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Article

Phenomena and Principles: Presenting Quantum Physics in a High School Curriculum

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Abstract: The goal of teaching quantum physics (QP) in high school is a problematic and highly turbulent area of divergent views, curricula studies, and claims. The innovative curricular approach of discipline-culture (DC) suggests a way of overcoming its significant difficulties. It suggests presenting QP as a fundamental theory structured in terms of the *nucleus*, *body*, and *periphery*. Applying this perspective in our study, we interviewed nine experts with respect to their view of how the *nucleus* of QP should be presented to high-school students. With the different viewpoints of the core essentials in hand, we compiled the *nucleus* of the QP. We also examined this subject using nine introductory university textbooks that might suit high school students and considered their coherence and suitability with regard to the specified *nucleus*. We found some confusion regarding the status of theoretical items: some fundamental principles, as perceived in the eyes of the experts, are presented as phenomena. Not only does this mismatch represent a special barrier for both the teachers and students to understand QP, it promotes an inadequate image of QP as well as a distorted view of the nature of science. Finally, we offer a framework for a DC-based QP curriculum free of the noted deficiencies.

Keywords: teaching; quantum physics; discipline-culture; curriculum design

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1. Introduction

Quantum physics (QP) exemplifies a central pillar of 20th century physics, and it represents a foundational stone for 21st century technological innovations. However, teaching QP at the high-school level is challenging, and, consequently, it is limited in many countries. The first steps of teaching-learning quantum mechanics are well-represented in a collection of articles [1]. This collection showed different attempts and aspects are still missing one inclusive perspective in the way to represent quantum theory. Recent years have seen attempts to add QP contents; however, the educational literature reveals a highly dissatisfying situation in schools around the world. Teaching QP is usually superficial and oversimplified, often emphasizing phenomena while barely touching on principles and fundamental theory [2,3]. For example, in 2016, the QP curriculum in the Netherlands depicted the phenomena of tunneling, the potential well, and the two-slit experiment as representing QP [4]. An experiment demonstrating the interference of a single photon was specifically developed for this program [5], but a discussion of the wave-particle dualism in this context was missing. The learning assessment tools emphasized the phenomenon, practically without stating the principle of superposition and its impact on measurement [6].

Teaching QP in high schools usually follows a historical approach and because of time constraints, it was abandoned in an early stage of history [3,7]. Although the role of the history of science is widely recognized [8–10], in the case of QP, such an approach can be problematic and sometimes unfeasible [11]. In fact, both a historical and a phenomenological approach fail to adequately represent fundamental theory as a basis of the curriculum. In such approaches, pupils are often presented separate pieces of the theory without a holistic view. Many curricula discuss Plank's explanation of black body radiation, the photoelectric effect, and Bohr's model of hydrogen, which is only semi-classical [2], but they often miss more substantial QP content, including problem solving. Note that the importance of these topics is mainly historical and they have little importance for understanding QP topics [3]. Teaching QP using a historical approach may cause students to believe that QP is nothing more than a certain modification to classical physics (CP), whereas it is actually an essentially different fundamental theory [12]. QP should not be taught by ignoring CP, and the differences between the theories should not be blurred. Teaching must emphasize the departure from classical logic [13,14], avoid oversimplification [15], and be extra careful in using classical analogies [11], which are often inaccurate and misleading. QP should be studied as a theory with a specific set of logic and laws, which require a deep understanding [16].

Another challenge in teaching QP in high school is the level of the mathematic formalism required. It is well above the standard math curriculum, surpassing most pupil's knowledge [2,17]. Visualization is often applied to overcome this deficiency [18,19]. Another approach in this direction involves simulations in order to illustrate the principles of QP, e.g., taking into account the Mach–Zehnder interferometer [16,20]. It should be recognized, however, that the transition to a phenomenological description excludes the possibility of using quantitative questions when teaching physics. This is a significant impediment in presenting physics as essentially a quantitative "exact science". Moreover, teaching QP non-quantitatively is in contrast with the previously taught domains of mechanics and electricity, where a computational approach is the standard method. Instead, students may perceive QP as a collection of stories, leading to an inadequate understanding [21]. In Germany, there was an attempt to introduce mathematical equations (aimed for 13th grade) that provide eigenvalues for simple differential operators [16]. In Italy, teachers tried using Feynman's path integrals, also addressing 13th graders, who are more mature than regular high-school students in other countries that have twelve-year schools [22]. The need to find a mathematical structure suitable for high-school students, which would allow at least some calculations in QP, was evident. Such a mathematical structure should represent quantum theory holistically, beyond a few simplified solutions. With this goal in mind, use of Dirac notations has been suggested as suitable for both computational and conceptual purposes [23].

Curricular designers seek a curriculum that addresses QP misunderstandings, resolving a confrontation with intuition using a more advanced teaching method. Several studies depicted the pertinent misconceptions of university (e.g., [24–26]) and secondary school students [2]. The alternative understandings of the Copenhagen interpretation have been investigated. For example, students may perceive an electron as a cloud in space [26]. However, beyond the required remedy of correcting misconceptions and alternative understandings, a new curriculum should present QP as a fundamental theory, and discuss its principles and applications.

An appropriate approach to building such a curriculum is to consider it as a discipline-culture (DC) [15,27–29]. The DC paradigm suggests presenting QP as a fundamental theory structured in a *nucleus*, *body*, and *periphery*. The *nucleus* includes basic principles and concepts, the *body* includes phenomena and applications, and the *periphery* includes the alternative physical accounts and the limits of validity. Thus, for example, in the classical mechanics picture, the *nucleus* includes Newton's laws, the *body* includes Kepler's laws, the associated subdued elements, solved problems, and applications, whereas the *periphery* includes alternative understandings, principles, including those of Aristotelian physics, relativity, and quantum theories [30]. The *periphery* is important because in order to understand what is correct, it is necessary to contrast it with the corresponding alternative [31]. This picture makes explicit the idea of historical dynamics, the transition from one theory to another, and from one *nucleus* to another [28,32].

The advantage of this approach lies in its holistic presentation. It emphasizes the basic principles of fundamental theory and connects them to their derived phenomena and experiments. We have applied this approach to construct a short course on QP for high school [23,33], which emphasized teaching an existing theory and not how it was historically developed. The principles and their illustrative experiments became more prominent. The *periphery* re-emphasized the principles and positioned the theory (as a whole) in relation to other fundamental theories such as classical mechanics. This approach tried to fit QP to the very limited curricular time-frame allotted by the contemporary school curriculum for modern physics. This constraint dictated a very different curriculum policy as a feasible and still meaningful alternative.

