physics

MDPI

Article

The Bose-Einstein Correlations and the Strong Coupling Constant at Low Energies

Gideon Alexander [1] and **Boris Blok** [2,*]

1 School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Tel-Aviv, Israel; gideona@post.tau.ac.il
2 Department of Physics, Technion, Israel Institute of Technology, 32000 Haifa, Israel
* Correspondence: blok@ph.technion.ac.il; Tel.: +972-4-829-3599

Received: 12 February 2019; Accepted: 6 March 2019; Published: 12 March 2019

Abstract: It is shown that $\alpha_s(E)$, the strong coupling constant, can be determined in the non-perturbative regime from Bose-Einstein correlations (BEC). The obtained $\alpha_s(E)$, where E is the energy of the hadron in the center of mass reference frame of the di-hadron pair, is in agreement with the prescriptions dealt with in the Analytic Perturbative Theory approach. It also extrapolates smoothly to the standard perturbative $\alpha_s(E)$ at higher energies. Our results indicate that BEC dimension can be considered as an alternative approach to the short-range correlations between hadrons.

Keywords: Bose-Einstein correlations; hadron physics; potential models

1. Introduction

In recent years Bose-Einstein (BEC) and Fermi-Dirac (FDC) correlations [1] have been extensively studied mainly with identical pion pairs produced in lepton-lepton and hadron-hadron reactions, as well as in heavy ion (AA) collisions. In the one-dimension (1D) correlation analyses of pion and hadron pairs it was found that the resulting R dimension depends on the particle mass and found to be proportional to $1/\sqrt{m}$ where m is the mass of the correlated particles (See e.g., Ref. [2]). It has been further shown [3] that this $R(m)$ behavior can be described in terms of the Heisenberg uncertainty relations and from a general QCD potential considerations.

The two identical particle correlation effect can be measured in terms of the correlation function

$$C(p_1, p_2) \;=\; \frac{\rho(p_1, p_2)}{\rho_0(p_1, p_2)}\,, \tag{1}$$

where p_1 and p_2 are the 4-momenta of the two correlated hadrons and $\rho(p_1, p_2)$ is the two-particle density function. The $\rho_0(p_1, p_2)$ stands for the two particle density function in the absence of the a BEC (or FDC) effect. This ρ_0 is often referred to as the reference sample against which the correlation effect is compared to. The BEC and FDC analyses had often different experimental backgrounds and have chosen various types of reference $\rho_0(p_1, p_2)$ samples. Thus, one must take this situation in account when judging the correlation results in terms of energy and/or mass dependence.

The function frequently used in the BEC and the FDC studies the evaluation of the R are:

$$C(Q) \;=\; 1 + \lambda e^{-Q^2R^2}\ \text{. for bosons and}\quad C(Q) \;=\; 1 - \lambda e^{-Q^2R^2}\ \text{. for fermions.} \tag{2}$$

These are the Goldhaber parametrizations [4,5] proposed for a static Gaussian particle source in the plane-wave approach which assumes for the particle emitter a spherical volume with a radial Gaussian distribution. The λ factor, also known as the chaoticity parameter, lies within the range of 0 to 1. Due to the fact that the major correlation experiments were carried out with identical bosons we will here focus our discussion on the BEC.

It has been noticed already some two decades ago that the R extracted from BEC and FDC analysis of hadron pairs produced in the decay of the Z^0 gauge boson [2,6] suggested a mass dependence roughly proportional to $1/\sqrt{m}$ [3]. This is illustrated in Figure 1 taken from reference [7] where a compilation of the R results was obtained from the Z^0 hadronic decay experiments at LEP, The difference between the R value at the pion mass to those of the proton and Λ baryons is indeed impressive. However presently no significant difference is seen between the R of the pions and the K-mesons produced in the Z^0 decay. Thus, this dimension data, deduced from the e^+e^- prompt interactions, cannot serve for a precise expression for the R dependence on energy. For that reason, we use here the BEC dimension results obtained in $Pb-Pb$ collisions experiments [8].

