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The Efficiency of a Ratio Product Estimator in the Estimation of the Finite Population Coefficient of Variation †

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Abstract: Ratio product estimators have been proposed by several authors for the estimation of the population mean and population variance, but very few authors have proposed ratio product estimators for the estimation of the population coefficient of variation. In this paper, we propose a ratio product estimator for the estimation of the population coefficient of variation. The mean square error of the proposed estimator was obtained up to the first order of approximation using the Taylor series technique. A numerical analysis was conducted, and the results show that the proposed ratio product estimator is more efficient.

Keywords: estimator; MSE; coefficient of variation; study variable; finite population

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1. Introduction

Various estimation strategies have been developed by many researchers in the field of sample surveys for the estimation of population parameters, including ([1–4]). Some of the estimation methods use auxiliary information for the precise estimate of the parameter. Auxiliary information is information on auxiliary variables, like the population mean, population variance, sample mean, sample variance, and so on, which are used to improve the efficiency of estimators. Authors such as ([5–14]) have worked in that direction.

To estimate the population coefficient of variation, ref. [15] were the first to propose an estimator for the coefficient of variation when samples were selected using SRSWOR. Other works include those of ([16–21]).

In the current study, we propose a ratio product estimator in the presence of the population mean, population variance, sample mean, and sample variance of X for the estimation of the population coefficient of variation for the study variable Y, with the aim of obtaining a precise estimate of the parameter.

Following the introduction is Section 2, which contains the methodology and some existing estimators in the literature, while Section 3 presents the proposed estimator, bias, and MSE of the proposed estimator. Section 4 discusses the efficiency comparisons of the proposed estimator, while the empirical study and conclusion are presented in Section 5 and Section 6, respectively.

2. Methodology

Let us consider a simple random sample size, n, drawn from the given population of N units. Let the value of the study variable Y and the auxiliary variable X for the i^{th} units (i = 1, 2,3, 4, . . ., N) of the population be denoted by Y_i and X_i, and let the i^{th} unit in the sample (i = 1, 2, 3, . . ., n) be denoted by y_i and x_i, respectively.

$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ are the population means of the study and auxiliary variables.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2$ represents the population variance of the study variable.

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2$ represents the population variance of the auxiliary variable.

$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})(Y_i - \overline{Y})$ represents the population covariance of the auxiliary and study variable.

$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the sample mean of the study and auxiliary variables.

$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$ represents the sample variance of the study variable.

$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ represents the sample variance of the auxiliary variable.

$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$ represents the sample covariance of the auxiliary and study variable.

Now, let us define sampling errors for both the mean and variance of Y and X:

$$e_0 = \overline{Y}^{-1}(\overline{y} - \overline{Y}), e_1 = \overline{X}^{-1}(\overline{x} - \overline{X}), e_2 = \left(S_y^2\right)^{-1}\left(s_y^2 - S_y^2\right), e_3 = \left(S_x^2\right)^{-1}\left(s_x^2 - S_x^2\right)$$

Such that

$$\overline{y} = \overline{Y}(1 + e_0), \overline{x} = \overline{X}(1 + e_1), s_y = S_y(1 + e_2)^{1/2}, s_x = S_x(1 + e_3)^{1/2}, s_y^2 = S_y^2(1 + e_2), s_x^2 = S_x^2(1 + e_3)$$

$$\begin{gathered} E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0 \\ E\left(e_0^2\right) = \gamma C_y^2, E\left(e_1^2\right) = \gamma C_x^2, E\left(e_2^2\right) = \gamma(\lambda_{40} - 1), E\left(e_3^2\right) = \gamma(\lambda_{04} - 1), \\ E(e_0 e_1) = \gamma\rho C_y C_x, E(e_0 e_2) = \gamma C_y \lambda_{30}, E(e_0 e_3) = \gamma C_y \lambda_{12}, \end{gathered}$$

$E(e_1 e_2) = \gamma C_x \lambda_{21}, E(e_1 e_3) = \gamma C_x \lambda_{03}, E(e_2 e_3) = \gamma(\lambda_{22} - 1)$. Here, $\gamma = n^{-1}(1 - f)$ and $f = nN^{-1}$ are sampling fractions. $C_y = \overline{Y}^{-1} S_y$ and $C_x = \overline{X}^{-1} S_x$ are the population coefficients of variation for the study variable Y and the auxiliary variable X. Also, ρ denotes the correlation coefficient between X and Y.

