

Investigating the Effect of Crack's Inclination on Strain Energy and Stress Intensity under Uniaxial Loading [†]

Salman Khan ^{1,*} , Massab Junaid ¹ and Fahd Nawaz Khan ²

¹ Faculty of Mechanical Engineering (FME), Ghulam Ishaq Khan (GIK) Institute of Engineering Sciences and Technology, Topi 23460, Pakistan

² Faculty of Material and Chemical Engineering (FMCE), Ghulam Ishaq Khan (GIK) Institute of Engineering Sciences and Technology, Topi 23460, Pakistan

* Correspondence: gme2185@giki.edu.pk

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Abstract: The component's strength and reliability are greatly influenced by the occurrence of cracks and flaws. Material surfaces or internal defects can create stress concentration, which causes failure of the materials. To quantify the stress intensity factor (SIF) and strain energy release rate (SERR) around the tips of the crack, numerical simulation has been performed on a plate of central crack with Comsol Multiphysics. The SIF and SERR with varying uniaxial stress loading (5, 10, 15, 20, 25 MPa) and crack's angle (0° – 180°) were determined using the J-integral method. The results show that the crack's maximum opening and sliding displacement occurred at an angle of 90° and 45° respectively. For mode I, the SIFs were maximum at the crack's inclination of 90° and for mode II, it was maximum at an angle of 45° . Similarly, the SERR was also found to be maximum when the crack was normal to the applied stress and observed to increase with the increase in applied stress.

Keywords: stress intensity factor (SIF); strain energy density (G); J-integral method; crack's angle; stress concentration; opening; and slide mode displacement



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1. Introduction

There are certain flaws or cracks either interior or at the surface of the components which were created due to high stresses induced during the manufacturing process. These cracks or flaws cause stress intensity when the components are under loading conditions. The stress distribution or intensity needs to be quantified around the tips of the crack because it creates a state of stress concentration (SC). SIF is a fracture mechanic parameter and is used to define the crack tips stress singularity and is denoted by K . When the SIF becomes equal to or greater than the fracture toughness, the component gets fractured and fails. Based on the crack surface displacement, the stresses near the crack tip can be divided into three basic types as shown in Figure 1 [1].

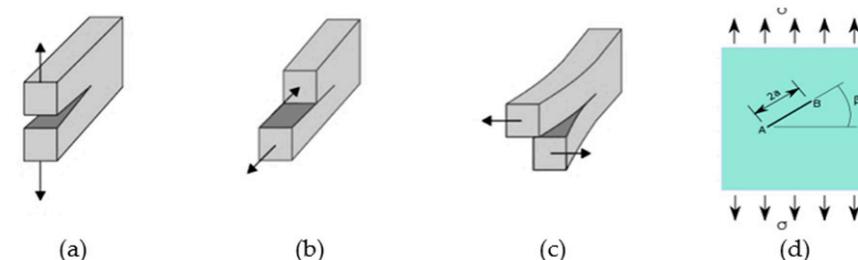


Figure 1. The three types of loading with respective crack' modes (a) Mode I (b) Mode II (c) Mode III [1] (d) The inclination of the crack with the load stress [2].

Research has been performed to develop fracture mechanics criteria to quantify the fracture toughness of different materials and developed different criteria. These fracture criteria can be classified into the following three groups which are further subdivided: stress, strain, and energy-based criterion [3].

For the uniaxial stress loading, the analytical equations for finding the SIFs within a rectangular plate of infinite length can be calculated by [4];

$$K_I = \sigma \times \sqrt{(\pi a)} \times \sin^2 \beta \quad (1)$$

$$K_{II} = \sigma \times \sqrt{(\pi a)} \times \sin \beta \times \cos \beta \quad (2)$$

The stress and strain around the crack tip are greatly affected by a constant term called T-stress and can lead to better results [5]. Energy-based theory cannot give a good prediction about the kink angle while the strain energy released rate (SERR) and maximum tangential stress (MTS) criteria give better predictions about the crack growth trajectories [6]. For a semi-infinite plate, the relationships between SIFs and the oblique edge fracture were investigated [7]. An experimental investigation was performed on the plate with a crack edge and find the effect on the SIFs (K_I and K_{II}) [8].

In the paper, a rectangular solid plate with an interior fracture is subjected to uniaxial stress. The fracture or crack is at an angle with the load direction, implying that it exhibits in both modes I and II. The J-integral approach is used to calculate the SERR at crack tips. As the stress intensity was caused at the tip of the crack and these SIFs must be quantified for both mode I and mode II using J- the integral method. The SIFs and SERR, the opening and sliding displacement with the crack's inclination and applied stress was studied using COMSOL Multiphysics software.

