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Aircraft Optimal Control for Longitudinal Maneuvers Using Population-Based Algorithm †

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Abstract: This report considers optimization of aircraft maneuvers in the vertical plane based on direct control methods. It proposes an object model for longitudinal motion suitable for optimal control, algorithms for control approximation and a numerical solution to the problem via a population-based optimization algorithm. The suggested method proves its applicability by forming the control signals for basic aircraft maneuvers in the vertical plane, including climb, speed increment, descent and speed decrement.

Keywords: optimal control; direct optimization; population-based algorithm; aircraft

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1. Introduction

Optimal control for a maneuverable aircraft is a sophisticated problem, since the control object belongs to a class of multidimensional nonlinear dynamic systems [1]. A common approach to control involves variation methods that require the solution of a two-point boundary value problem [2]. Recently, interest has arisen in direct methods for the formation of optimal control [3,4] due to their scalability, robustness and relative simplicity of implementation.

Direct methods assume that a finite set of parameters could determine the desired control signal on the required time interval with necessary precision. As a result, they reduce the initial problem of finding the optimal control to the unconditional parametric optimization problem.

However, depending on the type of control signal, the number of parameters may be quite large. This requires an approach other than widespread gradient methods. One of the possible solutions to this problem appears to be evolutionary or population-based optimization algorithms known in a variety of applications.

In this paper, we propose using a modification of the direct method developed by the authors [4] for the purpose of optimization of the typical aircraft movement in the longitudinal channel. This method is based on spline approximation of control signals and particle swarm optimization [5,6] (PSO) of the resulting problem.

2. General Formulation of the Problem

The mathematical model of the object could be generally described as

$$\dot{\mathbf{y}}(t) = \mathbf{f}(\mathbf{y}(t), \mathbf{u}(t), t),$$

where $\mathbf{y}(t)$ and $\dot{\mathbf{y}}(t)$—state vector and its time derivative, $\mathbf{u}(t)$—control vector, $\mathbf{f}$—vector-valued function of vector arguments, on time interval $t \in [t_0; T]$, where t_0—is the beginning and T—endpoint.

The initial conditions are assumed to be known

$$\mathbf{y}(t_0) = \mathbf{y}_0$$

and the target functional has the following form

$$J(\mathbf{y}(t), \mathbf{u}(t)) = \int_{t_0}^{T} F(\mathbf{y}(t), \mathbf{u}(t), t) dt. \tag{1}$$

The problem consists of finding control $\mathbf{u}(t)$, which provides a minimum of the target functional (1). The widespread approach which satisfies the necessary optimality conditions leads to the two-point boundary value problem. In the case of direct methods, this problem could be bypassed.

Let us assume that control $\mathbf{u}(t)$ may be presented as projections on some linear basis. Then, it could be uniquely determined by its coefficients in this basis. We denote the vector of these coefficients as $\mathbf{a}$

$$\mathbf{u} = \mathbf{u}(\mathbf{a})$$

Estimation of the coefficients' value allows us to determine the control signal. Replacement of the continuous signal by a set of parameters turns the former task into the problem of multidimensional parametric unconditional optimization, the solution of which provides required coefficients

$$a = \arg\min_{a \in A} J(\mathbf{y}(t), \mathbf{u}(\mathbf{a}(t))),$$

where A is the search domain in the parameter space.

Thus, to solve this problem, it is necessary to form the object model, determine the target functional and select the optimization algorithm.

3. Specification of Problem

3.1. Mathematical Model for Control Object

As a foundation for a control object model, we have chosen the system of equations for the spatial motion of an aircraft as a rigid body [1]. Restriction of the motion to single plane allows considerable simplification of the model. Then, the original model [1] can be reduced to a system of three equations determining the following output signals: angle of attack, velocity and flight altitude. This system takes the form presented below

$$\begin{aligned}
\dot{\alpha} &= \dot{\nu} - \frac{qS}{mV} c_{ye}(\alpha) - \frac{P_x}{mV} \sin(\alpha) + \frac{g}{V}(\sin(\alpha)\sin(\nu) + \cos(\alpha)\cos(\nu)) \\
\dot{V} &= -\frac{qS}{m} c_{xe}(\alpha) + \frac{P_x}{m} \cos(\alpha) + g(-\cos(\alpha)\sin(\nu) + \sin(\alpha)\cos(\nu)) \\
\dot{H} &= V(\cos(\alpha)\sin(\nu) - \sin(\alpha)\cos(\nu))
\end{aligned} \tag{2}$$

where

α—angle of attack, rad;
ν—angle of pitch, rad;
V—airspeed, m/s;
H—flight altitude, m;
$c_{xe}(\alpha)$—drag coefficient;
$c_{ve}(\alpha)$—lift coefficient;
m—aircraft mass, kg;
S—wing surface area, m^2;
P_x—projection of thrust force on axis Ox of body-fixed coordinate system, N;
g—gravity acceleration, m/s^2;
q—dynamic air pressure, Pa.

