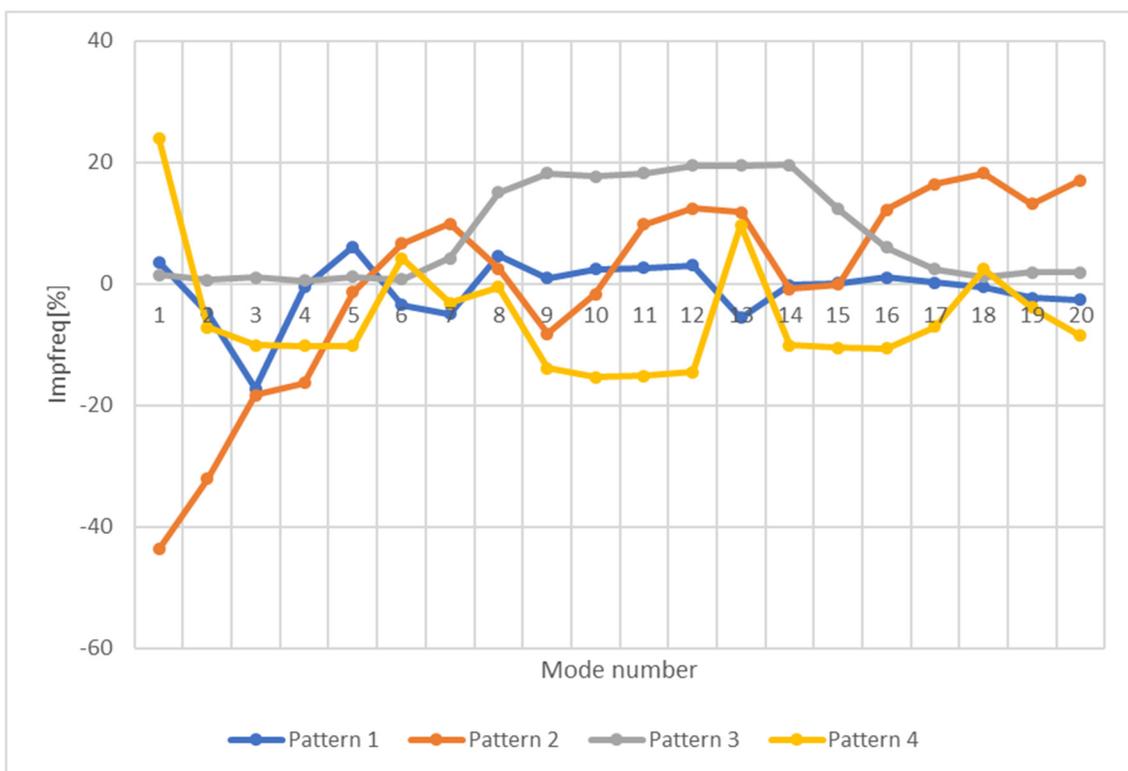


Figure 9. Improvement of statically optimized models in relation to the initial models, for beams of A1 type.

