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Inventory Improvement in Tyre Retail through Demand Forecasting †

Diana Neves [1], Magda Monteiro [1,2,*] and Maria José Felício [1,3]

1 ESTGA-Águeda School of Technology and Management, University of Aveiro, 3750-127 Águeda, Portugal; dcn@ua.pt (D.N.); mjfelicio@ua.pt (M.J.F.)
2 CIDMA–Center for Research and Development in Mathematics and Applications, University of Aveiro, 3810-193 Aveiro, Portugal
3 GOVCOPP-Research Unit on Governance, Competitiveness and Public Policies, University of Aveiro, 3810-193 Aveiro, Portugal
* Correspondence: msvm@ua.pt
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Abstract: The aim of this study is to develop the inventory planning system of a Portuguese tyre retailer based on forecasting sales models. Using sales history up to 2020, tyres were grouped into three levels of sales aggregation and different quantitative forecasting models were applied. The comparison of these models resorted to various evaluation measures to choose the most suitable one for each group. The study shows that for items with sales grouped monthly and for items with sales grouped by semester, Holt's method had a better performance on determining sales forecasts, while for tyres with sales grouped quarterly, it was Croston's method that stood out. The inventory policy outlined for each group of items reflects the results of the forecasted demand, and the review period depends on the sales group under analysis. In agreement with previous studies, the usefulness of statistical methods is corroborated. Additionally, the advantage of combining the said methods proved helpful, particularly as a starting point for tyre retail inventory planning.

Keywords: iventory; inventory planning; forecasting models; sales forecasting; intermittent demand

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1. Introduction

A retailer's inventory is extremely critical. Low liquidity caused by high inventory backlog or poor customer experiences ensuing from a shortage of inventory arises from decisions taken during inventory management [1]. Hence, it is necessary to improve the levels of inventory held, which can be achieved by forecasting the sales of the items sold or needed to provide the service.

The European Union represents 20% of the global market of light and commercial tyres [2]. However, this market is highly sensitive to external factors: namely, the advancement of the automobile industry or improvements in the economy and road transportation.

Mahama-Musah et al. [3] observed that the independent aftermarket (private workshops) is most popular for purchasing tyres and that the internet is widely consulted to search for locations to replace tyres and information about brands, prices, and timings. Concerning the choice of tyre brand, price and quality are the most common, with this choice also being highly influenced by the mechanic's opinion.

In Portugal, tyre retailing is quite fragmented and occurs mainly in small businesses, and, consequently, there is a high level of competition. Thus, this case study aimed to develop sales forecast models for tyres sold by a Portuguese retailer so as to improve the planning system and the inventory policy in place while also maintaining a compromise between the reduction of inventory related costs and the satisfaction of consumers.

To date, inventory planning in this retailer only resorts to human judgment without assistance from any forecasting models. Therefore, the aim was to develop simple models that will allow a user to forecast the sales of each tyre size so as as to upgrade the company's tyre inventory levels and lower the associated costs.

The study focused on applying several quantitative forecasting methods to each tyre size. Afterwards a comparative analysis was made to select the most adequate size for each one. The novelty of the study relies on the field of application, bringing the use of simple and commonly used quantitative forecasting methods, which allows the improvement of the current tyre inventory levels without much effort due to their easy implementation either in the usual demand context or in a context of intermittent demand.

This paper is organized as follows. The next section is devoted to a literature review followed by the presentation of the case study. Afterwards, the practical work carried out is described, namely, the selection of the data used, the methods followed, the results obtained, and the main conclusions drawn. Finally, the general conclusions are shown.

2. Literature Review

Inventory management is responsible for ensuring that the right amount of each item is always in stock and in a cost-effective way. To do so, it resorts to inventory control, which supports operational decisions on when and how much to replenish for each of the stock keeping units, as well as the parts and materials used to produce them. The operating costs involved also include inventory, holding, ordering, and stockout costs [4].

The time to place an order can follow one of three approaches [5,6]:

- Carry out a periodic review (R) and place orders of variable size at regular time intervals, bringing the inventory to a certain level (S) ((R,S) policy).
- Conduct an ongoing review and place a fixed quantity (S) order as soon as inventory levels fall below the defined threshold (s) ((s,S) policy).
- Link supply to demand by ordering sufficient stock to meet expected demand in a specific time period ((R,s,S) policy).

Once the frequency and the size of the orders are decided, aspects such as the average inventory level, safety inventory, and level of customer service are automatically defined.

The methods developed for making predictions can be divided into two broad groups: qualitative (by judgment) and quantitative.

The most common judgment forecasting methods include the manager's opinion, the panel of executive opinion, the sales force opinion, the market survey to consumers, the historical analogy, and the Delphi method [5,7].

These methods are valuable when there is little or no historical data or when changes in the market turn existing data unsuitable for forecasting purposes. Strategies for their improvement include combining with other judgmental or quantitative estimates [8].

In retail sales forecast, statistical methods are the most utilized—namely, simple moving averages, exponential smoothing, autoregressive integrated moving averages, and regression, applying the Box and Jenkins approach type [9–11].

The advantages of exponential smoothing methods are their simplicity, low cost, and easy implementation. Fildes et al. [12] concluded that the performance of these methods depends on how the smoothing parameters are estimated and on how they are initialized, so information should be taken from the time series itself.