We assume that at the initial moment state coordinates are known, the beginning and endpoints of the time interval are fixed.

The angle of pitch and projection of thrust become control signals. This choice is opportune for two reasons. Firstly, both of these signals are smooth enough that their approximation does not require introduction of many parameters. Secondly, nowadays the problem of tracking these signals in many cases could be delegated to the inner control loop, where it is solved by the automatic control system [7].

We use model (2) with the assumption that the angle of the engine installation is zero. Values of thrust force, drag $c_{xe}(\alpha)$ and lift $c_{ye}(\alpha)$ coefficients should be estimated based on the results of the identification of the model for motion of the aircraft in question.

3.2. Optimized Functional

Optimization of aircraft motion control in the longitudinal channel could focus on a set of basic maneuvers, including climb, descent, speed increment and decrement. In general terms, the performance of these motions could be thought of as a Lagrange problem with free end.

In this paper, we use as the target functional the sum of the Euclidean norms of the vector of the difference between the outputs of the model and the desired output values. This functional takes the form

$$J(V,\widetilde{V},H,\widetilde{H}) = \int_{t_0}^{T} \Big(J_V(V,\widetilde{V}) + J_H(H,\widetilde{H}) \Big) dt = \int_{t_0}^{T} \Big((V-\widetilde{V})^2 + (H-\widetilde{H})^2 \Big) dt \quad (3)$$

where $\widetilde{V}$, $\widetilde{H}$—desired values of airspeed and altitude.

In cases of maneuvers such as descent, an asymmetric functional can be used. Then, only when values of the variable become less than those specified are they taken into account, which should provide more opportunities for the optimization process to increase speed

$$J_V(V,\widetilde{V}) = \begin{cases} (V-\widetilde{V})^2, & V < \widetilde{V}; \\ 0, & V \geq \widetilde{V}. \end{cases} \quad (4)$$

3.3. Optimization Algorithm

Since the optimization problem requires the estimation of 20–30 parameters depending on the time interval and characteristics of the signal, in this article, we eschew common gradient methods in favor of population-based algorithms [6,8]. Within this class of optimization algorithms, a PSO algorithm was applied.

This algorithm benefits from its highly customizable nature, which allows it to be adjusted to meet the demands of the problem in question, striking a balance between examination of whole search domain and a more thorough investigation of promising solutions [9]. Nonetheless, it should be noted that this approach requires substantial time and computational resources and is less suitable for tasks of online optimization. The suboptimality of the obtained solution, as is commonly the case among population-based algorithms [10], should be pointed-out.

4. Direct Method of Program Control

At the core of direct control methods lies the assumption that the sought control could be projected on the basis of linearly independent functions. In this paper, cubic Hermitian splines [11] are considered on such a basis. Therefore, the approximation requires determining the values of the function and its derivative at the spline nodes. The nodes were evenly distributed throughout the entire time interval. The PSO algorithm requires specification of the boundaries of search spaces for the pitch angle and thrust; for example, $\pm 90^\circ$ in case of pitch angle.

To determine the initial values of parameters, particles were randomly thrown into the search space. After that, the model (2) could be integrated and the value of the targeted functional (3) for each particle obtained.

Then, the PSO algorithm finds those parameter values that provide the minimum for the functional, i.e., ensure that error between the desired signals and model output is minimal.

5. Results Description

Thus, the task consisted of applying the method of finding direct program control described above, optimization of the aircraft's maneuvers in the longitudinal channel. A mathematical model (2) was used to describe the control object's motion; the functional in general case took the form (3). Formula (4) was applied in case of descent problem.