Here, the aim was to refine some aspects of this approach to QP teaching, an approach that elicits and emphasizes principles and relates them to their applications rather than just mentioning them, by explaining such applications and related phenomena, which often occur in the context of teaching. There was an established need for practitioners [34] to be provided with a suitable teaching resource to rely on, learn, provide examples, and acquire competence. Equally important is to identify the principles of QP in parallel with other physics disciplines taught at school. In a way, this attempt is parallel to a similar attempt to teach classical mechanics [30]. There was a curricular implication that changed the regular emphasis with respect to the energy/momentum conservation laws of the *body* of knowledge, as derived from Newton's laws in the *nucleus*. In other words, the division into the *nucleus* and *body* implies certain ideological assumptions to be explicit in teaching: stating the basic principles and what is derived from them, specifically, how can a *nucleus* and a *body* be defined. Consequently, the picture for the students becomes transparent. In accordance, we now specify our research questions.

Here, first, we specify the *nucleus* in the context of QP to be taught in high school, which is not being explored sufficiently in an educational context. Therefore, we explored the views of pertinent experts, physicists directly dealing with QP research. Second, with the *nucleus* initially defined, we turned to the available textbooks for secondary education that can help in this regard.

The obtained answers to these research questions will lead us closer to a better understanding of how we can construct an appropriate DC-structured, QP curriculum for high school, or at least, present an essential step towards an option of this kind. Although there may probably be more than one solution, we will present a detailed proposal for such a curriculum. The effect of actually applying such a curriculum (an intervention study) will be presented elsewhere.

2. Methodology

2.1. Determining the Nucleus via Interviews with Experts

To elicit the QP *nucleus* applicable for high-school teaching, we first interviewed a sample of nine experts (Table 1). Other studies [35] showed that practitioners from different fields in physics have different perspectives on QP; therefore, it was important for us that the chosen experts were physicists engaged directly in the study and teaching of QP. Three of them teach or taught QP courses at the university level where they carry out their research. One was an expert in the philosophy of science with an emphasis on QP. To avoid possible institutional bias, experts from two research universities were selected, including an emeritus scholar and other younger researchers at the beginning of their careers, including a doctoral student with teaching experience. Although they were experts in the subject matter, all but one were very distant from teaching physics in school. Unlike other studies that often considered items of knowledge associated with all parts of DC theory and mostly to the *body* of knowledge [36], we conducted our interviews in order to identify the important concepts of the *nucleus*, revealing the fundamental meaning of QP as a theory. For the purpose of identification, personal interviews could better serve our goals.

Table 1. The interviewees and their domains of expertise.

No.	Position	Seniority (or Years)	Area of Expertise
1	Ph.D. student (physics)	Ph.D. Student	Quantum communication and entanglement
2	Physicist	4	Nuclear and hadronic physics
3	Physicist	40	High energy and strings theory, Lecturer (QP)
4	Physicist	12	Quantum entanglement, Lecturer QP
5	Physicist	36	Condensed matter; Lecturer QP
6	Physicist	14	Quantum coherence
7	Physicist	1	Nuclear astrophysics
8	Physicist	7	Nonlinear quantum optics
9	Philosopher of science	11	Philosophy of physics, classical and quantum statistical mechanics

First five interviews (Table 1) were conducted face-to-face and lasted about an hour each, in the format of semi-structured interviews (e.g., [37]). The rest were conducted at a distance. The experts were presented with the DC approach, and as an opening question, they were asked: "What do you consider as the fundamental QP principles, given that we want to teach them in high school?".

After the experts provided the concepts and principles that they considered as important, they were asked to refine them in order to see the hierarchy of the principles and their meanings. For example, after pointing to "duality" as an important component of the *nucleus*, we asked them to elaborate more about its meaning. Sometimes we asked about the relationship of certain concepts even if they were not mentioned by the interviewee. The duration of the interviews could vary according to the ability and interest shown by the interviewee to the claims made by other colleagues.

The experts defined the items they associate with the *nucleus*, exemplified by *body* items that correspond to the mentioned components of the *nucleus*, and sometimes they also referred to the *periphery*. The interviews were transcribed, and a thematic analysis was conducted: Because DC was presented to them explicitly, it was not a complicated task to analyze the experts' words and to categorize them in triadic terms. After the first author performed a preliminary categorization, the second author joined in its validation through a discussion and full agreement was easily reached. Only some of the data used in this report focused on the *nucleus*. Experts' examples and illustrations (associated with the *body* or *periphery*) are not presented here. Since each concept can have many illustrations for teaching [34], a Delphi study [36] would be a better way to study them.

2.2. QP Textbooks and the Nucleus

A sample of nine textbooks on "modern physics" was examined (Table 2). These textbooks, intended for colleges, or as service courses for engineering students, were examined because they should be more suitable for the high-school level, compared with "quantum physics" textbooks aimed at university physics students. Some of the textbooks considered contain much more advanced topics (e.g., nuclear physics, elementary particles), presented mainly phenomenologically with minimal formalism. Such parts, less relevant to high school are important for us in order to define the scope and limitations of our teaching at school, particularly in a DC perspective that connects various phenomena to basic principles. For the purpose of diversity, the sample included both new and old editions of textbooks. The textbooks were examined according to the saturation method for qualitative studies, in which the process ends when adding more items seems not to add new (qualitative) information [38].

Table 2. List of the physics textbooks examined.

(No)	Author(s) (Year of Edition)	Title
(1)	Beiser (2003 [39])	*Concepts of Modern Physics*
(2)	Weidner and Sells (1973 [40])	*Elementary Modern Physics*
(3)	Serway, Moses, and Moyer (2005 [41])	*Modern Physics*
(4)	Krane (1983 [42])	*Modern Physics*
(5)	Tipler and Llewellyn (2008 [43])	*Modern Physics*
(6)	Thornton and Rex (2013 [44])	*Modern Physics for Scientists and Engineers*
(7)	Nolan (2014 [45])	*Fundamentals of Modern Physics*
(8)	Noce, Ed. (2020 [46])	*Modern Physics; A Critical Approach*
(9)	Halliday, Resnick, Walker, and Taylor (2012 [47])	*Understanding Modern Physics* (Hebrew translation)

The assumption in looking for fundamental principles is that they are related to numerous items, experiments, and phenomena. The principles had to be referred to as such, and not as phenomena. The analysis was twofold, both quantitative and qualitative.