Moreover, there are several situations where items in inventory are infrequently requested and show great variability in demand values, which results in sporadic demand with a high risk of obsolescence. Simple exponential smoothing (SES) has proven to be a robust forecasting method and is probably the most used among statistical approaches to forecast intermittent demand [11,13]. However, Croston [14] observed that SES obtained negatively biased forecasts immediately before demand occurred and positively afterwards, which resulted in excessive inventory levels, and thus created a method where the forecast results from the ratio between the smoothed demand and the smoothed time between demands, using it for both SES with the same smoothing parameter [13,15,16].

Syntetos and Boylan [17] revised Croston's method, having developed a modified version (Syntetos–Boylan approximation, SBA) that theoretically eliminates the positive bias of the forecast.

Sbrana [18] has also suggested an intermittent model that considers that a time series switches between the state of a local level plus a constant and zero, reflecting the intermittency of demand. By doing this, it derives prediction intervals surpassing Croston's theory and its lack of an underlying stochastic model.

Nikolopoulos et al. [19] applied the aggregate-disaggregate intermittent demand approach, based on the theory that forecasts with higher levels of aggregation are more accurate and less variable. If after aggregation there is no demand equal to zero, any forecasting method can be used and the estimates can be disaggregated for a detailed analysis.

Artificial Neural Networks (ANN) are also involved in forecasting intermittent demand as they can model time series without assuming function models *a priori* [20,21].

The forecasting method to be used depends on several factors, from the forecast time window to the demand behaviour or its causes. Therefore, it is highly unlikely that a single model remains the best fit over time. Thus, estimates resulting from different methods may provide useful information, so combining them can be advantageous, as proven by Petropoulos et al. [22] and Hibon and Evgeniou [23].

It is also standard to assess the performance of forecasting methods by measuring their errors. These measures are useful to determine the model that best fits a time series and help in choosing and optimizing parameters, such as smoothing constants. However, choosing one measure over others may lead to completely different conclusions, and may even lead to disregarding one model that might be perfectly suitable for forecasting [24].

Syntetos and Boylan [13] argue that the presence of zeros needs to be considered in intermittent demand. So, studying the effect of the forecasts on inventory control parameters is more adequate, specifically on the resulting inventory and service levels [25,26].

Wallström and Segerstedt [27] compared several forecasting methods to show that a single error measure is not representative. Among others, they determined the number of stockouts and introduced the number of periods in inventory (PI) that considers the total number of periods in which the forecasted units remain in stock. Beyond the error, this also evaluates the time it takes to correct it.

As stated by several authors [28–30], the better accuracy of one forecasting model over others does not translate into better efficiency in inventory control, as what is crucial is how to use the forecast to achieve the targeted level of consumer service or to minimize the cost. A holistic understanding of the specific (and joint) nature of the inventory forecasting problem is required as it is furthered [4].

Regarding the forecast of tyre sales, the existing literature is very limited as performing these has been difficult, to some extent due to the impact that human psychology has on the decision of purchasing tyres [31]. The models used to this purpose are univariate, and rely only on past sales and estimates by experts [32].

More recent studies on forecasting retail sales have included macroeconomic factors, ANN, data mining models, hybrid models, or even extreme learning machine. Some papers have also combined expert judgement and statistical forecasts [9,32].

3. Data and Methods

This section describes the process of choosing the tyre sizes that will be targeted for inventory improvement by forecasting their sales volume as well as the methods used.

3.1. Selection of Items for Analysis

The company started selling tyres in May 2011, and by October 2020 had sold a total of 225 tyre sizes, including tyres for light, commercial, 4×4, and heavy vehicles. Currently there are only 181 of these references in inventory. The sizes corresponding to single orders from specific customers were also excluded.

For each of the 181 tyre sizes, monthly sales were aggregated in a total of 114 months. There were 99 references with highly intermittent sales (less than 20 non-zero observations over 114 months) and a very low impact on profit, reasons that led to their exclusion from the analysis. For each of the remaining 82 references, the percentage of months with non-zero observations was determined.

Items with less than 30% of non-zero observations were aggregated in semesters (28 references), items ranging from 30% to 50% non-zero observations were trimestral aggregated (23 references), and items with more than 50% of the months with non-zero observations were considered in this time unit (31 references).

The advantage of data aggregation is that it broadens the spectrum of forecasting methods that can be employed, as advocated by Nikolopoulos et al. [19].

3.2. Methods

To predict sales, exponential smoothing methods, such as SES and Holt's method, were tested, and so were Croston's method and SBA. Multiple linear regression (MLR) and generalized linear models (GLM) were also examined.

A training set was used that included the data, since sales first occurred in February 2020, and so different sales conditions were showcased. The test set comprised data from March 2020 until February 2021 with the aim of including all 12 months.

3.2.1. Smoothing Methods

The prediction formula for SES is given by:

$$P_{t+1} = \alpha Y_t + (1 - \alpha) P_t, \tag{1}$$

with Y_t representing the sales of the item at period t, P_{t+1} the forecast obtained at the next period and α the smoothing parameter, ranging from 0 to 1. To initialize the method, the first prediction was considered equal to the first observation [33].

Regarding the smoothing parameter, according to Hyndman and Athanasopoulos [33], it is more accurate to estimate it from the observed data, so the parameter was obtained by minimizing the root mean squared error (RMSE) using Microsoft's Excel Solver.

The application of Holt's method requires the following equations [34]:

$$\hat{a}_t = \alpha Y_t + (1 - \alpha)\left(\hat{a}_{t-1} + \hat{b}_{t-1}\right), \tag{2}$$

$$\hat{b}_t = \beta(\hat{a}_t + \hat{a}_{t-1}) + (1 - \beta)\hat{b}_{t-1}, \tag{3}$$

$$P_{t+h} = \hat{a}_t + \hat{b}_t \times h, \tag{4}$$

where the first and second equations define the level and slope at each time point, respectively. The smoothing parameters, α and β, range from 0 to 1. The third equation corresponds to the sales forecast h-step-ahead.

