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Modeling Contagion of Financial Markets: A GARCH-EVT Copula Approach †

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Abstract: To better assess the financial contagion through the VaR, several recent studies used copula models. In the same context, this paper addresses the inefficiency of the classical approach such as a normal distribution in modeling the tail risk, by using the conditional Extreme Value Theory (GARCH-EVT), in order to assess extreme risks with contagion effect. The GARCH-EVT approach is a two-stage hybrid method that combines a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) filter with the Extreme Value Theory (EVT). To implement our approach, we use macroeconomic time series from Morocco, Spain, France, and the USA.

Keywords: contagion effects; extreme value theory; GARCH-EVT; optimal tail selection; value at risk

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1. Introduction

Financial resilience in banking is considered a key pillar when discussing the strength of the international financial system and the world economy as a whole. Indeed, financial resilience becomes more puzzling and worrying in the context of increasingly frequent, significant, and complex events. These extreme events include the stock market crash of 1929, the stock market crash of 1987, the sudden devaluation of the Mexican peso against the U.S. dollar in December 1994, the 1997 Asian financial crisis, and the global financial crisis between mid-2007 and early 2009. All these crashes are characterized by a subsequent rapid spread, significant severe losses incurred by financial institutions, spillovers, and high contagion risks. These events revealed substantial weaknesses in the banking system and the prudential framework and thus, motivated many of the managers and researchers, to recover existing tools and to implement new management strategies that offer significant improvement, by taking into consideration the increased severity, the high frequency of extreme events and spillover effects.

One important suggestion is to reconsider the Value-at-Risk (VaR); the widely used risk management tool, in the context of extreme events and contagion effects, which are nonlinear, time-varying, and dependent in nature.

The VaR can be defined as the maximum potential change in the value of a portfolio of financial instruments with a given probability over a certain horizon. There are several approaches for the estimation of VaR, such as historical simulation, variance-covariance, and the Monte Carlo approaches. In addition, contagion can be empirically identified through the propagation of extreme negative returns, the increase in interdependence compared to normal times, and the distinction from common shocks [1]. The literature contains various definitions of financial contagion [2]. However, financial contagion is present if a statistically significant increase is observed in cross-market correlation after the occurrence of extreme shocks [3].

To better assess the financial contagion through the VaR, several recent studies used copula models to describe the multivariate dependence structure between financial markets, estimate the return period, and assess the corresponding losses. In the same context, this paper addresses the inefficiency of the classical approach such as a normal distribution in modeling the tail risk, by using the conditional Extreme Value Theory (GARCH-EVT), in order to assess extreme risks with contagion effect. The GARCH-EVT approach is a two-stage hybrid method that combines a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) filter with the Extreme Value Theory (EVT). The Peaks-Over-Threshold approach will be used for the pre-specification of the threshold that separates distribution tails from its middle part.

To implement our approach, we use time series retrieved by assessing the open-source records available on an international website. All statistical analyses were performed using R packages and our results provide important insights on risk management.

The remaining parts of the study are laid out as follows: Section 2 covers the research methodology and design, and results and findings are delineated in Section 3. Section 4 concludes the paper.

2. Materials and Methods

Our methodology is based on four main stages (Figure 1):