Within the quantitative analysis, we counted the occurrences of keywords of the *nucleus*, as it was defined from the first part of the study (the interviews). Most of the textbooks were available in digital form. They were searched by digital counting. First textbook (Table 2) was searched manually, using the index. Within the qualitative analysis, we examined how each item of the *nucleus* was referred to. The first and second authors categorized such connections as either a principle or phenomenon. Validation was carried out by comparing categorizations with a short discussion to reach full agreement. Similarly, conceptual definitions, explanations, and examples (including problems) were analyzed, as well as connections between the principles.

Another part of the analysis focused on the book headings (chapters or sub-chapters) and their relationship to the *nucleus*, *body*, or *periphery*.

3. Findings

3.1. Experts on the Nucleus of Quantum Physics

We obtained a significant amount of data from the experts regarding the QP *nucleus*. However, four of the experts (Nos. 1, 3, 6, and 9, see Table 1) hesitated somewhat in defining the *nucleus*. For example, expert No. 6 said: “Quantum physics does not have an agreed upon ontology... As in the parable of the blind examiners and the elephant, each sees quantum physics as something different” (Figure 1).

Figure 1. The parable of the blind and the elephant [48] (p. 89).

Two related difficulties can be seen here. One is the lack of consensus regarding the QP ontology and different controversial views. This revealed the second challenge of a pedagogical nature, namely, the possible approaches to teach QP and its principles, which may pose the question of whether defining the *nucleus* of QP is practical. Expert No. 9 suggested: “Students should be presented with the fact that, on the one hand, quantum physics does not have an agreed-upon worldview, but on the other hand, quantum physics possesses an excellent ability to predict results with an accuracy of 10 digits after the decimal point”.

3.1.1. The Items

The wave-particle duality, or the wavity as a property of a particle, is among the central principles that arose in all interviews. For example, expert No.3 said: "Axiomatically, there is a wave function that describes a particle".

Next, we refer to the meaning ascribed to this feature. The experts associated wavity with another principle they considered central: the principle of superposition. For example, expert No.9 stated: "The first principle is that nature includes state superposition. A state can include several probable options".

All experts mentioned the probabilistic aspect of the measurement results, for example, expert No.2 stated: "One of the fundamental principles is the probabilistic explanation of the wave function".

Other principles associated with the nucleus are non-locality and entanglement (experts Nos. 1, 8, and 9). Two of the experts (Nos. 1 and 8) emphasized that only when entanglement is present and realized, can one say that the system is "quantum", i.e., specific of QP. Expert No.1 indicated that quantum nature is essentially different, irreducible to the classical predecessors: " Waves we know, particles we know, but EPR doesn't have a classic analogue".

The EPR (Einstein–Podolsky–Rosen) experiment [49] deals with entangled particles: two particles produced together and comprising a single system. The experiment states the immediate effect of measuring one particle on the state of the other. Therefore, this experiment (realized in the 1980s [50]) demonstrated non-locality in the system described by the wave function unifying the objects. This way, "non-locality" entered into physics as a feature of reality. It implied the relationship between two distant measurements of the quantum objects in a single quantum state, which cannot be explained by any physical interaction between the parts of the system, and the interaction at a finite speed. Indeed, we saw the experts who consider non-locality and entanglement as an essential part of the *nucleus* of the QP as a fundamental theory.

Some experts mentioned Heisenberg's uncertainty principle (experts Nos. 2 and 4) and quantization of energy levels (experts Nos. 2, 3, and 7). For example, expert No. 2 mentioned: "The basic principles are duality, uncertainty, and discrete energy values".

Another principle, mentioned by two experts (Nos. 2 and 7), is the division of elementary particles into bosons and fermions. There are two kinds of indistinguishable particles with respect to their coexistence: the fermions, which obey the Pauli exclusion principle, and bosons, which are not committed to it (expert No. 2): "Fermions and bosons; fermions have the Pauli principle, as opposed to the bosons, which 'want' to be in the same state".

That is, the principles of central importance mentioned by the participants were as follows: duality, superposition, probability in measurement, entanglement, Heisenberg's uncertainty, quantization, as well as the division into bosons and fermions and their properties.

3.1.2. Scrutinizing with Respect to the *Nucleus*

In this Section, we consider what the experts meant when they referred to the *nucleus* concepts. In our opinion, duality was the gateway. When dealing with particles, e.g., an electron, they stated that the novelty lies in also considering the electron as a wave, i.e., while characterizing an electron, they added properties that exhibit wave phenomena. To furnish this idea and to clarify the meaning of "wavity" for matter, we asked the experts directly. In response, all the experts linked wavity to the principle of superposition. They ascribed to it the essence of quantum duality. For example, expert No. 9 said: "Superposition is the most fundamental, in quantum theory in general and in wavity in particular."

Just as a wave is a non-local phenomenon, i.e., it exists in different places and at different amounts, the quantum particle is also in a superposition of locations, as described by a wave function. In the words of expert No. 8: "Wave function [of a particle] consists of a collection of probabilities at each point, a superposition of several likelihoods. For physicists, it is more interesting to talk about discrete states and a superposition of any two states."

This account was generalized to the superposition of the states of other properties such as momentum states, the states of polarization, spin, and others. Expert 7 confirmed this explanation as a "legitimate interpretation". Another expert (No. 6) mentioned that the term "wave" in this context signifies a physical quantity having more than one value: " . . . wave function is superposition; it presents an extension of meaning that occurred in English and German. It is in a sense a borrowed semantic, not physical".

Expert No. 1 clarified the meaning of superposition as follows: "Waves also have superposition, but the novelty here [in QP] is the probability... The probabilistic features have no a classical analogue".

Thus, the important feature is probabilistic superposition, as expressed by the collapse of the wave function in a measurement. This is in contrast to the superposition in a classical case, e.g., in string, where several frequencies and amplitudes are simultaneous. Expert No. 7 stated: "As long as there is no measurement, there is a superposition. We cannot measure superposition. When measured, the state collapses, and if we do it [the measurement] many times, we get statistics [and the idea] of what the initial state was".

That is, according to the Copenhagen interpretation originally proposed by Max Born, quantum superposition means that before a measurement, a state consists of several eigenstates. A measurement produces a certain result, and the wave function collapses. Numerous identical measurements provide results initially described by the wave function.

Regarding measurement and collapse, some experts (Nos. 2, 7, 8, and 9) warned against the understanding of "turning a wave into a particle" (No. 7): "I wouldn't express it as measurement that turns a wave into a particle... It remains a wave function..."