This method also needs to be initialized, both in level and slope, so the average method was used, considering the recommendation of Fildes et al. [12]. For the tyre sizes where monthly forecasts were made, the first semester was used to this purpose. The first quarter was used for quarterly forecasts and two years for half-yearly forecasts.

The smoothing parameters were obtained through the minimization of the RMSE.

In Croston's method, the forecast is made using the following equations [14]:

$$P_{t+1} = \frac{a_t}{T_t}, \tag{5}$$

$$\begin{cases} a_{t+1} = \alpha Y_t + (1 - \alpha) a_t, & Y_t > 0 \\ a_{t+1} = a_t, & Y_t = 0 \end{cases}, \tag{6}$$

$$T_{t+1} = \alpha q_t + (1 - \alpha) T_t, \tag{7}$$

with P_t being the forecast of the demand at time t, a_t the correspondent level, T_t the time between the occurrence of two demands, and q_t the number of successive periods since the last demand occurrence. Thus, T_t is only updated when demand occurs.

Regarding the initialization of the method, a_1 corresponds to the first demand occurrence Y_1 and T_1 is equal to $Y_1 + 1$. The parameter α is the smoothing parameter, ranging from 0 to 1, and was obtained by minimizing the RMSE.

The SBA correction, which multiplies the forecasts resulting from the previous method (P_{t+1}) by $1 - \frac{\alpha}{2}$, was also tested.

3.2.2. Multiple Linear Regression and Generalized Linear Models

Having detected an apparent increase in tyre purchases in the months before and when rainfall typically occurs (September, October, November, and December) and in the months before summer travels (June, July, and August), simple linear regression (SLR) models were employed with time as the independent variable (t):

$$Y_t = \beta_0 + \beta_1 t + \varepsilon_t, \tag{8}$$

and MLR models, which also considered dummy variables:

$$Y_t = \beta_0 + \beta_1 t + \beta_2 c_t + \beta_3 v_t + \varepsilon_t, \tag{9}$$

that considered purchases in the rainy months (c_t, where 1 identifies September/October, October/November, or November/December, and 0 the remaining months) and others that account for purchases before summer trips (v_t, where 1 identifies June/July or July/August and 0 the remaining months), following Waters [5].

The parameter estimates of the models that minimize the MSE were obtained via SPSS software, version 27. The addition of the dummy variables to the SLR model increases the coefficient of determination, R^2, but the significant dummy variables are different between tyre sizes.

The use of these models in forecasting tyre sales should only be undertaken if the assumptions associated with them are met: namely, the normal distribution, homoscedasticity, and independence of the errors. In cases where these were compromised, GLM with Poisson and negative binomial distributions were considered due to the discrete nature of tyre demand. For the above distributions, the link function used is the logarithm [35]:

$$f(\mu) = log(\mu). \tag{10}$$

For a set of covariates, the tyre sales are conditionally independent with one of the above distributions whose mean, μ, relates with these covariates through the formula:

$$log(\mu) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3. \tag{11}$$

Note that these models have a set of premises that need to be validated. The response variable must be non-negative, its observations independent, and the log mean a linear function of the covariates. In Poisson's distribution, equidispersion must also be validated.

3.2.3. Assessment Measures

To compare the accuracy and efficiency of the tested forecasting models, several measures were determined in accordance with Wallström and Segerstedt's [27] conclusion. The first was the RMSE, which measures the deviation of the estimates from the real value, advocated as appropriate by Bretschneider [36].

The average final inventory level, the percentage of shortages (which occurs whenever the final inventory of the period is equal to or lower than zero,) and PI were also calculated. The formulas applied were:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2}; \tag{12}$$

$$\begin{gathered}\text{Average Final Inventory: } \overline{FI} = \tfrac{1}{n}\textstyle\sum_{t=1}^{n} FI_t \times \mathbf{1}_{FI_t \geq 0}, \\ \text{where } FI_t = FI_{t-1} + \hat{Y}_t - Y_t, \text{ and } FI_1 = \hat{Y}_1 - Y_1;\end{gathered} \tag{13}$$

$$\text{Shortage Percentage: } SP = \frac{1}{n}\sum_{t=1}^{n} \mathbf{1}_{FI_t \leq 0} \times 100\%; \tag{14}$$

$$\begin{gathered}\text{Average Period in Inventory: } \overline{PI} = \tfrac{1}{n}\textstyle\sum_{t=1}^{n} PI_t, \\ \text{where } PI_t = PI_{t-1} + \textstyle\sum_{i=1}^{t}\left(\hat{Y}_i - Y_i\right), \text{ and } PI_1 = \hat{Y}_1 - Y_1.\end{gathered} \tag{15}$$

Finally, the estimates of the two forecasting methods with lower RMSE were combined through simple average according to Makridakis and Winkler [37], and following Aiolfi and Timmermann [38], a weighted combination of the same two methods was also performed for each tyre size according to the following formula:

$$Weighted\ Prediction_t = \frac{\text{RMSE}_2}{\text{RMSE}_1 + \text{RMSE}_2} Prediction_{1t} + \frac{RMSE_1}{\text{RMSE}_1 + \text{RMSE}_2} Prediction_{2t} \tag{16}$$

4. Results

This section presents the results obtained, divided into the levels of the aggregation of the tyre. First, the values determined for a specific tyre size are revealed, and then a global analysis of the remaining sizes is carried out.