2. Numerical Modeling

A rectangular Aluminum plate with a central slanted crack is subjected to a stress in the Y direction. The crack's inclination varies from 0° – 180° while the applied stress field varies from 5–25 MPa. The dimension of the plate is given in Table 1. The SIF and the SERR are to be calculated at each angle of the crack. The tips of the crack under the mixed mode (Tensile, compressive and shear) of stress loading can be in opening mode with the tensile mode (denoted by K_I), in plan shear mode (denoted by K_{II}) [9]. For this purpose, the Comsol Multiphysics was used to quantify the effect of loading stress and crack's inclination on the SIFs, SERR, opening and sliding displacement.

Table 1. The dimensions of the rectangular plate with central crack.

S. No	Parameters	Symbol	Values
1	Plate's length	L	100 mm
2	Plate's width	W	80 mm
3	Crack length	2a	40 mm
4	Crack angle	β	0° – 180°
5	Load in Y-axis	σ	5–25 MPa

3. Results and Discussion

A 2D rectangular plate with a central crack was studied using Comsol Multiphysics. The load was applied in the uniaxial direction in Y-direction. The crack' angle was varied from 0° to 180° and the respective opening and sliding displacements, the SERR and the corresponding SIFs were determined. The slanted crack can create two types of SIFs, one for mode I (opening) and other for mode II (shear). As the crack is at center of the plate and it has two sharp edges, so at each edges the SIF was calculated for each mode. The variation of the opening and sliding displacement of the crack with the inclination and varying load is shown in Figure 2a,b respectively. Since the plate is two-dimensional plate, there will no tearing mode of crack. The SERR at both the crack tip 1 and tip 2 are given in

Figure 2c,d. The SERR was high when the crack is normal to the applied load. For mode I, the SIFs at both the crack's tip are given in Figure 2e,f. It will be high when the applied load and crack are normal to each other. Similarly, for mode II, the SIF is maximum when the applied load is at 45° to the crack as given in Figure 2g,h. The numerical simulation results are shown in Figure 3.