As for Heisenberg's uncertainty and quantization, experts were not unanimous. Expert No. 6 stated: "Quantization, tunneling, and uncertainty are not fundamental principles, but the phenomena arise from fundamental principles, and they can be derived from these principles".

Other experts (Nos. 1 and 5) have also argued that quantization can be a phenomenon in classical waves and that it is not a quantum principle. Another expert (No. 2), to whom these words were presented, addressed the pedagogical aspect of this point: "Even if something derives from the core, it could not always be shown to the pupils immediately... In classical physics, we teach conservation of momentum, but only later, it derives from the Noether theorem".

In other words, if one cannot derive a phenomenon as implied from a principle, perhaps a school teacher can treat it as a principle.

Regarding the meaning of Heisenberg's principle of uncertainty, expert No. 2 commented: "The uncertainty principle is erroneously presented using Heisenberg's microscope... The "uncertainty" is related to a conscious viewer, and this is a problem".

It should be emphasized that the Heisenberg principle refers to the nature of things, rather than depending on the observer.

3.1.3. Interim Summary of the *Nucleus*

There is a consensus among the experts regarding the centrality of the superposition principle in QP. This principle possesses probabilistic meaning, as evident in the collapse of the wave function in measurement. Beyond that, there was some agreement relating wavity to superposition. This approach seems to be suitable for high-school teaching and it should be presented as the greatest conceptual innovation of QP. The wavity of matter constitutes the gateway (historical and conceptual) to QP. Looking at wavity as a superposition of states establishes a unified view essentially different from the classical view. In contrast, the concept of quantization (possessing discrete, non-continuous values) appears to be suitable for the *body*, rather than for the *nucleus*. There are several reasons for that:

A. Methodological: Several experts explicitly argued that quantization represents a phenomenon and not a principle. In addition, the phenomenon of quantization is also observed in CP.
B. Conceptual: Quantization derives from the solution of the wave equation. It can be illustrated by standing waves. Quantization cannot be a fundamental principle in QP, since it characterizes all waves, both classical and quantum.

C. Pedagogical: Pupils encounter quantization while studying about the Bohr model, which is semi classical; it belongs to the *periphery* of QP. Considering it as a fundamental principle is misleading regarding the essential features of QP.

The Heisenberg uncertainty principle stems from a more basic principle (Non-commuting nature of certain observables such as the momentum and coordinate of a particle), which is beyond high-school students' mathematical level. However, the Heisenberg uncertainty principle is related to the state of superposition, determining the kind of states that can be superposed. Therefore, in high school, this state of matter can be presented as claiming the existence of pairs of physical quantities (e.g., position and momentum, spin and polarization in different directions), for which eigenstates cannot coexist in a superposition. In this sense, uncertainty can appear as a principle in teaching and can be referred to with regard to the *nucleus*. Presented as an inequality relating to the accuracies of location and momentum (or life time and energy), it may be used as a phenomenological constraint rather than a basic principle.

We wish to note that the experts did not explicitly mention the complementarity principle. However, they talked about uncertainty and duality in relation to complementarity. Some of them had reservations regarding conceptualizing measurement (and the related collapse) as a transition of the quantum objects from a wave to a particle.

Regarding non-locality and entanglement, it was associated with the *nucleus*. To establish a logical sequence of teaching the *nucleus*, it should be preceded with such concepts as quantum state (eigenstate) and superposition of states. Quantum non-locality could be illustrated in the interference of a single photon with two paths of transition available, two slits experiment or the Mach–Zehnder interferometer (both affiliated to the *body* in the quantum theory). Considering these cases can strengthen students' understanding of superposition and non-locality, also demonstrated by the realization of the EPR scenario as a paradigmatic experiment (In the advanced presentation of QP, the EPR experiment belongs to the *body* knowledge. Yet, in the introductory course, which only touch on non-locality, EPR can be sent to the *periphery*, posing a 'paradox' and promising the students its resolution in their further learning of Quantum theory).

Another item of the nucleus further expands the theory to many particles. After the principle of the indistinguishability of elementary particles in QP, the claim of dividing the micro-world particles into bosons and fermions may follow Pauli's principle for fermions and the possible coexistence of bosons in the same state. The principles of the *nucleus* can be associated with the phenomena of the *body* knowledge—the periodical table of elements for fermions and the laser for bosons.

3.2. Analysis of Textbooks

We will first address the textbooks from a more general perspective, followed by how the books refer to the concepts of the QP *nucleus*.

3.2.1. Blurring the Structure of the *Nucleus* and *Periphery*

All the textbooks of the sample begin by presenting special relativity theory. At the introductory level, QP is not relativistic. Thus, the two fundamental theories are essentially different. This implies that their *nuclei* are located in each other's *periphery* [27,28]. In fact, the two theories are combined under the roof of Modern Physics developed at the beginning of the 20th century. Importantly, however, if the curriculum does not deal with the status of a fundamental theory in physics and there is no such theory called Modern Physics, the inclusion of both theories in a text may confuse both the students and teachers regarding the boundaries of the QP *nucleus*.

Modern Physics texts often follow a historical narrative. In it, Planck's account of the black body radiation, Einstein's account of the photoelectric effect, and the Bohr model of an atom follow each other in a line. The Bohr model is semi-classical and belongs to the *periphery* of QP. This historical evolution in no way represents QP as a theory. For instance, in Krane's book [42] Planck's and Einstein's explanations are sub-headed under

"The quantum theory of...". Certainly, this approach represents neither the *nucleus* of the theory nor its *periphery*. The presented history is also inaccurate, since the Bohr model is located after dealing with de Broglie waves. Using de Broglie waves in the Bohr model was heuristic, and it does not explain wavity in the quantum sense as was discussed above.

A review of chapter headings and their subsections reveals a problem when addressing principles (*nucleus*) versus phenomena (*body*), let alone the *periphery*. For example, in Thornton and Rex's text [44], there is a chapter providing a historical introduction, a chapter on Bohr's atom, and a chapter (i.e., with the same status) on the wavity of matter, its meaning, the uncertainty principle, and more. Within the latter chapter, wavity is not associated with a superposition of states. There is no section that defines the basic principles of QP; however, several sections deal with phenomena (scattering, "a particle in a box"). The basic principles do not appear in titles, which would indicate their centrality. Other books in the sample are similar in this respect. Not identifying a *periphery* eliminates any contrast with the *nucleus*, blurring the meaning of different historical discoveries and missing the major principles of QP, which becomes clear by explicitly identifying the *nucleus*, *body*, and *periphery*.