4.1. Sales Grouped by Month

As for tyres with sales grouped monthly, the number of observations stands between 93 and 106, while the percentage of non-zero demand varies from 57% to 100%.

4.1.1. The Case of the 185/55R15 Tyre Size

The 185/55R15 tyre size has a total of 391 units sold, showing sales in 83% of the months. Table 1 shows the results obtained with each of the methods tested and in relation to each evaluation measure. The values of the combination of the two methods with the lowest RMSE value are also presented.

Table 1. Results of the evaluation measures for five forecasting methods for the 185/55R15 tyre size.

Measures	SES	Holt	GLM	Croston	SBA	Combined Forecast	
						Simple Average	Weighted Average
RMSE	3.039	3.170	**2.978**	3.252	3.156	3.004	3.004
$\overline{FI}$	16.248	**10.406**	19.535	34.347	12.822	16.248	16.248
SP	7.767%	17.476%	**6731%**	14.563%	15.534	7.767%	7.767%
$\overline{PI}$	**187**	−1127	−232	−708	−821	−22	−4

The method with the lowest RMSE value is the Poisson regression model, followed by SES and SBA. In this case, the estimated GLM model (ANOVA p-value < 5%) is:

$$log(\mu_t) = 1.266 + 0.298c_t, \tag{17}$$

with the independent variable months of October/November (c_t) being statistically significant, considering a 5% significance level. Time (t) and months of July/August (v_t) are not statistically significant.

The premises associated with the Poisson model were assessed and validated.

Figures 1 and 2 allow the visualization of the adjusted *p*-value of the determined models to the real observations, reflecting some differences, which consequently translate into different values in the evaluation measures, as shown in Table 1.

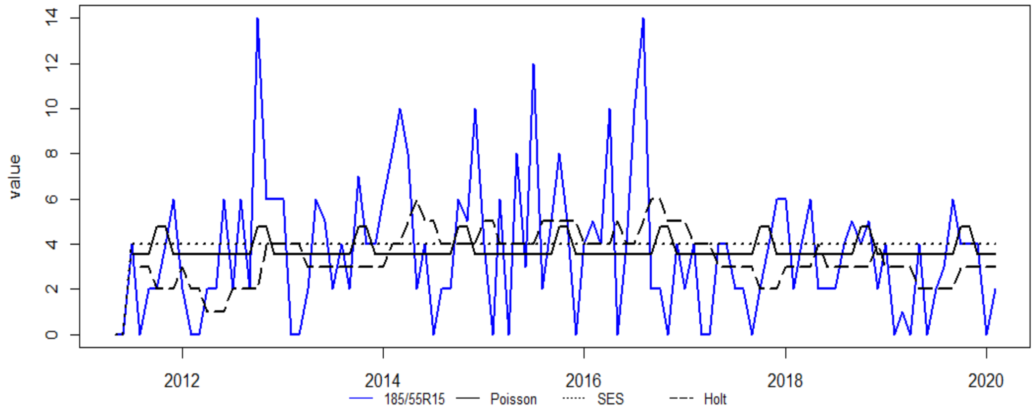

Figure 1. Poisson, SES and Holt's models for the 185/55R15 tyre size.

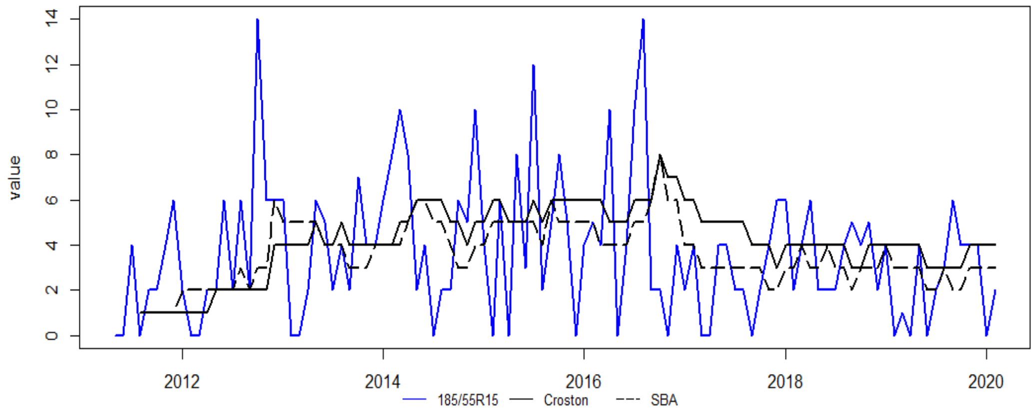

Figure 2. Intermittent models for the 185/55R15 tyre size.

Regarding the remaining assessment measures, the lowest $\overline{FI}$ value is obtained with Holt's method, but this method shows the highest SP, as seen in Table 1. Poisson's model, on the other hand, has the lowest SP, and SES shows the lowest $\overline{PI}$ value (not considering the weighted forecast).

Thus, based on Table 1, the model with the appropriated adjustment for the 185/55R15 tyre size results from the weighted combination of two methods with the lowest RMSE value, SES and GLM, as suggested by Petropoulos et al. [22]. In fact, the weighted model presents the second smallest value of RMSE and SP, the third smallest $\overline{FI}$ and the lowest $\overline{PI}$ value, being considered suitable to forecast.

Through the analysis of Table 2, one can observe that Holt's method and SES are the methods with the lowest value of RMSE when making estimates. Furthermore, the table shows that the optimal period for reviewing the data is monthly, which is expected since this item's sales are high and show variability, requiring constant revision.