In general,

$$\mu_{rs} = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \overline{y})^r (x_i - \overline{x})^s \text{ and } \lambda_{rs} = \mu_{rs}\left(\mu_{20}^{r/2}\mu_{02}^{s/2}\right)^{-1}, \text{ respectively.}$$

Some Existing Estimators in the Literature

The estimator used to estimate the population coefficient of variation in the absence of the auxiliary variable is given by

$$\hat{C}_y = \frac{s_y}{\overline{y}} \quad (1)$$

The mean square error (MSE) expression of the estimator $\hat{C}_y$ is given by

$$MSE\left(\hat{C}_y\right) = C_y^2\gamma\left(C_y^2 + 0.25(\lambda_{40} - 1) - C_y\lambda_{30}\right) \quad (2)$$

Also, [18] introduced estimators for calculating the finite population coefficient of variation. These estimators were designed specifically for estimating the coefficient of variation for one component of a bivariate normal distribution by considering prior knowledge about the second component. They established a Cramer–Rao-type lower bound based on the mean square error of these estimators. Through extensive simulations, they compared 28 estimators and found that 8 of them exhibited higher relative efficiency compared to the sample coefficient of variation. They also provided the asymptotic mean square errors for the most effective estimators, offering valuable insights for users in calculating the coefficient of variation. Thus, the estimators are given as follows:

$$t_{AR1} = \hat{C}_y \left(\frac{\overline{X}}{\overline{x}} \right) \tag{3}$$

$$t_{AR2} = \hat{C}_y \left(\frac{\overline{x}}{\overline{X}} \right) \tag{4}$$

$$t_{AR3} = \hat{C}_y \left(\frac{S_x^2}{s_x^2} \right) \tag{5}$$

$$t_{AR4} = \hat{C}_y \left(\frac{s_x^2}{S_x^2} \right) \tag{6}$$

The mean square error (MSE) expressions of the estimators are given by the following:

$$MSE(t_{AR1}) = C_y^2 \gamma \Big[C_y^2 + 0.25(\lambda_{40} - 1) + C_x^2 - C_x\lambda_{21} - C_y\lambda_{30} + 2\rho C_y C_x \Big] \tag{7}$$

$$MSE(t_{AR2}) = C_y^2 \gamma \Big[C_y^2 + 0.25(\lambda_{40} - 1) + C_x^2 + C_x\lambda_{21} - C_y\lambda_{30} - 2\rho C_y C_x \Big] \tag{8}$$

$$MSE(t_{AR3}) = C_y^2 \gamma \Big[C_y^2 + 0.25(\lambda_{40} - 1) + (\lambda_{04} - 1) - (\lambda_{22} - 1) - C_y\lambda_{30} + 2C_y\lambda_{12} \Big] \tag{9}$$

$$MSE(t_{AR4}) = C_y^2 \gamma \Big[C_y^2 + 0.25(\lambda_{40} - 1) + (\lambda_{04} - 1) + (\lambda_{22} - 1) - C_y\lambda_{30} - 2C_y\lambda_{12} \Big] \tag{10}$$

Thus, [20] introduced three estimators that combine difference and ratio approaches for estimating the coefficient of variation in a finite population. These estimators utilize the known population mean, population variance, and population coefficient of variation of an auxiliary variable. They also investigated the biases and mean square errors (MSEs) associated with these proposed estimators. By comparing their performances with existing estimators using information from two populations, they demonstrated that their proposed estimators were superior in efficiency compared to various other estimators, including unbiased, ratio type, exponential ratio type, and difference type estimators. Thus, the estimators are as follows:

$$T_{M1} = \left[\frac{\hat{C}_y}{2} \left(\frac{\overline{X}}{\overline{x}} + \frac{\overline{x}}{\overline{X}} \right) + w_1 (\overline{X} - \overline{x}) + w_2 \hat{C}_y \right] \left(\frac{\overline{X}}{\overline{x}} \right) \tag{11}$$

$$T_{M2} = \left[\frac{\hat{C}_y}{2} \left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2} \right) + w_3 \left(S_x^2 - s_x^2 \right) + w_4 \hat{C}_y \right] \left(\frac{S_x^2}{s_x^2} \right) \tag{12}$$