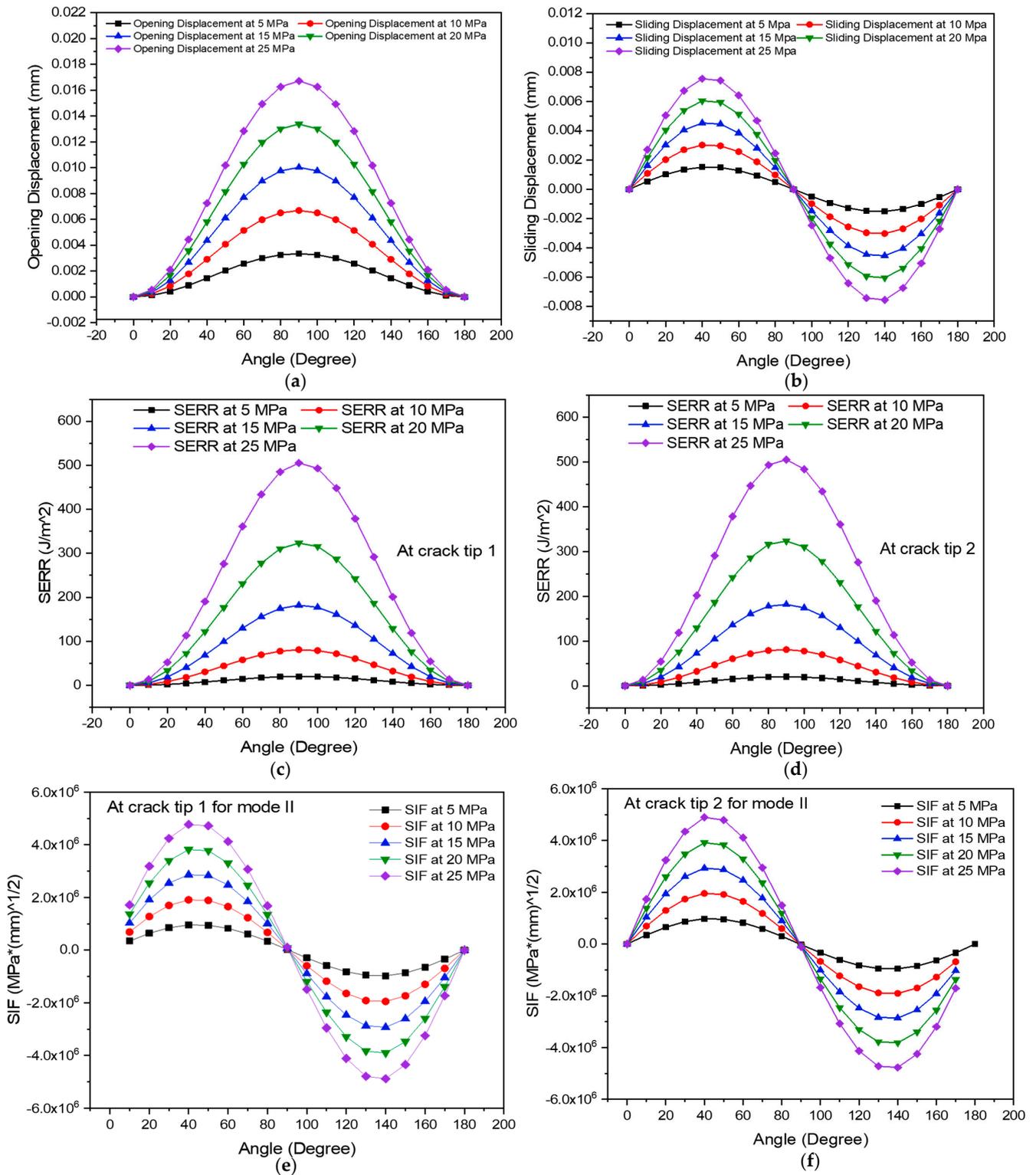


Figure 2. Cont.

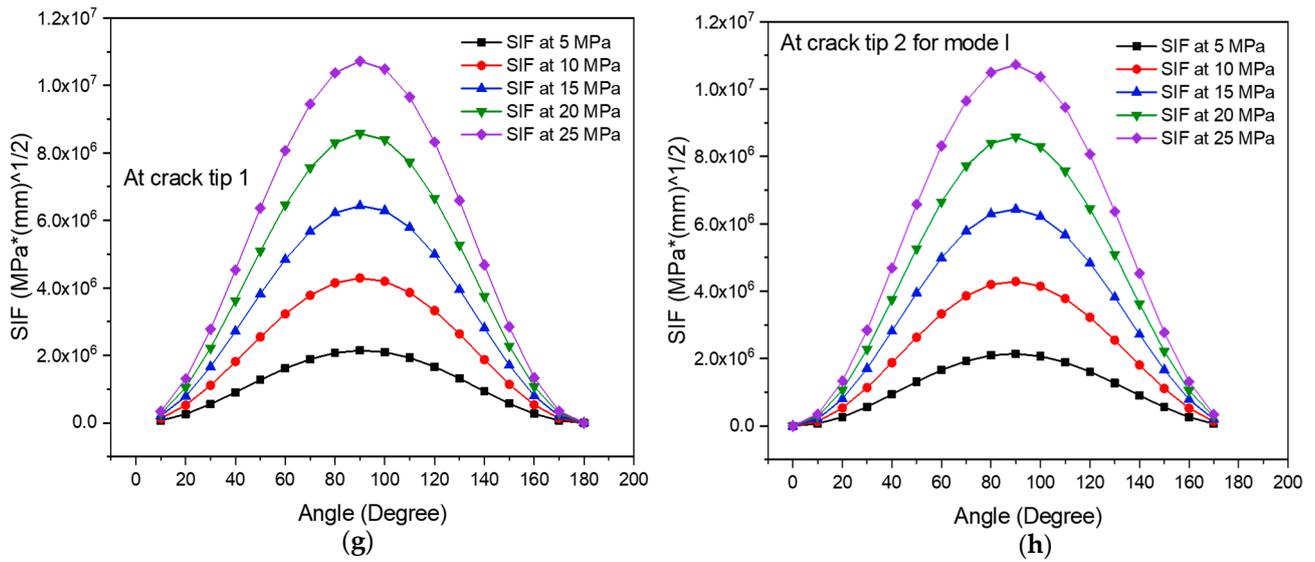


Figure 2. The variation of crack opening displacement with angle and loads (a) The variation of crack sliding displacement with inclination and loads (b) The SERR for mode I at crack tip 1 (c) The SERR for mode I at crack tip 2 (d) The SIF for mode II at crack tip 1 (e) The SIF for mode II at crack tip 2 (f) The SIF for mode I at crack tip 1 (g) The SIF for mode I at crack tip 2 (h).