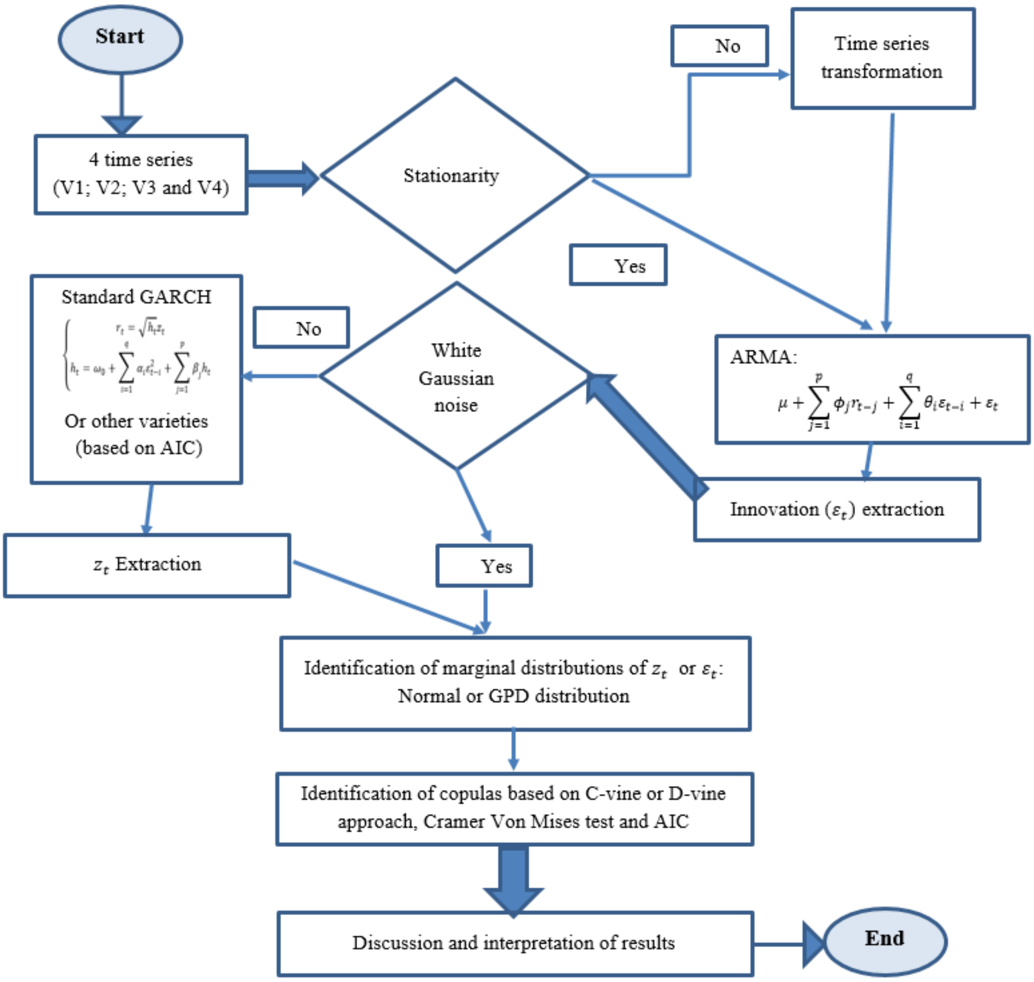

Figure 1. Proposed methodology flowchart.

Stage 1: Modeling time series with ARMA models. We first check whether the used times series are stationary or not, using visualization or analytical approach such as the Augmented Dickey-Fuller test (ADF). When the stationarity is not accepted, transformation is needed and thus, an ARMA model is identified [4]. A mixed autoregressive moving average process of order (p, q) process is a stationary process $\{Y_t\}$ which satisfies the relation:

$$r_t = \mu + \sum_{j=1}^{p} \phi_j r_{t-j} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t, \tag{1}$$

where $\phi_j, j = 1, 2, \ldots, p, \theta_i, i = 1, 2, \ldots, q$ are parameters of the ARMA model to be estimated.

Stage 2: Innovations extraction and Gaussian white noise assumption checking. Innovations ε_t are extracted from the ARMA model and the Ljung and Box portmanteau test is used to examine if ε_t can be considered Gaussian White Noise.

Stage 3: Use of GARCH and identification of marginal distributions. If the assumption of the Gaussian White Noise is not validated, a standard sGARCH model or other GARCH varieties, such as GJR-GARCH [5], are identified, and then, z_t are extracted. The dynamics of the conditional volatility of the GARCH(p,q) model are given by:

$$\begin{cases} r_t = \sqrt{h_t} z_t \\ h_t = \omega_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_t \end{cases} \tag{2}$$

The GJR-GARCH is given by

$$\begin{cases} r_t = \sqrt{h_t} z_t \\ h_t = \omega_0 + \sum_{i=1}^{q} (\alpha_i + \chi_i I(\varepsilon_{t-i} < 0)) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \end{cases} \tag{3}$$

where z_t is normalized white noise and h_t is the conditional variance of the innovation ε_t, ω is the intercept, and the parameters α_i, χ_i and β_j are the autoregressive coefficients of the variance. Marginal distribution is identified for ε_t and z_t.

Stage 4: Copulas fitting based on C-Vine and D-vine approach. The main idea of vine copulas is the modeling of copulas in high dimensions, based on a structure of interconnected trees of bivariate copula. This construction approach makes it possible to model complex dependencies in high dimensions by bivariate copulas [6].