3.2.2. Lack of Stating the Basic Principles

We found that the textbooks examined did not state the basic principles of QP. For example, in one of the books [43], in the chapter *The wavelike properties of particles*, the topic of particle-waves is discussed but no principle was stated, nor were there any fundamental interpretations of wavity in connection to the superposition of states. This makes the claim regarding the wave function totally unclear regarding its connection to a wave (what wave? why a wave?). The statement "$|\Psi|^2$ is proportional to the probability of detecting an electron in a unit volume" [43] (p. 204) appears arbitrary and is not explained.

Another example comes from Nolan [45]. This book presents and postulates (a sort of *nucleus*) both in the context of special relativity and in the context of Bohr's model of the atom. In light of this, the lack of such important definitions, which are central to QP, is very prominent.

3.2.3. Treating Principles as Phenomena

Another observed feature of the presentations in the sampled books is reference to principles as phenomena. Such a reference is made to superposition, the Heisenberg uncertainty principle, and wavity. For instance, wavity is referred to as the ability of an electron to perform diffraction and interference [43] (p. 187), as if in a classical sense. The meaning by addressing the superposition of different states (which does not exist in CP) is totally missed.

1. The same is true in Krane's textbook [42], that is, wavity is treated in the context of de Broglie's hypothesis, the focus is on diffraction and interference (p. 104), and there is no reference to the superposition of states. Later (p. 110), the double slit experiment is described with an examination of the slit in which an electron passes; however, there is no mention of wave function collapse during a measurement. The description mentions that it is related to particle-wise behavior, and if it is a particle, it loses its wavity (the complementarity principle). In this case, the phenomenon is indeed explained by a principle (the *body* is explained by the *nucleus*); however, a few deficiencies can be mentioned: The measurement not only revealed the particle property. It caused the collapse of the wave function from a two-state superposition to a single state (particle-wise). This is the same for a measurement made in the slits and a measurement made on the screen. This perspective is absent in the textbook.
2. The principle of complementarity is not explained here. Here, the principle is not distinguished from phenomena. The title *Through Which Slit Does the Particle Pass?* focuses on the phenomenon. The principle is not mentioned.
3. The principle of complementarity rarely appears in the book. It was referred to in the claim of duality; the wave and particle properties complement each other. The only

other case is where it is mentioned in relation to Bohr's contribution to QP, too little for the central principle of this theory.

Recall that the interviewed experts did not mention the complementarity principle as among the fundamental principles of QP. Complementarity has several faces, among which is the uncertainty principle. In the textbooks, complementarity is presented in the aspect of wave-particle duality, and the transition from wave to particle at the moment of measurement. This is unlike the experts who referred to the wave function collapse in the measurement but did not relate that to complementarity. This point, the impact of measurement on a quantum state, seems to us as important and was frequently missing.

Thornton and Rex's book [44] has more emphasis on principles. For example, there are two pages (pp. 191–192) under the title *Probability, Wave Functions, and the Copenhagen Interpretation*, but there is still no comparison between the Copenhagen interpretation and any other. The discussion mainly addresses the historical evolution of the QP theory and less on its holistic image.

Below, in Sections 3.2.4–3.2.7, we present how the books deal with some major components of the QP *nucleus*.

3.2.4. Superposition

In seven of the books chosen, the term "superposition" appears exclusively in addressing the interference between wave functions causing an interference pattern (in one of them, the word does not appear even once in the index). The definition of superposition as a fundamental principle that allows a particle to be simultaneously in several different states in superposition—the core feature of QP—is lacking.

In textbook [42], the term "superposition" appears only four times. The Bohr model appears after particle waves. The Bohr model is semi-classical and deterministic. It determines well-defined orbits and the energies of the electron in the hydrogen atom. It ignores superposition and is certainly inconsistent with the probabilistic aspect of quantum superposition. Its introduction after "matter waves" positions it as an advanced quantum model. This is not the case, mainly because it lacks quantum wavity and superposition as its essence.

In another textbook [43], the term superposition addresses classical waves (5 times), but its use in a quantum context (3 times) lacks a substantive explanation of superposition as a unique principle that allows several states to exist simultaneously. The phenomenon of the wave function collapse in measurement is absent (the term "collapse" does not appear). The term superposition appears in the context of interference and diffraction but never regarding the ability to coexist in multiple states. Nonetheless, superposition is mentioned in the more advanced topics, such as the Bose–Einstein condensate and elementary particles, but without an explanation presenting it as a principle.

Another example comes from Beiser's textbook [39], where superposition appears in two locations (four times). It addresses classical waves (p. 55) as a principle of combined waves. The other case is when considering the Schrödinger equation (pp. 169–170). It is used to present the mathematical aspects of the linearity of the equation, allowing its solutions to be combined. Superposition does not appear as a principle of the possibility of co-existing quantum states (e.g., in considering non-locality).

Only in book [46] does the superposition appear as a physical principle that addresses the combining of two states. In other textbooks in the sample, superposition is mentioned only as a mathematical feature, and not as a central and unique physical principle with probabilistic meaning.

3.2.5. Measurement and the Collapse of the Wave Function

The probabilistic aspect of QP is an established item of the *nucleus* (according to experts' interviews). It is multiply referred to in all the textbooks. To detect problems, we will address two topics linked to this concept: measurement and collapse of the wave function.

In eight of the textbooks, the term "collapse" does not appear at all. The books ignore it, and also lacked any other interpretation of measurement, similar to the "many worlds" interpretation. The issue of the essence of QP is lacking. In addressing measurement, the textbooks did not dedicate chapters or sections to define measurement as projecting to a new state. Seemingly, in the context of measurement, probabilistic results appear as a phenomenon: using the wave function, one calculates the probability of certain results in measurement. The books do not state the fundamental difference between the quantum and classical probabilities. The former presents the intrinsic feature of the QP, its *nucleus*, and differs from classical probability in statistical mechanics, technically imposed by the lack of information regarding the states of all particles. For example, Serway, Moses, and Moyer [41] (p. 192) refer to the wave function as a probability function describing the Born Interpretation of the phenomenon. The matter is not related to a state's superposition, a subject of intrusion in the course of measurement, and a probabilistic transfer to a single state. In other words, the *nucleus* is not contrasted with the *periphery* (quantum vs. classical physics). This terminology could be useful for pointing to the principles on which other ideas are based.