Given the above, the most suitable inventory policy for the 185/55R15 tyre size is the (R,s,S) policy, where the review period (R) must be monthly and the quantity to order (S) will aim to satisfy the expected demand, which is around 3 units (average S), as shown in Table 3, considering the final inventory of the previous month and the safety inventory (s). The latter must be established in two units in order to satisfy unforeseen demand because of the variability observed in sales, ensuring that it is possible to satisfy one customer's needs (in general, two tyres are always supplied at a time).

Table 2. RMSE values from March 2020 to February 2021, for the 185/55R15 tyre size, without reviewing the previously obtained model and updating it every 1, 2, 4, and 6 months with real data.

		SES	**Holt**	**GLM**	**Croston**	**SBA**
DETERMINED MODEL		3.039	3.170	2.978	3.252	3.156
REVISION	No review	3.109	2.692		3.004	2.842
	Monthly	3.109	**2.475**	3.132	3.141	3.140
	2 months	3.109	2.795		3.187	3.226
	4 months	3.109	2.934		3.169	3.161
	6 months	3.109	2.737		2.955	2.883

Table 3. Determination of the monthly quantity to order (S) for the 185/55R15 tyre size.

	Real Sales	**Forecast**		
		Holt	**SES**	**Weighted**
MAR/20	0	2	4	3
APR/20	4	2	4	3
MAY/20	2	2	4	3
JUN/20	0	2	4	3
JUL/20	2	2	4	3
AUG/20	4	2	4	3
SEP/20	10	2	4	3
OCT/20	2	3	4	4
NOV/20	0	3	4	4
DEC/20	0	3	4	3
JAN/21	2	2	4	3
FEB/21	4	2	4	3

4.1.2. Global Analysis of Remaining Tyre Sizes

The results of the forecasting methods allow us to conclude that SES is the method with the lowest RMSE in 48% of tyre sizes, followed by Holt's method in 26% of the sizes, as shown in Table 4.

Table 4. Summary of the evaluation measures applied to each forecasting method, considering tyre sizes with sales grouped monthly.

	SES	**Holt**	**MLR/GLM**	**Croston**	**SBA**
LOWEST RMSE	**15 (48%)**	8 (26%)	2 (6%)	2 (6%)	4 (13%)
2ND LOWEST RMSE	8 (26%)	**9 (29%)**	0 (0%)	**9 (29%)**	5 (16%)
SMALLEST $\overline{FI}$	6 (19%)	7 (23%)	0 (0%)	6 (19%)	**12 (39%)**
LARGEST $\overline{FI}$	7 (23%)	**13 (42%)**	0 (0%)	11 (35%)	0 (0%)
LOWER ***SP***	**17 (55%)**	11 (35%)	1 (3%)	1 (3%)	1 (3%)
HIGHER ***SP***	2 (6%)	4 (13%)	0 (0%)	11 (35%)	**14 (45%)**
LOWEST $\overline{PI}$ (ABSOLUTE VALUE)	7 (23%)	**16 (52%)**	2 (6%)	2 (6%)	4 (13%)
$\overline{PI} < 0$	14 (45%)	17 (55%)	12 (39%)	**30 (97%)**	**30 (97%)**

The small presence of MLR and GLM in this general analysis results from the fact that once the validity of the assumptions of the different models and the statistical significance of the independent variables was analysed, it was determined that in only two tyre sizes, one with normal and other with Poisson distribution, can these models be considered valid, and so they were abandoned for the remaining sizes.

Croston's method and SBA only performed better in six sizes, which show demand in less than 70% of the months, and in the remaining four sizes with the same percentage of demand, SES and Holt's method show lower RMSE.

According to Table 4, concerning the average inventory, SBA and Holt's method present the lowest $\overline{FI}$ in 39% and 23% of the sizes, respectively. However, Holt's and

Croston's methods show, in 42% and 35% of the analysed sizes, the largest $\overline{FI}$. On the other hand, the highest SP is verified in Croston's method and SBA (35% and 45%, respectively), with SES being the method with the most intermediate values of $\overline{FI}$ and SP.

Observing the $\overline{PI}$ values, in absolute value, the smallest numbers are found with Holt's method in 52% of the sizes. The greatest underestimation of inventory occurs with Croston's method and SBA in 97% of the sizes, with SES and Holt's method showing a similar percentage of underestimation and overestimation of demand.

When the validated models with the two smallest RMSE are grouped, the combination of SES and Holt's method is the winner in 45% of the sizes, followed by Croston's method and SBA in 16% of the sizes, according to Table 5.

Table 5. Crossover of the models with the lowest RMSE for tyre sizes with sales grouped monthly.

	Holt	**GLM**	**MLR**	**Croston**	**SBA**
SES	**14 (45%)**	1 (3%)	1 (3%)	3 (10%)	4 (13%)
HOLT		0 (0%)	0 (0%)	3 (10%)	0 (0%)
CROSTON		0 (0%)	0 (0%)		5 (16%)

After obtaining the forecasts by weighting the two methods with the lowest RMSE, one observes that the resulting model presents the lowest RMSE, average inventory, SP, and $\overline{PI}$ values in 97%, 39%, 65%, and 100% of the sizes, respectively.

Once the forecasts from March 2020 to February 2021 were determined, the accuracy of the forecasting methods regarding the RMSE (and shown in Table 6) was studied.

Table 6. Evaluation of the RMSE value obtained for each model in different review periods, considering tyre sizes with sales grouped monthly.