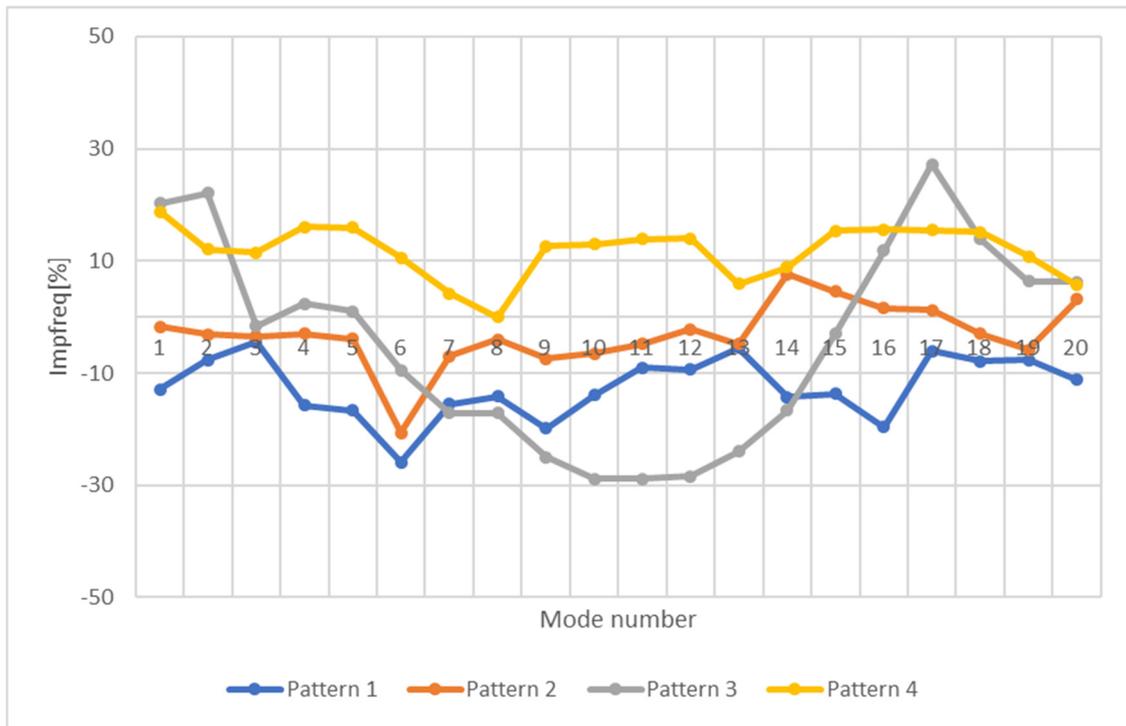


Figure 10. Improvement of statically optimized models in relation to the initial models, for beams of A2 type.