The main issue in the field of financial contagion is to analyze the underlying process and to emphasize the main variables that could indicate a financial crisis in a country. This has led researchers to broaden the scope of the investigation and thus, several categories such as common shocks, trade spillovers, and financial linkages are identified. In this context, the empirical studies suggested by Račickas and Vasiliauskaitė [7] present a number of determinants of a financial crisis. It is worth noting that some explanatory variables are exclusive for currency crises, banking crises, or debt crises; others are informative for more than one type of crisis.

To implement our approach, we identify four time-series associated with financial contagion from the World Development Indicators (WDI) website. More details are provided in the following table (Table 1).

Table 1. Used data.

Variables		Unit	Countries	Covered Period
V1	GDP per capita growth	Annual %	France	1970–2021
V2	Trade	% of GDP	Morocco	
V3	Inflation, consumer prices	Annual %	Spain	
V4	Exports as a capacity to import	Constant LCU	US	

3. Results and Discussions

A descriptive analysis shows that there is a strong interconnection between Morocco and Spain. These two neighboring partners are linked by more than 16 billion euros of trade and; Morocco is the third economic partner of Spain outside the EU. In addition, Spanish exports to Morocco have increased by 29% in 2020/2021, 17,000 Spanish companies have trade relations with Morocco and 700 are established in the neighboring country. It is also worth noting the increase in the range of Moroccan exports to Spain in recent years, reflecting the modernization of the national productive fabric.

France remains one of Morocco's leading economic partners, despite growing competition in the areas of trade and investment. The relationship between the two countries makes France the first partner of Morocco on the level of commercial exchange, tourist arrivals, and direct investments.

A detailed analysis, from Figure 2, shows a simultaneous trend in terms of inflation, exports, trade, and inflation in Morocco, France, Spain, and the US. In addition, the trade and the exports time series exhibit an upward trend, while the GDP time series are more or less stable, with the exception of Morocco for which more fluctuations are noted.

The analysis of inflation leads to discerning two distinguishable periods. The first one, before 1985, was characterized by high levels of inflation (% annual) with a maximum of 17.6%, 13.6%, 13.5%, and 24.5% for Morocco, France, the US, and Spain, respectively. The second period is characterized by controlled inflation.