The problem under consideration was solved in two main stages. At first, the problem was solved to determine the solution with the best stabilizing time. As experiments have shown, when we shift the initial time taking the functional into account, the resulting solution changes. With a time shift equal to the stabilization time, it is possible to achieve improvements in fuel consumption and mode of maneuver. Thus, the problem was calculated taking into account this shift.

For the problem of climb and speed increment, results were calculated depending on the initial conditions of altitude, as shown in Figure 1.

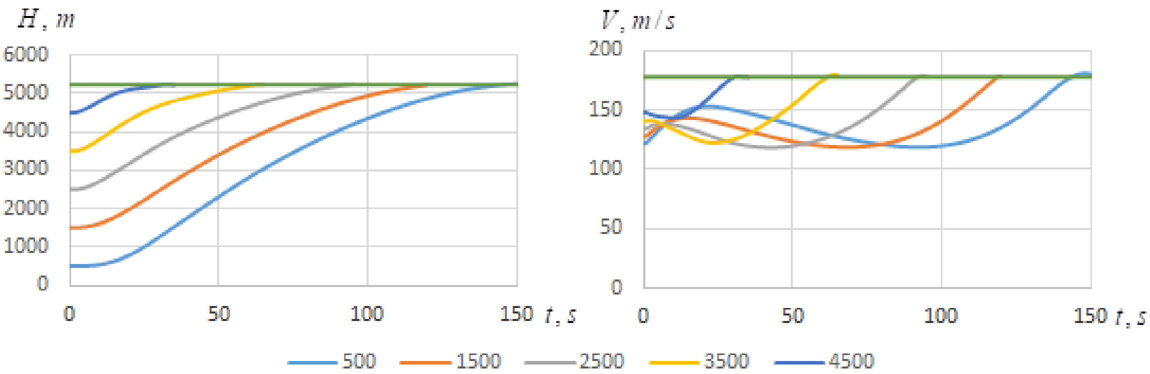

Figure 1. Values of altitude and speed obtained by variation of initial condition for altitude.

Analysis of the descent maneuver shows that taking into account the increase in speed during the maneuver makes it possible to improve the final results. To obtain this, the asymmetrical part (4) is included into the target functional. The problem was considered in a way similar to one described above by variation of initial altitude value. As Figure 2 shows, a family of solutions was found.

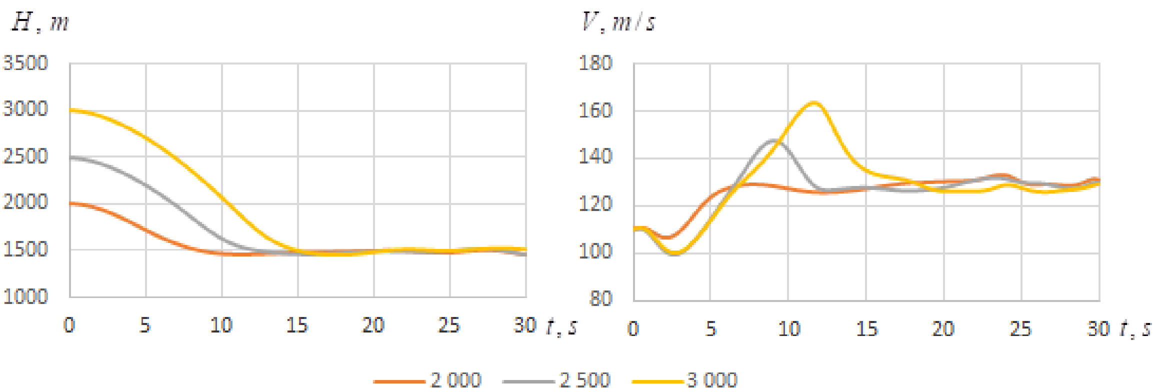

Figure 2. Values of altitude and speed obtained by variation of initial condition for altitude in descent problem.

In all three cases, the maneuver provides a similar picture of performance. The aircraft goes into a dive, losing altitude but gaining speed. When reaching a predetermined altitude, it levels off, trying to keep the pitch angle sufficient to ensure level flight, while gradually losing speed.

Similar results could be observed for the speed decrement problem. The target functional corresponds to Formula (3). The aircraft reduces its speed mainly by increasing its angle of attack, which naturally leads to an increase in altitude. Nonetheless, Figure 3 shows that the requirements for altitude increase have little effect on the overall behavior of the speed on the time interval under consideration. This means that addition of stricter requirements for altitude values could be accommodated without corresponding worsening of the result.