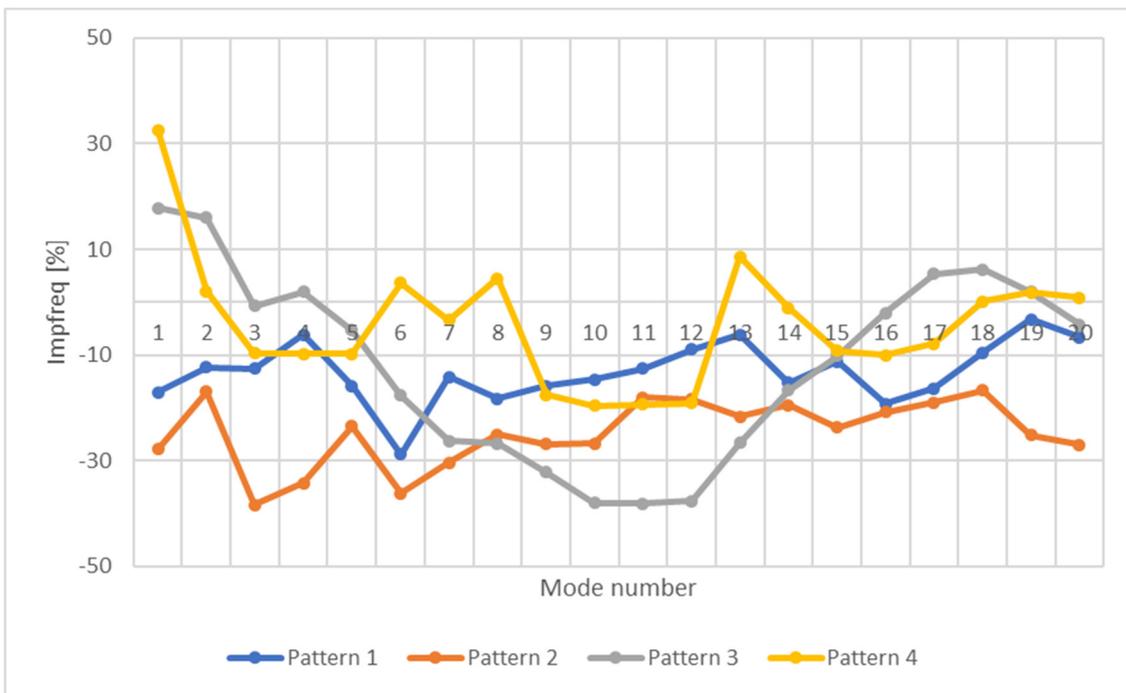


Figure 11. Improvement of statically optimized models in relation to the initial models for beams of A3 type.

It can be seen that all the beams behave similarly in terms of their dynamic response. The frequency range surpasses 1800 Hz for beam A1 pattern 3 final (optimized), shown in Figure 6. It is the maximum value of frequency obtained for all the models. Beam A3 pattern 4 (Figure 8) is the model that behaves best, as the highest maximization of frequencies can be obtained considering all models. The dynamic response of a structure,

in terms of natural frequencies, is very important in structural analysis. It is known that lower frequencies are more energetic, and, therefore, they are known to be more prone to disrupt the adequate operation of applications involving high accelerations and comprising lightweight mobile parts. The dynamic behavior can be improved if the natural frequencies of the optimized models can be maximized in comparison with the initial ones. It can be seen from beam A1, whose results are shown in Figure 9, there is an improvement for Patterns 1 and 3. Pattern 4 presents an advantage only for some modes, and Pattern 2 is causing an overall worsening. For beams of the A2 type, shown in Figure 10, there is an overall improvement for Pattern 4, while Pattern 3 gives that only for some modes, and Patterns 1 and 2 present an overall worsening. Beams of the A3 type, shown in Figure 11 present an improvement for some modes for Patterns 3 and 4 and an overall worsening for Patterns 1 and 2.

4. Conclusions

Although the internal reinforcements are useful in improving the static behavior, as shown in [12–14,16,17], the improvement they originate do not appear to be worth the price of increasing the mass. All the 12 studied beams were already subjected to optimization routines for the improvement of the static behavior [12–14,16,17]. The initial and optimized models were then subjected to modal analysis. When comparing the modal behavior of A3 pattern 4 with a simple hollow-box beam, it can be seen that the improvements are quite good. It is shown that overall, the improvement in the dynamic behavior originated by the static optimization is significant, albeit mild.

Author Contributions: Conceptualization, H.M.S.; methodology, H.M.S.; software, H.M.S.; validation, H.M.S.; formal analysis, H.M.S.; investigation, H.M.S.; resources, H.M.S.; data curation, H.M.S.; writing—original draft preparation, H.M.S.; writing—review and editing, J.W.; visualization, H.M.S.; supervision, J.W.; project administration, J.W.; funding acquisition, J.W. All authors have read and agreed to the published version of the manuscript.