Forecast	**SES**	**Holt**	**MLR/GLM**	**Croston**	**SBA**
SAME AS MARCH 2020	10 (32%)	**13 (42%)**	0 (0%)	3 (10%)	5 (16%)
MONTHLY REVIEW	6 (19%)	**22 (71%)**	1 (3%)	0 (0%)	2 (6%)
2-MONTH REVIEW	8 (26%)	**20 (65%)**	0 (0%)	0 (0%)	3 (10%)
4-MONTH REVIEW	13 (42%)	**14 (45%)**	0 (0%)	1 (3%)	3 (10%)
6-MONTH REVIEW	10 (32%)	**16 (52%)**	0 (0%)	3 (10%)	2 (6%)

Note that if the forecast calculated for March 2020 does not change during the following year, the method with the smallest error becomes Holt's method, with SES being the second-best option. If the forecasts are revised monthly, Holt's method remains the one with the lowest RMSE, as when the revision is carried out every 2, 4, or 6 months. The second method with the lowest RMSE in all revision periods remains SES.

As for the review period, the lowest RMSE value is obtained when the data is reviewed monthly (77% of the sizes), followed by the constant forecast equal to March 2020 or revision every 6 months (10% of the sizes) and the 4-month review (3% of the sizes).

Thus, for tyre sizes with sales grouped by month, one concludes that the smallest error is achieved by combining the weighted forecasts resulting from SES and Holt's method, with monthly data revision.

Table 7 presents the parameters of the (R,2,S) inventory policy specific to three tyre sizes. It is possible to verify the variability of the sales units foreseen for each month and between sizes.

Table 7. (R,2,S) Inventory policy parameters for 175/65R14, 195/65R15 and 385/65R22.5 tyre sizes.

R–Monthly		175/65R14	195/65R15	385/65R22.5
AVERAGE S		13	17	15
Weighted Forecast	20 March	14	17	12
	20 April	13	16	12
	20 May	13	16	13
	20 June	13	16	13
	20 July	13	17	14
	20 August	14	17	14
	20 September	14	18	14
	20 October	13	18	15
	20 November	13	19	16
	20 December	13	17	16
	21 January	14	17	18
	21 February	13	17	18

4.2. Sales Grouped by Quarter

Considering the tyre sizes with sales grouped quarterly, the number of observations ranges between 31 and 35, while the percentage of demand different from zero fluctuates from 54% to 89%.

A similar analysis was carried out for the monthly cases, and the results of the forecasting methods show that SES is the method with the lowest RMSE in 30% of the sizes, followed by Croston's method in 26% of the sizes.

The MLR and GLM models, except for one tyre size, were abandoned since the premises of the models were not met.

Regarding the $\overline{FI}$, SBA presents the lowest inventory level in 43% of the sizes, contrary to SES and Croston's method, with 48% and 35%, respectively, of the largest inventory. The highest SP happens with Croston's method and SBA (in 39% of the sizes).

In absolute value, the lowest $\overline{PI}$ numbers are obtained with Holt's method. The largest underestimation of inventory occurs with Croston's method and SBA.

When combining the validated models with the two smallest RMSE, grouping SES and Holt's method show the best values in 43% of the sizes, followed by Croston's method and SBA in 39% of the sizes.

After obtaining the forecasts and weighing the two methods with the lowest RMSE, the resulting model shows the lowest RMSE, $\overline{FI}$, SP, and $\overline{PI}$ values in most tyre sizes.

Once the forecasts from the second quarter of 2020 to the first quarter of 2021 were determined, one verifies that when keeping the forecast for the second quarter of 2020 constant during the analysis period, the method with the lowest MSE continues to be SES, followed by Croston's method. If the forecasts are revised quarterly, SES remains the method with the minimum error and the second place is divided by Croston's method and SBA. However, when the review is performed every two quarters, in 30% of the sizes, SES obtains the lowest RMSE, as does SBA.

As for the review period, the lowest RMSE value is obtained when there is no revision, followed by the review every two quarters and, finally, the quarterly review.

Therefore, the weighted combination of SES and Croston's method appears to be the most adequate for data grouped quarterly, keeping the forecast constant for one year.

4.3. Sales Grouped by Semester

Concerning sales grouped every six months, the number of observations ranges between 11 and 19, while the percentage of non-zero demand varies from 57% to 100%.

As in the previous cases, a thorough analysis of the forecasting methods under study was undertaken. After applying the forecasting methods, Holt's method reveals the lowest RMSE in 46% of the tyre sizes, followed by SES in 29% of the sizes.

Regarding the $\overline{FI}$, SBA presents the lowest inventory level in 71% of the sizes, contrary to SES and Holt's method, with 39% and 46%, of the largest inventory. However, the highest SP is also verified with SBA.

Concerning the $\overline{PI}$, in absolute value, the lowest numbers are obtained with Holt's method, followed by SES and MLR/GLM. The greatest underestimation of inventory occurs with Croston's method and SES.

When combining the forecasting models with the two smallest RMSE, the combination of SES and Holt's method is preferred in 46% of sizes, followed by SES and Croston's method in 14% of sizes.

Having determined the weighted forecasts with the two methods with the lowest RMSE, the resulting model achieves the lowest RMSE, $\overline{FI}$, SP, and $\overline{PI}$ values in the majority of the sizes.

After obtaining the forecasts from the second half of 2020 to the first half of 2021, one notices that when keeping the forecast for the second semester of 2020 constant during the year, the method with the smallest error becomes SES, followed by Holt's. If the forecasts are revised every six months, SES remains the method with the lowest error, with Holt's method following.