The mean square errors of the estimators are given by

$$MSE(T_{M1}) = C_y^2 \Big(A + w_1^2 B + w_2^2 C + 2w_1 D - 2w_2 E - 2w_1 w_2 F \Big) \tag{13}$$

$$MSE(T_{M2}) = C_y^2 \Big(A_1 + w_3^2 B_1 + w_4^2 C_1 + 2w_3 D_1 - 2w_4 E_1 - 2w_3 w_4 F_1 \Big) \tag{14}$$

where $A = \gamma\left(C_x^2 + C_y^2 + 2\rho C_y C_x - C_x\lambda_{21} - C_y\lambda_{30} + \frac{(\lambda_{40}-1)}{4}\right)$, $B = \gamma\delta^2(\lambda_{04}-1)$ for $\delta = \frac{\overline{X}}{C_y}$, $C = 1 + \gamma\left(3C_x^2 + 3C_y^2 + 4\rho C_y C_x - 2C_x\lambda_{21} - 2C_y\lambda_{30}\right)$, $D = \gamma\delta\left(C_x^2 + \rho C_y C_x - \frac{C_x\lambda_{21}}{2}\right)$, $E = \gamma\left(\frac{3C_x\lambda_{21}}{2} - 3\rho C_y C_x - \frac{5C_x^2}{2} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right)$, $F = \gamma\delta\left(\frac{C_x\lambda_{21}}{2} - \rho C_y C_x - 2C_x^2\right)$; and $A_1 = \gamma\left((\lambda_{04}-1) + C_y^2 + 2C_y\lambda_{12} - (\lambda_{22}-1) - C_y\lambda_{30} + \frac{(\lambda_{40}-1)}{4}\right)$, $B_1 = \gamma\delta_1^2(\lambda_{22}-1)$ for $\delta_1 = \frac{S_x^2}{C_y}$, $C_1 = 1 + \gamma\left(3(\lambda_{04}-1) + 3C_y^2 + 4C_y\lambda_{12} - 2(\lambda_{22}-1) - 2C_y\lambda_{30}\right)$, $D_1 = \gamma\delta_1\left((\lambda_{04}-1) + C_y\lambda_{12} - \frac{(\lambda_{22}-1)}{2}\right)$, $E_1 = \gamma\left(\frac{3(\lambda_{22}-1)}{2} - 3C_y\lambda_{12} - \frac{5(\lambda_{04}-1)}{2} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right)$, $F_1 = \gamma\delta_1\left(\frac{(\lambda_{22}-1)}{2} - C_y\lambda_{12} - 2(\lambda_{04}-1)\right)$; and $w_1 = \frac{CD-EF}{F^2-BC}$, $w_2 = \frac{DF-BE}{F^2-BC}$, $w_3 = \frac{C_1D_1-E_1F_1}{{F_1}^2-B_1C_1}$ and $w_4 = \frac{D_1F_1-B_1E_1}{{F_1}^2-B_1C_1}$.