Similarly, Krane's book [42] (p. 110), when describing the measurement on the screen in the double slit experiment, does not mention the collapse of the electron's wave function and its probabilistic nature being valid for the flux of many particles. A missed opportunity was to state that the same collapse occurs in the detector instead of on the screen when the former was added next to the slits.

The book [46] differs from the rest. It relates to collapse (26 times), the interpretation of many worlds (5 times), and emphasizes the uniqueness of quantum measurement [46] (p. 8–39): "The postulate that translates operationally this view [the Copenhagen interpretation] is the one that takes into account the so-called collapse of the wave function. According to this hypothesis, when a quantum system is measured it interacts with a classical instrument, which forces the measured quantity of the quantum system to assume only one of the possible eigenvalues for that observable, so that the wave function instantly changes, i.e., collapses, in the eigenfunction associated with the selected eigenvalue."

All together, most of the textbooks lack a discussion on the meaning of measurement in QP as a unique feature through which we obtain knowledge of the micro-world.

3.2.6. Heisenberg's Uncertainty Principle

All the books refer to Heisenberg's uncertainty principle, and it does not suffer from a lack of emphasis (more than 100 references). At the same time, there are a number of problematic aspects in this regard. It has been often treated as a phenomenon and not as a principle. For example, [39] proclaims: "This principle... is one of the most significant physical laws"; yet it states [39] (p. 108): "It is impossible to know both the exact position and exact momentum of an object at the same time".

It appears in a separate line, highlighted as a definition of a principle. In a way, avoiding an explicit reference to uncertainty as an intrinsic feature in nature may be understood by learners as a difficulty, insurmountable, but it does not exclude the existence of the exact values. Next, one reads (p. 112): "A measurement establishes the position of a proton with an accuracy of $\pm$ 10^{-11} m. Find uncertainty in the proton's position 1s later. Assume $v << c$".

Presented as a phenomenon, the difficulty of *our ability* to know is emphasized. A similar motif appears in the explanation of "Heisenberg's microscope" (p. 113). The uncertainty in momentum is caused by the photon that measured the position. This explanation is deficient. As was mentioned in the experts' views, it refers to the uncertainty in our knowledge (the observers who need a photon) rather than a fundamental limitation. Next, on pp. 114–116, in addressing the minimal kinetic energy, the uncertainty appears independent of the observer. Thus, the two references to Heisenberg's principle are sometimes mixed: as the limitation introduced by observation and as intrinsic feature of reality. This blurs the difference between principle and phenomenon being potentially confusing at any level of teaching, especially in high school.

The uncertainty is often stated in the unclear status as our inability to be precise [41] (p. 174): "If a measurement of position is made with precision Δx, and a simultaneous measurement of momentum in the x direction is made with precision Δp_x, then the product of the two uncertainties can never be smaller than $\frac{\hbar}{2}$. That is, $\Delta p_x \Delta x \geq \frac{\hbar}{2}$." Here, $\hbar$ is the reduced Planck's constant.

The definition is presented as a highlighted paragraph. The origin of uncertainty is ignored. This fuzziness of its origin is common: our limitation, a phenomenon, is not a principle. It is never mentioned in the context of the superposition of states. In the whole sample, uncertainty is addressed only regarding the momentum-position or time-energy.

Another example we encounter in the definition appears in [42], which starts with (p. 116): *It is not possible to make a simultaneous determination of the position and the momentum of a particle with unlimited precision.*

A formula that links Δx to Δp follows. Yet the author proceeds with emphasis on our ability to be accurate. He states that inaccuracy of Δx implies inaccuracy of momentum and clarifies the essence [42] (p. 116):" The Heisenberg relationships are sometimes called "indeterminacy" rather than "uncertainty" principles, because the idea of uncertainty may suggest an experimental limit that can be reduced by using better equipment or technique".

The emphasis is explicit; it is a matter of principle. The author warns of confusion. Yet, the principle is never stated as implying the absence of being a particle in eigenstates of position and momentum simultaneously.

Thus, the uncertainty principle receives the status of a physical property and not the human inability to be precise, but even in these cases, it does not go beyond addressing the momentum-position and time-energy. In addition, it is often presented as a phenomenon, described by a formula, and in most textbooks, it ignores the eigenstates and the superposition states. In other words, it does not appear as a principle, it never mentions superposition, but rather, it refers to it as a curious phenomenon related to position and momentum, which have a certain relationship.

3.2.7. Entanglement and Non-Locality

The authors perceive entanglement and non-locality as an advanced topic. In seven books, these concepts were not mentioned. Non-locality appeared among the advanced topics in the book [46], in the context of the EPR paradox (p. 8–43). Nolan's text [45] did not address non-locality as a physics feature, but employed it in describing quantum teleportation (six times, pp. 354–355).

In conclusion, our findings indicate that the textbooks suitable for teaching QP in high school do not emphasize enough the principles, often present them as phenomena, and do not provide a set of principles on which the considered theory is built.

4. Discussion and Conclusions

Designing a curriculum for teaching QP at the high-school level is challenging in terms of the conceptual difficulty, the peculiarity of the theory, and the required mathematics (e.g., [35,51]). A special barrier is a lack of a pedagogical tradition to present a uniform theory instead of detached topics. Quantum theory essentially differs from the classical one and is not a refinement of it. An additional difficulty stems from the very limited time reserved for it in the curriculum. We claim that the DC paradigm of physical theory serves as an adequate tool in facing these difficulties. Its advantage is a well-defined hierarchical structure that carefully defines the components of the theory starting from its *nucleus*. This new requirement is demanding because such a tradition is lacking in the context of high-school teaching. We have arrived at a set of conceptual fundamental principles through interviews with experts in QP. We then examined the way in which these principles are expressed in textbooks at the relevant level and extracted additional insights.

In our defining the basic principles feasible for teaching QP in high school, we consider as central the wave-particle duality, with particle wavity defined as the ability of being in a superposition of states. This is consistent with the mathematical claim of linearity from which the superposition principle derives [12,51,52]. The quantum superposition is probabilistic, which makes the particle wavity specific in QP. This nature is manifested in measurements in which the collapse of the wave function reduces a quantum object into a particle-like state. Despite its centrality, we found this discussion extremely rare in the available teaching materials.