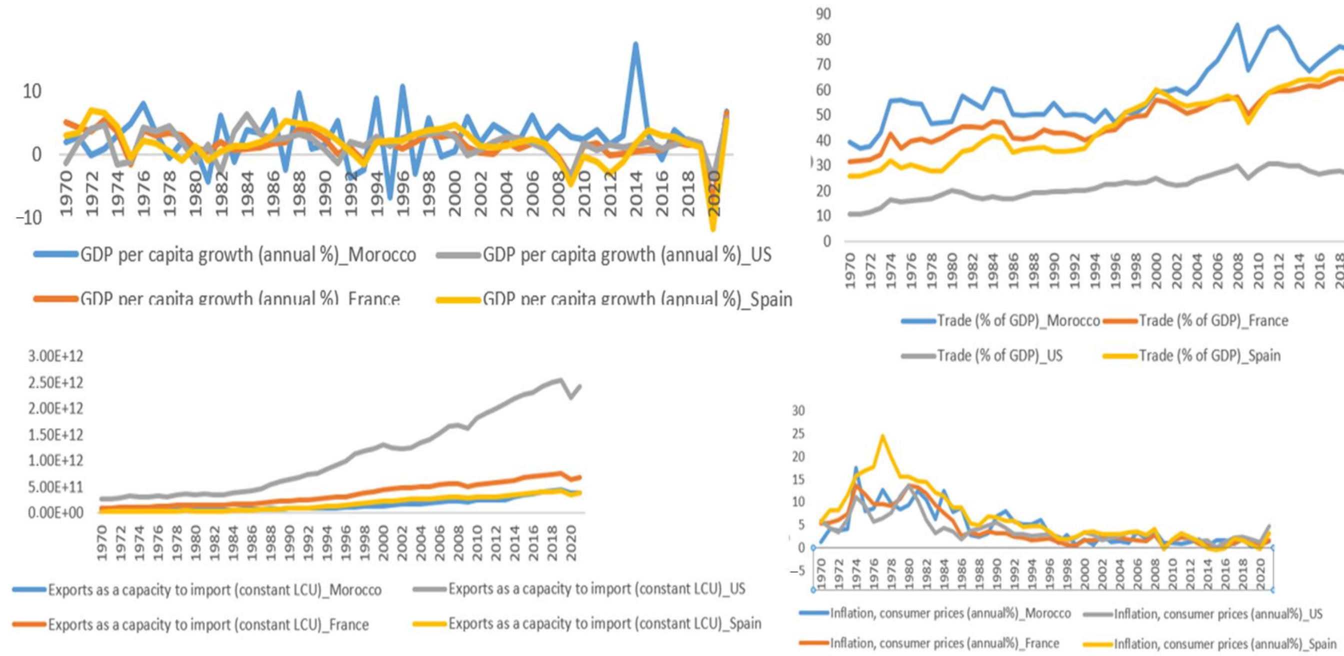

Figure 2. Trends comparison in terms of inflation, exports, trade, and GDP.

The analysis of the inflation leads to discerning two distinguishable periods. The first one, before 1985, was characterized by high levels of inflation (% annual) with a maximum of 17.6%, 13.6%, 13.5%, and 24.5% for Morocco, France, the US, and Spain, respectively. The second period is characterized by controlled inflation.

It is worth noting that all computations are performed using R software. To implement our methodology, we transform the raw data to have stationary time series. Results show also that most fitted models are ARMA(1,1) which is characterized by autocorrelation functions that decline dramatically and ARMA(1,0) which predicts the present value of a time series, using the immediately prior value in time.

Results (Table 2) indicate that most of the analyzed time-series in this study have a non-homogenous variance, so there is a GARCH effect. Except for the US time series, the assumption of Gaussian white noise is not satisfied, thus SGARCH or GJR-GARCH are estimated for those time-series.

Table 2. ARMA and GARCH modeling.

Indicators		Time Series	Model
V1	France	Raw	ARMA (1,1) $\mu = 0; \varphi = -0.03; \theta = -0.99$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 0.31; \alpha = 0.43; \beta = 0.94; \gamma = 0.76$
	Morocco	Raw	ARMA (1,0) $\mu = 0; \varphi = -0.72$
		Residuals	BB Gaussian
	Spain	Raw	ARMA (1,1) $\mu = 0; \varphi = 0.45; \theta = -1$
		Residuals	sGARCH(1,1):$\alpha_0 = 0; \alpha = 0.31; \beta = 0.68$
	US	Raw	ARMA (1,1) $\mu = -0.01; \varphi = 0.11; \theta = -0.99$
		Residuals	BB Gaussian
V2	France	Raw	ARMA (1,1) $\mu = 4.53; \varphi = -0.17; \theta = -0.99$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 1.67; \alpha = 0.017; \beta = 0.99; \gamma = 0.18$
	Morocco	Raw	ARMA (1,1) $\mu = 1.19; \varphi = 0.74; \theta = -0.99$
		Residuals	BB Gaussian
	Spain	Raw	$\sqrt{h_t} z_t : \mu = 0; \varphi = 0; \theta = 0$
		Residuals	sGARCH(1,1):$\alpha_0 = 0; \alpha = 0; \beta = 0.888$
	US	Raw	ARMA (1,0) $\mu = 1.69; \varphi = 0.05$
		Residuals	BB Gaussian
V3	France	Raw	ARMA (1,0) $\mu = -41.13; \varphi = -0.22$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 932.71; \alpha = 0.98; \beta = 0; \gamma = 0.04$
	Morocco	Raw	ARMA (1,0) $\mu = -1.14; \varphi = -0.56$
		Residuals	BB Gaussian
	Spain	Raw	ARMA (1,1) $\mu = -2.77; \varphi = -0.90; \theta = 1$
		Residuals	sGARCH(1,1):$\alpha_0 = 0; \alpha = 0; \beta = 0.83$
	US	Raw	ARMA (1,1) $\mu = -0.01; \varphi = -0.44; \theta = 0.76$
		Residuals	BB Gaussian
V4	France	Raw	ARMA (1,1) $\mu = 11.73; \varphi = 0.99; \theta = -0.94$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 2.93; \alpha = 0; \beta = 0.99; \gamma = 0.26$
	Morocco	Raw	ARMA (1,1) $\mu = 5.22; \varphi = 0.56; \theta = -0.99$
		Residuals	BB Gaussian
	Spain	Raw	$\mu + \sqrt{h_t} z_t : \mu = 13.78; \varphi = 0; \theta = 0$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 3.54; \alpha = 0.21; \beta = 0.62; \gamma = 0.32$
	US	Raw	ARMA (1,1) $\mu = 10.57; \varphi = -0.79; \theta = 1$
		Residuals	GJR-GARCH(1,1):$\alpha_0 = 2.66; \alpha = 0; \beta = 1; \gamma = 0.26$