The minimum mean square errors of the estimators are given by

$$MSE(T_{M1})_{\min} = C_y^2\left[A + \frac{(CD^2 + BE^2 - 2DEF)}{(F^2 - BC)}\right] \tag{15}$$

$$MSE(T_{M2})_{\min} = C_y^2\left[A_1 + \frac{\left(C_1{D_1}^2 + B_1{E_1}^2 - 2D_1E_1F_1\right)}{\left({F_1}^2 - B_1C_1\right)}\right] \tag{16}$$

3. Proposed Estimator

Having studied the estimators developed by [18,20] for the estimation of the finite population coefficient of variation, we therefore proposed a new ratio product estimator in the presence of the population mean, population variance, sample mean, and sample variance of X for the estimation of the population coefficient of variation of the study variable Y, with the aim of obtaining a precise estimate of the parameter. As such, the proposed estimator is given as follows:

$$T_g = \hat{C}_y\left[k_1\left(\frac{\overline{X}}{\overline{x}}\right)\left(\frac{S_x^2}{s_x^2}\right) + k_2\left(\frac{\overline{x}}{\overline{X}}\right)\left(\frac{s_x^2}{S_x^2}\right)\right] \tag{17}$$

where k_1 and k_2 are unknown constants to be determined.

Expressing Equation (17) in terms of error terms, we obtain the following:

$$T_g = \frac{S_y(1+e_2)^{\frac{1}{2}}}{\overline{Y}(1+e_0)}\left[k_1\left(\frac{S_x^2}{S_x^2(1+e_3)}\right)\left(\frac{\overline{X}}{\overline{X}(1+e_1)}\right) + k_2\left(\frac{S_x^2(1+e_3)}{S_x^2}\right)\left(\frac{\overline{X}(1+e_1)}{\overline{X}}\right)\right] \tag{18}$$

After simplifying Equation (18) to the first order of approximation, we obtain

$$T_g = C_y\left[k_1\begin{pmatrix} 1 - e_3 + e_3^2 - e_1 + e_1e_3 + e_1^2 - e_0 \\ +e_0e_3 + e_0e_1 + e_0^2 + \frac{e_2}{2} - \frac{e_2e_3}{2} \\ -\frac{e_1e_2}{2} - \frac{e_0e_2}{2} + \frac{e_2^2}{8} \end{pmatrix} + k_2\begin{pmatrix} 1 + e_3 + e_1 + e_1e_3 - e_0 - e_0e_3 \\ -e_0e_1 + e_0^2 + \frac{e_2}{2} + \frac{e_2e_3}{2} + \frac{e_1e_2}{2} \\ -\frac{e_0e_2}{2} - \frac{e_2^2}{8} \end{pmatrix}\right] \tag{19}$$

By subtracting C_y from both sides of Equation (19), we obtain

$$T_g - C_y = C_y\left[k_1\begin{pmatrix} 1 - e_3 + e_3^2 - e_1 + e_1e_3 + e_1^2 - e_0 \\ +e_0e_3 + e_0e_1 + e_0^2 + \frac{e_2}{2} - \frac{e_2e_3}{2} \\ -\frac{e_1e_2}{2} - \frac{e_0e_2}{2} + \frac{e_2^2}{8} \end{pmatrix} + k_2\begin{pmatrix} 1 + e_3 + e_1 + e_1e_3 - e_0 - e_0e_3 \\ -e_0e_1 + e_0^2 + \frac{e_2}{2} + \frac{e_2e_3}{2} + \frac{e_1e_2}{2} \\ -\frac{e_0e_2}{2} - \frac{e_2^2}{8} \end{pmatrix} - 1\right] \tag{20}$$

Expectations on both sides of Equation (20) are taken to obtain the bias of the estimator as follows:

$$Bias(T_g) = C_y\left[k_1\left(1+\gamma\begin{pmatrix}(\lambda_{04}-1)+C_x\lambda_{03}+C_x^2\\+C_y\lambda_{12}+\rho C_yC_x+C_y^2\\-\frac{(\lambda_{22}-1)}{2}-\frac{C_x\lambda_{21}}{2}\\-\frac{C_y\lambda_{30}}{2}-\frac{(\lambda_{40}-1)}{8}\end{pmatrix}\right)+k_2\left(1+\gamma\begin{pmatrix}C_x\lambda_{03}-C_y\lambda_{12}\\-\rho C_yC_x+C_y^2\\+\frac{(\lambda_{22}-1)}{2}+\frac{C_x\lambda_{21}}{2}\\-\frac{C_y\lambda_{30}}{2}-\frac{(\lambda_{40}-1)}{8}\end{pmatrix}\right)-1\right] \quad (21)$$