As for the review period, the lowest RMSE value is obtained when no review is performed, compared to the biannual review (in 54% and 46% of sizes, respectively). Note that there are only two forecasts under analysis.

In summary, concerning the tyre sizes with sales grouped by semester, the smallest RMSE is achieved by combining the weighted forecasts resulting from SES and Holt's method. Keeping the forecast constant for one year or revising it every semester presents a similar error value.

5. Discussion and Conclusions

The importance of having a clearly defined inventory policy in a company is revealed by the present work. In fact, determining when to place orders and what is the optimal quantity to order for each item not only improves the use of the retailer's financial resources, but also guarantees consumer satisfaction, as it is possible to answer their needs in a faster and more diversified way.

However, it is not always clear which of the sales forecasting method is the most suitable—the one that allows to reduce the uncertainty of demand—and it may even be difficult to find one that can be adjusted.

After testing five forecasting models with 82 tyre sizes, some differences arose according to the level of sales aggregation. Specifically, for tyre sizes with sales grouped monthly, the method considered the most suitable when determining the model was SES, which was surpassed by Holt's method after determining the forecasts, which agrees with some authors about being unrealistic that a single model is predominant over time.

Likewise, it appears that while the second method with the lowest RMSE in the models determined for tyre sizes with sales grouped quarterly was Holt's method, once the forecasts were calculated, it was surpassed by Croston's method.

It is also determined that the revision periods are different and that the quantities to order are quite diverse between sizes and, in multiple situations, between months/quarters/semesters for the same size.

For the most sold tyre sizes (those grouped by month), the review period must be monthly. For sizes gathered by quarter, the review period should be annual. As for the tyres grouped by semester, it seems to be more prudent to review sales every semester.

Regarding the quantity to order, and since it was possible to adjust demand forecasting models to the different tyre sizes, supply must be related to demand—that is, one ought to order a sufficient quantity in order to meet the expected demand during the revision period and ensure that there is always a safety inventory to satisfy unexpected demand (at least, a pair of tyres of each size sold frequently).

This is defined as an (R,s,S) inventory policy, considered adequate for items with faster output, but it is also slower, and the model to be used results from the weighted combination of the forecasts of the two methods that presented the lowest RMSE for each level of data aggregation.

In summary, for data grouped monthly and semesterly, the forecasts obtained with SES and Holt's method should be combined, while for data grouped quarterly, the estimates resulting from SES and Croston's method ought to be weighted.

SES seems to be suitable to forecast items with intermittent demand. However, it is worth mentioning the importance of analysing the models at each review period and of adjusting accordingly to new information that is acquired. The need for evaluating the error measures is also evident in order to detect biases as soon as possible.

As for future work, it can be useful to carry out a survey to assess the consumer's purchasing behaviour and decision-making process regarding tyres and to collect complementary information, whether concerning economic aspects or the proximity of competitors, so as to add practical information and make the forecasting models more accurate.

Furthermore, the application of other forecasting methods, such as neural networks and bagged forecasts, should be evaluated, analysing the possible benefits of improved inventory system versus increased complexity in forecasting methods given the retail sector we are working in.

Consequently, the practical implications of this study ought to be analysed after the implementation of the proposed inventory plan in the company's inventory levels and associated costs.

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References

1. Tian, X.; Wang, H.; Erjiang, E. Forecasting intermittent demand for inventory management by retailers: A new approach. *J. Retail. Consum. Serv.* **2021**, *62*, 102662. [CrossRef]
2. European Tyre and Rubber Manufacturers' Association. *The ETRMA Statistics Report*; ETRMA: Brussels, Belgium, 2019.
3. Mahama-Musah, F.; Vanhaverbeke, L.; Gillet, A. The impact of personal, market- and product-relevant factors on patronage behaviour in the automobile tyre replacement market. *J. Retail. Consum. Serv.* **2020**, *57*, 102206. [CrossRef]
4. Goltsos, T.E.; Syntetos, A.A.; Glock, C.H.; Ioannou, G. Inventory–forecasting: Mind the gap. *Eur. J. Oper. Res.* **2022**, *299*, 397–419. [CrossRef]
5. Waters, D. *Inventory Control and Management*; Wiley: Hoboken, NJ, USA, 2003.
6. Babiloni, E.; Cardós, M.; Albarracín, J.M.; Palmer, M. Demand Categorisation, Forecasting, and Inventory Control for Intermittent Demand Items. *South Afr. J. Ind. Eng.* **2012**, *21*, 101–113. [CrossRef]
7. Hillier, F.; Lieberman, G. *Introduction to Operations Research*; McGraw-Hill: New York, NY, USA, 2001.
8. Blattberg, R.C.; Hoch, S.J. Database models and managerial intuition: 50% model + 50% manager. *Perspect. Promot. Database Mark. Collect. Work. Robert C Bl.* **2010**, *36*, 228–240. [CrossRef]
9. Beheshti-Kashi, S.; Karimi, H.R.; Thoben, K.-D.; Lütjen, M.; Teucke, M. A survey on retail sales forecasting and prediction in fashion markets. *Syst. Sci. Control Eng.* **2015**, *3*, 154–161. [CrossRef]
10. Ma, S.; Fildes, R. Retail sales forecasting with meta-learning. *Eur. J. Oper. Res.* **2021**, *288*, 111–128. [CrossRef]