Heisenberg's uncertainty principle is commonly known as the limit of the accuracies of two quantities that cannot coexist as exact values. In high school physics, this relationship cannot be derived from the more general claim. We argue for teaching, in addition to the common form of the uncertainty principle, the conceptually equivalent claim with regard to the same pair of quantities prohibiting their simultaneous being in eigenstates. In other words, given that one quantity is in an eigenstate implies the other to be in a superposition of its eigenstates. Such an implication of the uncertainty principle strengthens our qualitative understanding of QP.

We observed that the experts did not mention the complementarity principle. Similarly, it appears to be not popular in the textbooks. (There is a different approach to preserving the centrality of the principle of complementarity [12]). This is an educational phenomenon, given that complementarity is a central philosophical principle of quantum theory. In fact, the complementarity principle (historically related to Niels Bohr) is closely related to Heisenberg's uncertainty principle, and the latter can be seen as a quantitative refinement of the former. Both principles involve measurement in the quantum world. Bohr's complementarity is not quantitative. It might explain its strange absence in introductory textbooks and teaching discourses. Yet, the conceptual relationship between the two principles would be definitely conceptually rewarding and it represents an area that requires an extra effort from physics educators.

4.1. Principles and Phenomena: Teaching the Body as the Nucleus

The division between *nucleus* and *body* is not always unequivocal. In the context of high school, which is given significant limitations of different kinds, some phenomena that actually arise from a fundamental but inaccessible principle can be referred to as a *nucleus*. A prominent example of this is Heisenberg's uncertainty principle, which is not fundamental in the advanced course, but it can be so in high school. Thus, some canonical phenomena, such as quantization, tunneling [53,54], and interference [55] might sometimes be treated as a "concept" or "principle".

In some previous studies, there was a search for key-topics in QP for high school, through various research methods (e.g., [3,35,54]). They categorized their findings to "fundamental principles" (or "concepts"), "examples" (sometimes include "experiments" as a different category), and "applications". Other categories were "philosophical aspects", "atomic theory" (which is sometimes included under "examples") and "mathematical representations". Certainly, what we found and define as the *nucleus* is congruent with their first category, but a refinement according to the triadic framework of the topics they consider as fundamental would show that it includes concepts that we define as the *body* (e.g., quantization, tunneling); and they are silent about the *periphery*. This may be because of the lack of distinction between "what is important to teach in high school QP" and "what is the nucleus of QP". The first, should definitely include aspects of the *body* (examples, applications), therefore canonical phenomena might get a principal status. However, although "duality" is unanimously a fundamental concept, in many cases it is described according to the behavior as a wave or as a particle in different experiments with no connection to superposition, as we mentioned above. That is, the *body* is presented as the *nucleus*. In addition, whereas some experts thought that teaching superposition is too complex (e.g., [54], p. 316), we consider it as the gateway for understanding QP and immanent to high-school QP curriculum. Yet, we do not exclude that there could be some circumstances to make a different choice of *nucleus*.

What is pedagogically important is that the DC approach requires a clear distinction between the *nucleus* and the *body* and *periphery*. In a way, one may put it as distinguishing between principles and the corresponding illustrative phenomena. Anything that derives from basic principles and can demonstrate or explain them can be categorized as the "*body*" knowledge. That includes interference, tunneling, quantized spectra, and more. Such an organization of the curriculum by DC can effectively overcome the challenges of teaching described above, assist pupils [30,33] and teachers [56] in understanding the subject matter and can significantly improve students' perceptions of the nature of science and scientific knowledge [30,57,58].

4.2. Treating Principles as Phenomena: Teaching the Nucleus as the Body

Unawareness of the DC structure may blur the boundary between the *nucleus* and the *body* by referring to principles as phenomena. Introductory QP textbooks usually introduce matter waves mainly through the phenomena of interference and diffraction of electrons. However, there is another way by addressing waves through the fundamental definition of the superposition of states [33]. A clear example of the former could be by presenting single photons in an experiment [4]. The alternative instruction, although describing interference, refers to the principle of superposition, and the collapse of the wave function in measurement. Similarly, presenting the uncertainty principle as our "inability" to know focuses on the phenomenon instead of the principle. In the DC approach, the phenomena demonstrate the principles and do not replace them; interference is not wavity. Instead of equating wavity with interference, the teacher states that wavity leads to interference, even if historically the order of the principle-phenomenon could be reversed.

Moreover, stating that the double-slit experiment is a phenomenon not only demonstrates wavity by interference, it also demonstrates a whole set of principles, a superposition of states, the non-locality of an electron that interferes with itself, the role of measurement (the screen as a detector), and the collapse of the wave function. Taken as a phenomenon, this experiment represents several basic principles. This is not obvious for different stakeholders of QP [34], but to appreciate this, one needs to distinguish between principles and their application.

In the DC approach, we try not to present a phenomenon and explain it through a principle, but rather, we claim the main thing, the *nucleus*–the principle, through the *body*–the phenomenon. In that way, we avoid confirming that wavity *is* interference or vice versa, stating that nature behaves according to the *principle* of wavity; hence, we observe a *phenomenon* of interference. We found this teaching suitable for the kind of challenges and severe limitations that we face in teaching QP at school [23,33,58].

We observed that the textbooks do not identify certain knowledge as the *periphery* of QP. Historical introductions often mix the items we identify as the *nucleus* in contrast to others identified as the *periphery.* The image of quantum theory loses clarity, which appears in contrast with the *periphery* and supports the understanding of principles—the *nucleus*. This effect is known, and an understanding of the wrong creates an understanding of the right. If one skips a comparison with the previous knowledge, we reduce the chances that the cognitive processes regarding conceptual change, consequently constructing new knowledge [59,60]. The absence of a comparison deprives the *nucleus* of its status, principles, and consequently, phenomena become indistinguishable.

4.3. Reference to the Principles at a Lower Level

Sometimes a principle is treated at a lower level than it actually deserves. Thus, teaching superposition solely as a mathematical feature while discussing interference, without emphasizing that it allows the superposition of states, greatly reduces the centrality of this principle. Likewise, Heisenberg's uncertainty is often referred to as a limitation in the relationship between position and momentum, without regard to the fundamental split among observables with direct implications to the kind of observable superposition states, which devaluates the importance of these fundamental principles.

Construction of a curriculum requires determining the content considered as central, regardless of the accuracy of its form. This should be backed up by theoretical and empirical considerations appropriate for the considered population. It is rather common to believe that QP cannot be taught in high school, and that it is a subject only taught at the university level. In effect, this challenge implies the need for a specific approach, a special perspective, and adapted tools, which are technically and philosophically feasible for this goal [23,36]. We believe that the findings of this study help in that endeavor. Specifically, we seek to bridge the gap between the teaching of QP at high school and higher levels. There is much in common in the *nucleus* (principles), *periphery,* and *body*, even if the way to teach and to construct the content must be different.