Expectations on both sides of Equation (20) are squared and taken to obtain the mean square error (MSE) of the estimator as follows:

$$MSE(T_g) = C_y^2\left(1+k_1^2A_2+k_2^2B_2+2k_1k_2C_2-2k_1D_2-2k_2E_2\right) \quad (22)$$

where,

$$A_2 = 1+\gamma\left(3(\lambda_{04}-1)+4C_x\lambda_{03}+3C_x^2+4C_y\lambda_{12}+4\rho C_yC_x+3C_y^2-2(\lambda_{22}-1)-2C_x\lambda_{21}-2C_y\lambda_{30}\right),$$
$$B_2 = 1+\gamma\left(4C_x\lambda_{03}-4C_y\lambda_{12}-4\rho C_yC_x+3C_y^2+2(\lambda_{22}-1)+2C_x\lambda_{21}-2C_y\lambda_{30}+(\lambda_{04}-1)+C_x^2\right),$$
$$C_2 = 1+\gamma\left(3C_y^2-2C_y\lambda_{30}\right),$$
$$D_2 = 1+\gamma\begin{pmatrix}(\lambda_{04}-1)+C_x\lambda_{03}+C_x^2+C_y\lambda_{12}+\rho C_yC_x+C_y^2-\frac{1}{2}(\lambda_{22}-1)-\frac{1}{2}C_x\lambda_{21}-\frac{1}{2}C_y\lambda_{30}\\-\frac{1}{8}(\lambda_{40}-1)\end{pmatrix},$$
$$E_2 = 1+\gamma\left(C_x\lambda_{03}-C_y\lambda_{12}-\rho C_yC_x+C_y^2+\frac{1}{2}(\lambda_{22}-1)+\frac{1}{2}C_x\lambda_{21}-\frac{1}{2}C_y\lambda_{30}-\frac{1}{8}(\lambda_{40}-1)\right).$$

By differentiating Equation (22) partially with respect to k_1 and k_2 and equating the terms obtained to zero, we obtain $A_2k_1+C_2k_2=D_2$ and $C_2k_1+B_2k_2=E_2$, and by solving these simultaneously, we obtain the optimum values of k_1 and k_2,

$k_1 = \frac{B_2D_2-C_2E_2}{A_2B_2-C_2^2}$ and $k_2 = \frac{A_2E_2-C_2D_2}{A_2B_2-C_2^2}$, and putting these expressions into Equation (22) gives the following minimum mean square error $(\mathrm{MSE})_{\min}$:

$$MSE(T_g)_{\min} = C_y^2\left[1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D\ E_2\right)}{\left(A_2B_2-C_2^2\right)}\right] \quad (23)$$

4. Efficiency Comparisons

In this section, the efficiency conditions of T_g over the sample coefficient of variation, $\hat{C}_y$, t_{AR1}, t_{AR2}, t_{AR3}, t_{AR4}, T_{M1}, and T_{M2}, were established.

i. T_g is more efficient than $\hat{C}_y$ if

$$MSE(T_g)_{\min} < MSE(\hat{C}_y) \quad (24)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right) < \gamma\left(C_y^2+0.25(\lambda_{40}-1)-C_y\lambda_{30}\right) \quad (25)$$

ii. T_g is more efficient than t_{AR1} if

$$MSE(T_g)_{\min} < MSE(t_{AR1}) \quad (26)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right) < \gamma\left(C_y^2+0.25(\lambda_{40}-1)+C_x^2-C_x\lambda_{21}-C_y\lambda_{30}+2\rho C_yC_x\right) \quad (27)$$

iii. T_g is more efficient than t_{AR2} if

$$MSE(T_g)_{\min} < MSE(t_{AR2}) \quad (28)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\gamma\left(C_y^2+0.25(\lambda_{40}-1)+C_x^2+C_x\lambda_{21}-C_y\lambda_{30}-2\rho C_yC_x\right) \quad (29)$$