Funding: H.M. Silva gratefully acknowledge the support provided by the Foundation for Science and Technology (FCT) of Portugal, within the scope of the project of the Research Unit on Materials, Energy and Environment for Sustainability (proMetheus), Ref. UID/05975/2020, financed by national funds through the FCT/MCTES.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Meirovitch, L. *Fundamentals of Vibrations*, reprinted ed.; McGraw Hill: New York, NY, USA, 2001.
2. Jonsson, J.; Andreassen, M. Distortional eigenmodes and homogeneous solutions for semi-discretized thin-walled beams. *Thin-Walled Struct.* **2011**, *49*, 691–707. [[CrossRef](#)]
3. Gonçalves, R.; Camotim, D. The vibration behaviour of thin-walled regular polygonal tubes. *Thin-Walled Struct.* **2014**, *84*, 77–188. [[CrossRef](#)]
4. Pagani, A.; Boscolo, M.; Banerjee, J.; Carrera, E. Exact dynamic stiffness elements based on one-dimensional higher-order theories for free vibration analysis of solid and thin-walled structures. *J. Sound Vib.* **2013**, *332*, 6104–6127. [[CrossRef](#)]
5. Bebianoa, R.; Basagliab, C.; Camotima, D.; Gonçalves, R. GBT buckling analysis of generally loaded thin-walled members with arbitrary flat-walled cross-sections. *Thin-Walled Struct.* **2018**, *123*, 11–24. [[CrossRef](#)]
6. Yang, F.; Zhong, Y.; Liu, R.; Cao, H. Effective performance analysis of stiffened honeycomb sandwich panels using VAM-based equivalent model. *Thin-Walled Struct.* **2023**, *185*, 110590. [[CrossRef](#)]
7. Du, Y.; Zou, T.; Pang, F.; Hu, C.; Ma, Y.; Li, H. Design method for distributed dynamic vibration absorbers of stiffened plate under different boundary constraints. *Thin-Walled Struct.* **2023**, *185*, 110494. [[CrossRef](#)]
8. Wang, Y.-J.; Zhang, Z.-J.; Xue, X.-M.; Zhang, L. Free vibration analysis of composite sandwich panels with hierarchical honeycomb sandwich core. *Thin-Walled Struct.* **2019**, *145*, 106425. [[CrossRef](#)]

9. Franco, C.; Chesnais, C.; Semblat, J.-F.; Giry, C.; Desprez, C. Finite element formulation of a homogenized beam for reticulated structure dynamics. *Comput. Struct.* **2022**, *261–262*, 106729. [[CrossRef](#)]
10. Silva, H.M.; Meireles, J.F. Comparative effectiveness of the mechanical behaviour of sandwich beams under uncoupled bending and torsion loadings. *Mech. Mech. Eng.* **2017**, *21*, 855–869.
11. Silva, H.M.; Wojewoda, J. Determination of the product of inertia of stiffened plates based on Finite Element Method results. *Eng. Struct.* **2020**, *207*, 1102. [[CrossRef](#)]
12. Silva, H.M.; Meireles, J.F. Feasibility of internally reinforced thin-walled beams for industrial applications. *Appl. Mech. Mater.* **2015**, *775*, 119–124. [[CrossRef](#)]
13. Silva, H.M.; Meireles, J.F. Numerical study on the mechanical behaviour of hollow-box beams subjected to static loading. *Mech. Mech. Eng.* **2017**, *21*, 871–884.
14. Silva, H.M.; Meireles, J.F. Structural Optimization of internally reinforced beams subjected to uncoupled and coupled bending and torsion loads for industrial applications. *Mech. Mech. Eng.* **2017**, *21*, 731–753.
15. Silva, H.M.; Meireles, J.F.; Wojewoda, J. Structural optimization coupled with materials selection for stiffness improvement. *Mech. Mech. Eng.* **2018**, *22*, 831–844. [[CrossRef](#)]
16. Silva, H.M.; Meireles, J.F.; Wojewoda, J. Experimental validation of a novel thin-walled beam prototype. *Mech. Mech. Eng.* **2018**, *22*, 7–23. [[CrossRef](#)]
17. Silva, H.M. Optimization of the Mechanical Behavior of Hollow-Box Beams. Ph.D. Thesis, Lodz University of Technology, Lodz, Poland, 2018.
18. Meireles, J.F. Análise Dinâmica de Estruturas por Modelos de Elementos Finitos Identificados Experimentalmente. Ph.D. Thesis, University of Minho, Braga, Portugal, 2008.
19. Silva, H.M. Determination of the Material/Geometry of the Section Most Adequate for a Static Loaded Beam Subjected to a Combination of Bending and Torsion. Master's Thesis, University of Minho, Braga, Portugal, 2011.
20. Silva, H.M.; Meireles, J.F. Determination of material/geometry of the section most adequate for a static loaded beam subjected to a combination of bending and torsion. *Mater. Sci. Forum* **2012**, *730–732*, 507–512. [[CrossRef](#)]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.