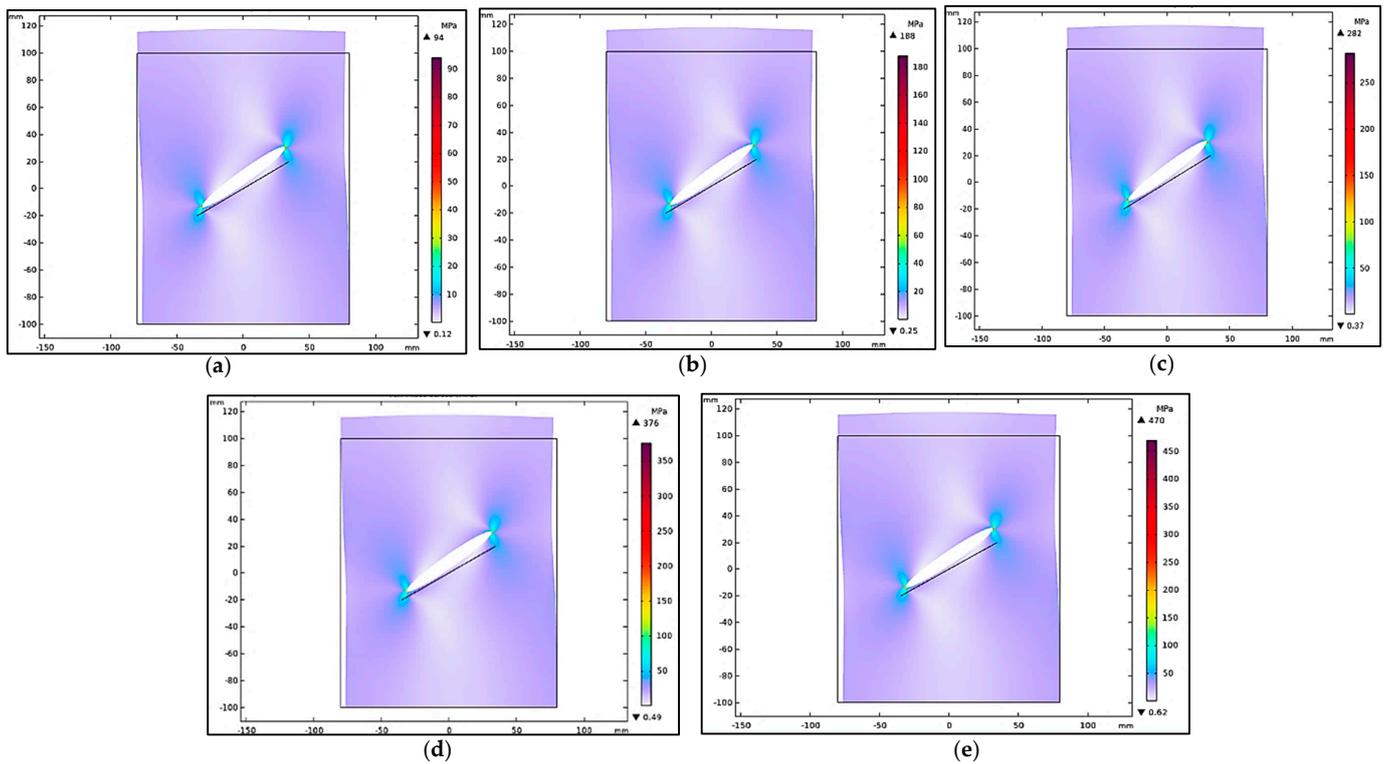


Figure 3. Von mises stress distribution at different applied loads (a) 5 MPa (b) 10 MPa (c) 15 MPa (d) 20 MPa (e) 25 MPa.

4. Conclusions

From the numerical investigation, it was concluded that:

4.1. For Mode I (Opening)

- i. The opening displacement of the crack was maximum when the stress was applied normally (at 90°) to the crack and increased with the increase in the applied stress field.
- ii. The SERR was high when the crack was normal to the applied stress.
- iii. SIF was also high when the angle is 90° between applied stress and crack and was directly related to the applied stress.

4.2. For Mode II (Sliding)

- i. The crack sliding displacement was maximum when the applied stress and the crack were at an angle of 45° and was directly related to the stress field.
- ii. The SERR was high when the crack was at 45° to the applied stress.
- iii. SIF was also high when the angle was 45° between applied stress and crack.

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