11. Willemain, T.R.; Smart, C.N.; Schwarz, H.F. A new approach to forecasting intermittent demand for service parts inventories. *Int. J. Forecast.* **2004**, *20*, 375–387. [CrossRef]
12. Fildes, R.; Hibon, M.; Makridakis, S.; Meade, N. Generalising about univariate forecasting methods: Further empirical evidence. *Int. J. Forecast.* **1998**, *14*, 339–358. [CrossRef]
13. Syntetos, A.A.; Boylan, J.E. The accuracy of intermittent demand estimates. *Int. J. Forecast.* **2005**, *21*, 303–314. [CrossRef]
14. Croston, J.D. Forecasting and Stock Control for Intermittent Demands. *Oper. Res. Q. (1970–1977)* **1972**, *23*, 289. [CrossRef]
15. Gardner, E.S., Jr. Exponential smoothing: The state of the art—Part II. *Int. J. Forecast.* **2006**, *22*, 637–666. [CrossRef]
16. Kourentzes, N. On intermittent demand model optimisation and selection. *Int. J. Prod. Econ.* **2014**, *156*, 180–190. [CrossRef]
17. Syntetos, A.; Boylan, J.E. On the bias of intermittent demand estimates. *Int. J. Prod. Econ.* **2001**, *71*, 457–466. [CrossRef]
18. Sbrana, G. Modelling intermittent time series and forecasting COVID-19 spread in the USA. *J. Oper. Res. Soc.* **2022**, *74*, 465–475. [CrossRef]
19. Nikolopoulos, K.; Syntetos, A.A.; Boylan, J.E.; Petropoulos, F.; Assimakopoulos, V. An aggregate–disaggregate intermittent demand approach (ADIDA) to forecasting: An empirical proposition and analysis. *J. Oper. Res. Soc.* **2011**, *62*, 544–554. [CrossRef]
20. Gutierrez, R.S.; Solis, A.O.; Mukhopadhyay, S. Lumpy demand forecasting using neural networks. *Int. J. Prod. Econ.* **2008**, *111*, 409–420. [CrossRef]
21. Mitrea, C.A.; Lee, C.K.M.; Wu, Z. A comparison between neural networks and traditional forecasting methods: A case study. *Int. J. Eng. Bus. Manag.* **2009**, *1*, 19–24. [CrossRef]
22. Petropoulos, F.; Makridakis, S.; Assimakopoulos, V.; Nikolopoulos, K. 'Horses for Courses' in demand forecasting. *Eur. J. Oper. Res.* **2014**, *237*, 152–163. [CrossRef]
23. Hibon, M.; Evgeniou, T. To combine or not to combine: Selecting among forecasts and their combinations. *Int. J. Forecast.* **2005**, *21*, 15–24. [CrossRef]
24. Koutsandreas, D.; Spiliotis, E.; Petropoulos, F.; Assimakopoulos, V. On the selection of forecasting accuracy measures. *J. Oper. Res. Soc.* **2022**, *73*, 937–954. [CrossRef]
25. Strijbosch, L.W.; Syntetos, A.A.; Boylan, J.E.; Janssen, E. On the interaction between forecasting and stock control: The case of non-stationary demand. *Int. J. Prod. Econ.* **2011**, *133*, 470–480. [CrossRef]
26. Teunter, R.H.; Duncan, L. Forecasting intermittent demand: A comparative study. *J. Oper. Res. Soc.* **2009**, *60*, 321–329. [CrossRef]
27. Wallström, P.; Segerstedt, A. Evaluation of forecasting error measurements and techniques for intermittent demand. *Int. J. Prod. Econ.* **2010**, *128*, 625–636. [CrossRef]
28. Eaves, A.H.C.; Kingsman, B.G. Forecasting for the ordering and stock-holding of spare parts. *J. Oper. Res. Soc.* **2004**, *55*, 431–437. [CrossRef]
29. Syntetos, A.A.; Boylan, J.E. On the stock control performance of intermittent demand estimators. *Int. J. Prod. Econ.* **2006**, *103*, 36–47. [CrossRef]
30. Syntetos, A.A.; Boylan, J.E. Demand forecasting adjustments for service-level achievement. *IMA J. Manag. Math.* **2008**, *19*, 175–192. [CrossRef]
31. Natarajan, K.; Soundararajan, K.; Jayakrishnan, J. Buyer Behaviour Patterns and Satisfaction Trends of Commercial Vehicles Owners in Replacement of Tyres. *Annamalai Int. J. Bus. Stud. Res.* **2013**, *5*, 79–86.
32. Sagaert, Y.R.; Aghezzaf, E.-H.; Kourentzes, N.; Desmet, B. Temporal big data for tactical sales forecasting in the tire industry. *INFORMS J. Appl. Anal.* **2018**, *48*, 121–129. [CrossRef]
33. Hyndman, R.; Athanasopoulos, G. *Forecasting: Principles and Practice*; Springer: Berlin/Heidelberg, Germany, 2018.
34. Holt, C.C. Author's retrospective on 'Forecasting seasonals and trends by exponentially weighted moving averages'. *Int. J. Forecast.* **2004**, *20*, 11–13. [CrossRef]
35. Turkman, M.A.A.; Silva, G.L. *Modelos Lineares Generalizados: Da Teoria à Prática*; Edições SPE: Lisboa, Portugal, 2000.
36. Bretschneider, S. Estimating forecast variance with exponential smoothing Some new results. *Int. J. Forecast.* **1986**, *2*, 349–355. [CrossRef]
37. Makridakis, S.; Winkler, R.L. Averages of Forecasts: Some Empirical Results. *Manag. Sci.* **1983**, *29*, 987–996. [CrossRef]
38. Aiolfi, M.; Timmermann, A. Persistence in forecasting performance and conditional combination strategies. *J. Econ.* **2006**, *135*, 31–53. [CrossRef]