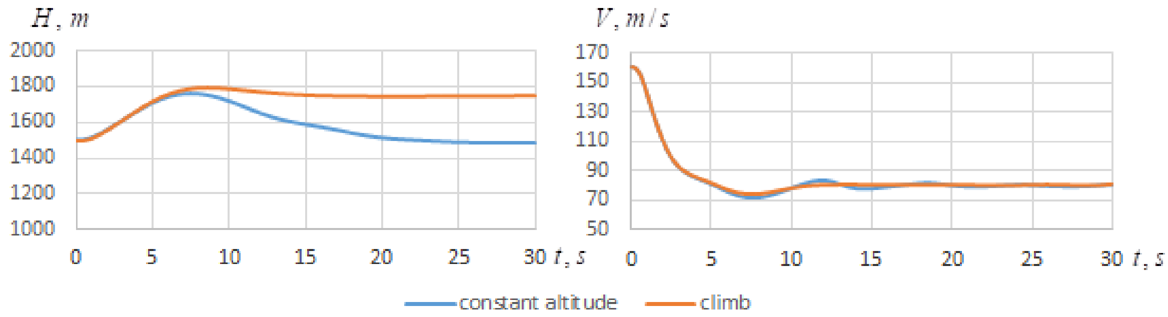

Figure 3. Values of altitude and speed obtained in speed decrement problem.

Above was presented the application of the model for the optimization of relatively simple maneuvers in the longitudinal channel under varying initial conditions. The proposed method could also be used in the case of more dynamic and complex movements, such as, for example, pulling up. Such a problem appears among different applications, including avoiding a collision with the ground.

Within the framework of this task, the idea implemented in the functional (4) is further developed. In this case, the functional becomes asymmetric not only in speed, but also in altitude, including the following expression

$$J_H(H, \widetilde{H}) = \begin{cases} (H - \widetilde{H})^2, & H < \widetilde{H}; \\ 0, & H \geq \widetilde{H}. \end{cases}$$

Pulling up with different initial values of speed was considered. Figure 4 presents the resulting signals for speed and altitude.

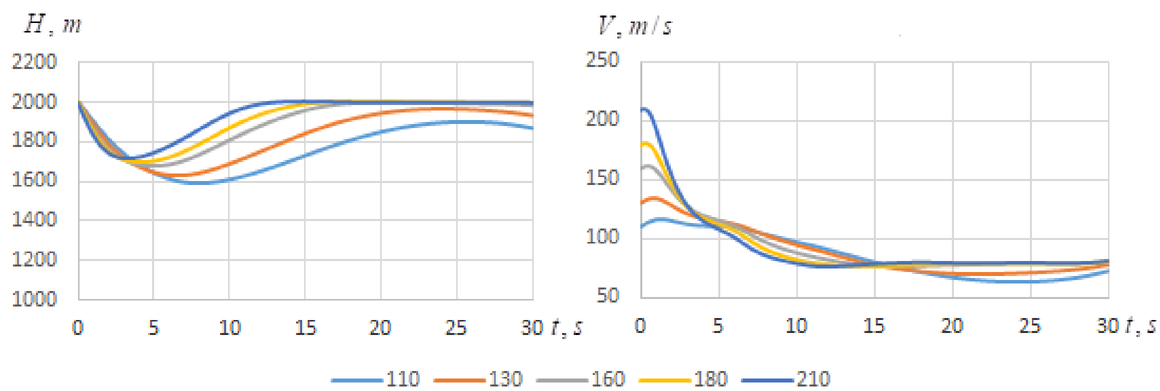

Figure 4. Values of altitude and speed obtained through variation of initial condition for speed in pulling-up problem.

The research showed how an increase in the initial speed leads to a decreasing loss of altitude, but at the same time increases the maximum value of the normal force that accompanies the maneuver. However, it should be noted that extreme values of normal force are not observed over the entire time interval, but are localized during the time of a sharp change in the pitch angle, which means that they last only a few seconds.

6. Conclusions

The paper considers a direct method for optimization of aircraft's maneuvers in the longitudinal channel. The method proposes representation of the desired signals via the cubic Hermitian splines and the solution of the resulting numerical optimization problem via the PSO method. Application of the method to the problems of climb, descent, speed increment and decrement is shown.

The results obtained can be used both for the optimization of individual maneuvers and for determining the reachable areas for the aircraft.

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