iv. T_g is more efficient than t_{AR3} if

$$MSE(T_g)_{\min}<MSE(t_{AR3}) \quad (30)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\gamma\left(\begin{array}{l}C_y^2+0.25(\lambda_{40}-1)+(\lambda_{04}-1)-(\lambda_{22}-1)\\-C_y\lambda_{30}+2C_y\lambda_{12}\end{array}\right) \quad (31)$$

v. T_g is more efficient than t_{AR4} if

$$MSE(T_g)_{\min}<MSE(t_{AR4}) \quad (32)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\gamma\left(\begin{array}{l}C_y^2+0.25(\lambda_{40}-1)+(\lambda_{04}-1)+(\lambda_{22}-1)\\-C_y\lambda_{30}-2C_y\lambda_{12}\end{array}\right) \quad (33)$$

vi. T_g is more efficient than T_{M1} if

$$MSE(T_g)_{\min}<MSE(T_{M1}) \quad (34)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\left(A+\frac{\left(CD^2+BE^2-2DEF\right)}{\left(F^2-BC\right)}\right) \quad (35)$$

vii. T_g is more efficient than T_{M2} if

$$MSE(T_g)_{\min}<MSE(T_{M2}) \quad (36)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\left(A+\frac{\left(C_1D_1^2+B_1E_1^2-2D_1E_1F_1\right)}{\left(F_1^2-B_1C_1\right)}\right) \quad (37)$$

5. Empirical Study

In this section, an empirical study will be carried out to demonstrate the performance of the proposed estimator over the existing ones. Data from the books by Murthy (1967) [22] and Sarjinder Singh (2003) [23] will be used.

Population 1: (Source: [22])

X: Area under wheat in 1963; Y: area under wheat in 1964

$$N=34, n=15, \overline{X}=208.88, \overline{Y}=199.44, C_x=0.72, C_y=0.75, \rho=0.98, \lambda_{21}=1.0045, \lambda_{12}=0.9406,$$
$$\lambda_{40}=3.6161, \lambda_{04}=2.8266, \lambda_{30}=1.1128, \lambda_{03}=0.9206, \lambda_{22}=3.0133$$

Population 2: (Source: [23])

X: Number of fish caught in the year 1993; Y: number of fish caught in the year 1995

$$N=69, n=40, \overline{X}=4591.07, \overline{Y}=4514.89, C_x=1.38, C_y=1.35, \rho=0.96, \lambda_{21}=2.19, \lambda_{12}=2.30,$$
$$\lambda_{40}=7.66, \lambda_{04}=9.84, \lambda_{30}=1.11, \lambda_{03}=2.52, \lambda_{22}=8.19$$

Table 1 shows the mean square error (MSE) and the percentage relative efficiency (PRE) of the proposed estimator. The results revealed that the proposed estimator has a minimum mean square error and a higher percentage relative efficiency. This implies that the suggested estimator is more efficient than the existing ones.

Table 1. MSEs and PREs of proposed and existing estimators.

	Population 1	Population 1	Population 2	Population 2
Estimators	**MSE**	**PRE**	**MSE**	**PRE**
$\hat{C}_y$	0.008003575	100	0.03808827	100
t_{AR1}	0.02589068	30.91296	0.08517984	44.71512
t_{AR2}	0.01184353	67.57761	0.06393314	59.57516
t_{AR3}	0.03365777	23.77928	0.188603	20.19494
t_{AR4}	0.05890541	13.58716	0.2261359	16.84309
T_{M1}	0.006737495	118.7916	0.03533973	107.77748
T_{M2}	0.006013652	133.09009	0.02810758	135.5089
T_g	**0.004943499**	**161.901**	**0.01718988**	**221.5738**

6. Conclusions

In this study, we proposed a ratio product estimator for the estimation of the finite population coefficient of variation. This estimator utilized information on the sample and population mean as well as the sample and population variance of the auxiliary variable X. The results from the numerical analysis show that the proposed estimator is more efficient than the conventional estimators with the evidence of having a minimum mean square error; hence, it should be applied for estimation in real-life situations.

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