Once the innovations are extracted, marginal distributions are identified and copulas are fitted. It is worth noting that Copulas are fitted with two marginal; the Normal and the Generalized Pareto distribution (GPD) and by using C-Vines and D-vines approaches.

Tables 3 and 4 present the multivariate dependencies between the different retained indicators (Trade, GDP, Inflation, and Export) among countries (Morocco, Spain, France, and the US), using D-Vines and C-Vines copulas. From these results, we can have a clear idea about the different structures of the dependencies, and thus contagion mechanisms, such as the dependence between exportation in Morocco and exportation in the US given information on exportations in Spain and France (Copula ($U_{\text{Export Morocco}}$; $U_{\text{Export US}}/U_{\text{Export France}}$; $U_{\text{Export Spain}}$). For this example, the identified Copula is the survival of Clayton Copula. The structure of dependence between the exportations in France and the exportations in Morocco, given the information on the exportations in Spain (Copula ($U_{\text{Export France}}$; $U_{\text{Export Morocco}}/U_{\text{Export Spain}}$) is as Gumbel Copulas. Both Gumbel and Survival Clayton are considered as extreme value Copulas. The consequence of these findings is that the impact of the contagion is remarkable at the extremes, characterized by subsequent rapid spread, significant severe losses, spillovers, and high contagion risks.

Table 3. Copulas fitting (marginals are considered Normal distributions).

Copulas	Designation (D-Vines)	Copulas Family	Designation (D-Vines)	Copulas Family
c_{32}	$U_{\text{Export spain}}$; $U_{\text{Export Morocco}}$	t	$U_{\text{GDP Spain}}$; $U_{\text{GDP Morocco}}$	t
c_{13}	$U_{\text{Export France}}$; $U_{\text{Export spain}}$	t	$U_{\text{GDP France}}$; $U_{\text{GDP Spain}}$	t
c_{41}	$U_{\text{Export US}}$; $U_{\text{Export France}}$	Normal	$U_{\text{GDP US}}$; $U_{\text{GDP France}}$	Survival Gumbel
$c_{12,3}$	$U_{\text{Export France}}$; $U_{\text{Export Morocco}}$ /$U_{\text{Export spain}}$	Gumbel	$U_{\text{GDP France}}$; $U_{\text{GDP Morocco}}$/$U_{\text{GDP Spain}}$	Rotated Gumbel 90 degrees
$c_{43,1}$	$U_{\text{Export US}}$; $U_{\text{Export Spain}}$/$U_{\text{Export France}}$	Rotated Clayton 270 degrees	$U_{\text{GDP US}}$; $U_{\text{GDP Spain}}$/ $U_{\text{GDP France}}$	Joe
$c_{24,13}$	$U_{\text{Export Morocco}}$; $U_{\text{Export US}}$/$U_{\text{Export France}}$; $U_{\text{Export Spain}}$	Survival Clayton	$U_{\text{GDP Morocco}}$; $U_{\text{GDP US}}$ /$U_{\text{GDP France}}$; $U_{\text{GDP Spain}}$	Rotated Clayton 270 degree
Copulas	**Designation (D-vines)**	**Copulas Family**	**Designation (C-vines)**	**Copulas Family**
c_{32}	$U_{\text{Inflation spain}}$; $U_{\text{Inflation Morocco}}$	Normal	c_{12}: $U_{\text{trade France}}$; $U_{\text{trade Morocco}}$	Normal
c_{13}	$U_{\text{Inflation France}}$; $U_{\text{Inflation spain}}$	Frank	c_{13} : $U_{\text{trade France}}$; $U_{\text{trade spain}}$	Survival Gumbel
c_{41}	$U_{\text{Inflation US}}$; $U_{\text{Inflation France}}$	Guassian	c_{14} : $U_{\text{trade France}}$; $U_{\text{trade US}}$	Survival BB7
$c_{12,3}$	$U_{\text{Inflation France}}$; $U_{\text{Inflation Morocco}}$/$U_{\text{Inflation spain}}$	Frank	$c_{24;1}$: $U_{\text{trade Morocco}}$; $U_{\text{trade US}}$/$U_{\text{trade France}}$	Survival Clayton
$c_{43,1}$	$U_{\text{Inflation US}}$; $U_{\text{Inflation Spain}}$/$U_{\text{Inflation France}}$	t	$c_{34;1}$: $U_{\text{trade spain}}$; $U_{\text{trade US}}$/$U_{\text{trade France}}$	Survival Clayton
$c_{24,13}$	$U_{\text{Inflation Morocco}}$; $U_{\text{Inflation US}}$/ $U_{\text{Inflation France}}$; $U_{\text{Inflation Spain}}$	Frank	$c_{23,14}$: $U_{\text{trade Morocco}}$; $U_{\text{trade Spain}}$/$U_{\text{trade France}}$; $U_{\text{trade US}}$	Rotated Joe 90 degree