4.4. Making a DC Curriculum for High School

All of the above supports the need to make a curriculum based on a *nucleus*, *body*, and *periphery*. One can see that that this program will differ from other programs regarding its structure, content, and in the emphasis provided to various details.

First, because of the short time allocated for teaching, the curriculum should focus on QP and less on its historical introductions such as black body radiation and Bohr's model of the atom [3]. Second, reviewing worldwide curricula reveals that topics such as "Discrete energy levels", "Wave-particle duality", "Interaction between light and matter", "Technical applications", and "Matter waves, quantitative (calculations with the De Broglie wavelength)" are the ones that are taught the most [3]. In these topics, the emphasis is on phenomena much more than the principles of QP. (Even though duality is considered in Stadermann et al. [3] as a fundamental principle, it is often regarded as phenomena like the double slit experiment or the Mach-Zehnder interferometer.) Therefore, we do not claim that teaching the phenomena is not recommended; after all, for the purpose of learning, the abstract should be anchored to the concrete. Nevertheless, the focus and organization of QP curricula should differ.

We begin with a single particle and its state. It can be an eigenstate of a physical observable or a superposition of states. The *nucleus* defines wavity as a superposition whose coefficients possess probabilistic meaning, as realized in the collapse of the wave function in measurement. Heisenberg's uncertainty claims the existence of pairs of physical quantities, such as the position-momentum pair, in which being in an eigenstate implies that the mate is in a state of superposition. This is the *nucleus*.

The *body* includes the double slit experiment for electrons. It clarifies the principles of the *nucleus*. Its account includes interference of the wave functions of a single electron. The collapse of the wave function takes place on a screen, producing a specific position with the probability determined by the wave function. The interference pattern emerges after the screen is hit numerous times. Another case considers the output with a detector placed in one of the slits. The new pattern on the screen is compared with the one without the detector, illustrating the role of measurement, in the slits or on the screen.

The following example considers spin and polarization. The case of three measurements in succession is considered. It appears that each measurement deletes the "memory" of the electron or photon regarding its previous state. The Mach–Zehnder interferometer can follow further by illustrating the principles of superposition and the wave function collapse. As a special example, we used the BB84 quantum encryption protocol.

The items of the *periphery* emphasize the *nucleus*. For instance, we refer to the historical idea that the electron in an atom presents a cloud spread in space (an element of the periphery). Another example is the nature of quantum uncertainty essentially different from the uncertainty in classical statistical physics (an element of the periphery). Another misconception, a particle moving in a wave-like path can be also affiliated to the periphery of QP.

Table 3 illustrates the idea of inserting curricular content along with its tripartite organization into a DC structure. The content expands in the cases of single, double, and multiple particles (bosons and fermions). For each item of the *nucleus*, there are examples from the *body* and alternative conceptions of the *periphery*. We argue that such organization of the curriculum is not only representative but also seems beneficial [33] in the reality of a highly limited number of teaching hours. The corresponding teaching should include active learning (e.g., using worksheets and simulations). Being innovative in creating the content of the new curriculum requires organized and guided training for teachers [56].

Table 3. An example of DC-type of curricular structure of QP in high school. See text for details.

Nucleus	Body	Periphery
State–eigenstate and the principle of the superposition of states; the wave function The wavity of matter and superposition. Probabilistic interpretation and measurement Heisenberg's uncertainty and complementarity principle	Dirac notations The double-slit experiment with electrons Spin and polarization The Stern–Gerlach experiment Mach–Zehnder interferometer The BB84 protocol	Classical state and probability in mechanics and thermodynamics Classical measurement without disturbance Electron as a cloud Classic uncertainty—the lack of knowledge
Entanglement	An experiment to examine Bell's inequality	Hidden variables
Quantum indistinguishability Bosons and fermions	Laser, Pauli's exclusion principle, The Mendeleev periodic table	Particles distinguishable in classical statistics. Unification of matter.

4.5. Dirac Notation

An important part of designing the curriculum is adapting proper mathematical tools of presentation. There is certainly room to combine the teaching of the Schrödinger equation and its solutions, and the teaching of simple differential operators. However, subject to a limited timetable, it is necessary to consider the feasibility of such methods, which may be very mathematical and may disregard important conceptualizations. It appears possible, however, to use a special symbolic language, namely, the formalism of Dirac notations [23], to circumvent the challenge of mathematical representation and efficiently deliver the conceptualizations of QP. Such formalism allows representing much of the contents of Table 3, such as the principles of superposition, measurement, and collapse of the wave function of a few state systems (spin half and one).

An expression such as $|\psi\rangle = \frac{1}{2}|\uparrow\rangle - \frac{\sqrt{3}}{2}|\downarrow\rangle$ denotes the superposition of spin states; it represents a superposition state of several eigenstates and the simplest wave function of two states. It indicates the amplitudes allowing the calculation of probabilities to emerge in a measurement as well as the relative minus sign, which distinguishes this superposition state from another state with identical probabilities. That is, writing the state in such a way allows one to visualize the content often hidden in a complex mathematical form that normally prevents any computational context in high school. In our class, it enabled students to consider numerical problems requiring a simple calculation of probabilities and to deal with important cases involving spin or polarization.

5. Coda and Future Research

In this study, we discussed teaching QP in high school within the new educational paradigm of discipline-culture. In particular, we elaborated on the content of the *nucleus*—the core essential principles and concepts of quantum theory as a fundamental physics theory. We made all of this possible in a form suitable for high-school teaching. Within this perspective, we interviewed a group of experts, who were university researchers in QP, and examined how their ideas regarding the *nucleus* are (and are not) reflected in textbooks relevant for high-school teaching. We presented the idea of a curriculum structure based on the three components of the DC structure. Further research should examine the results of utilizing the DC approach for teaching and its impact on both the students' comprehension of QP and more generally, on their perceptions of the nature of science. Specifically, we are curious regarding the impact of the quantitative aspect of teaching with the use of Dirac notation, which seems very promising. Further research should accompany teachers' training when teaching QP and should elucidate students' difficulties, as well as determine the possibility of teaching additional content such as the Schrödinger equation, available in the same formalism, which was not mentioned as one of the components of the *nucleus*, but whose importance (in the *body*) is understood.

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