Table 4. Copulas fitting (marginals are considered GPD).

Copulas	Designation (C-Vines)	Copulas Family	Designation (C-Vines)	Copulas Family
c_{31}	$U_{\text{Export Morocco}}$; $U_{\text{Export France}}$	t	c_{21}: $U_{\text{Inflation Spain}}$; $U_{\text{Inflation France}}$	Joe
c_{32}	$U_{\text{Export Morocco}}$; $U_{\text{Export Spain}}$	t	c_{32} : $U_{\text{Inflation US}}$; $U_{\text{Inflation spain}}$	Joe
$c_{21;3}$	$U_{\text{Export Spain}}$; $U_{\text{Export France}}$/$U_{\text{Export Morocco}}$	Survival Joe	$c_{31;2}$: $U_{\text{Inflation US}}$; $U_{\text{Inflation France}}$/$U_{\text{Inflation Spain}}$	Survival Gumbel
$c_{2,1}$	$U_{\text{GDP Spain}}$; $U_{\text{GDP France}}$	Survival BB7	$c_{1;2}$: $U_{\text{Trade France}}$; $U_{\text{Trade Spain}}$	Joe
$c_{3,2}$	$U_{\text{GDP US}}$; $U_{\text{GDP Spain}}$	Joe	$c_{3;1}$: $U_{\text{trade US}}$; $U_{\text{trade France}}$	Survival BB7
$c_{31,2}$	$U_{\text{GDP US}}$; $U_{\text{GDP France}}$/$U_{\text{GDP Spain}}$	Joe	$c_{32,1}$: $U_{\text{trade US}}$; $U_{\text{tradeSpain}}$/$U_{\text{tradeFrance}}$	Joe

For Inflation, it is worth noting that most fitted Copulas are Gaussian Copulas, which means that the structure of the dependence is not strong at the extremes such as different crises. This is because the inflation is controlled by central banks that have different visions and implement different adequate policies to maintain the inflation controlled.

4. Conclusions

The dependence on financial markets during a period of extreme fluctuations has received considerable attention within the literature. In the same context, the main contribution of this work is to understand the structure of dependence between different pertinent variables that can be used, to explain the contagion of financial Markets. Financial contagion can be defined as the spread of an economic crisis from one market or region to another so, events in one market can affect other markets.

In this study, the used methodology is based on the GARCH-EVT Copula approach. Copula modeling is a popular tool for analyzing the dependencies between variables.

It allows the investigation of tail dependencies and the specification of models for the marginal distributions separately from the dependence structure and more specifically, the Copula co-movements capture how shocks in a particular market may transcend to other currency markets. These implications are of particular interest in risk, survival applications, and prediction of financial contagion. More recently, there are different empirical applications of Copula-based methods in economics, due to the flexibility of the approach and the gain in terms of the computational complexity of estimation. Our findings starkly highlight the adequacy of two copulas. The first one; the Normal Copula, is appropriate for the inflation while the second is suitable for trade, exportations, and the GDP. It is worth noting that the normal copula provides a general linear form of the dependence and captures a general form of the dependence, while the survival Clayton Copula characterizes the dependence in the extremes.

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Conflicts of Interest: The authors declare no conflict of interest.

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