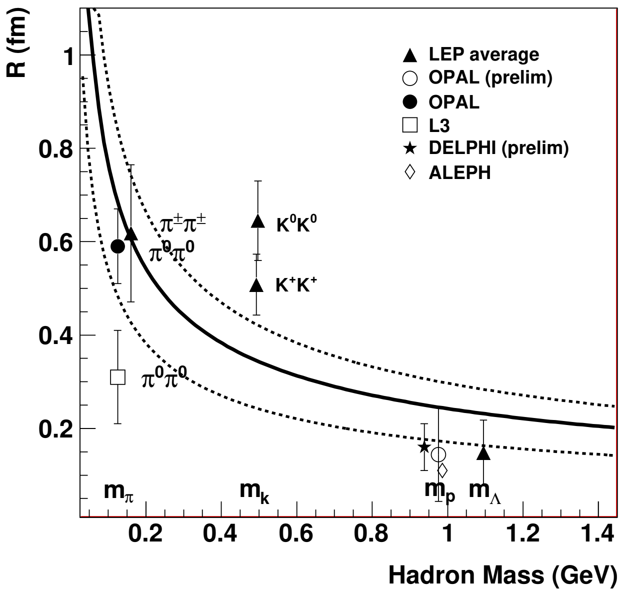

Figure 1. A compilation of R versus the hadron mass obtained from BEC and FDC analyses of the Z^0 hadronic decays of the LEP experiments [2,6] taken from Ref. [7]. The solid line and the dotted lines represent respectively Equation (7) with $\Delta t = 10^{-24}$ s and $\Delta t = (1 \pm 0.5) \times 10^{-24}$ s. Here $\hbar = c = 1$.

In this letter we show that the BEC can serve in the evaluation of the strong coupling constant α_s at the non-perturbative region of the scales $E \leq 1$ GeV (where E is the energy of the hadron in the center of mass reference frame of the di-hadron pair). The resulting coupling constant is shown to be in good qualitative agreement with the one obtained from solving the Bethe-Saltpeter equation to determine the effective potential of the quarkonium which in turn is consistent with the α_s deduced from an Analytic Perturbative Theory (APT) prescription.

In Section 2 we discuss the mass dependence of the BEC source scale and in Section 3 we derive an analytic formula relating the strong coupling constant α_s and the BEC source radius. Finally, in Section 4 we present numerical values for α_s and show that the BEC derived source dimension corresponds to strong overlapping hadrons where the BEC source radius is of the order of the distance between them.

2. The Mass Dependence of the BEC Dimension R

Since the maximum of the BEC enhancement of two identical bosons of mass m occurs when $Q \to 0$, the three-vector momentum difference of the bosons approaches zero. Thus, we can link the BEC effect to the Heisenberg uncertainty principle [3], namely

$$\Delta p \Delta r \;=\; 2\mu v r \;=\; m v r \;=\; \hbar c \,, \tag{3}$$

where μ is the reduced mass of the di-hadron system and $r = \Delta r$ is the distance between them. Here we use for Δp the GeV unit while r is given in fm units so that $\hbar c$ = 0.197 GeV fm. Thus, one obtains:

$$r \;=\; \frac{\hbar c}{m v} \;=\; \frac{\hbar c}{p} \,. \tag{4}$$

We also apply the uncertainty relation expressed in terms of time and energy

$$\Delta E \Delta t \;=\; \frac{p^2}{m} \Delta t \;=\; \hbar \,, \tag{5}$$

where the energy and Δt are given respectively in GeV and seconds. Thus, one has

$$p \;=\; \sqrt{\hbar m / \Delta t} \,. \tag{6}$$

Inserting this expression for p into Equation (4) one finally obtains

$$r(m) \;=\; \frac{hc}{\sqrt{m}} \sqrt{\frac{\Delta t}{h}} \;=\; \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m}} \,. \tag{7}$$

Comparing values of r in Equation (7) and experimental data for R (see Figure 1 we are led to identify r with R .

As mentioned above the R values, deduced from the BEC and FDC analyses of the Z^0 hadronic decays are shown in Figure 1. These results provided the first clue that the R may depend on the mass of the two identical correlated particles [3]. As can be seen, the measured R values of the pion pairs are located at $\sim$0.6 fm except for one $\pi^0\pi^0$ result where its R value lies significantly lower. The R Kaon pairs values are seen to be near to those of the charged pions. Impressive however are the R values obtained from the Λ hyperon and proton baryon pairs which lie close together in the vicinity of 0.15 fm. The solid line in the figure was calculated from Equation (7) with $\Delta t = 10^{-24}$ s representing the strong interactions time scale. The dashed lines are derived from Equation (7) setting $\Delta t = (1 \pm 0.5) \times 10^{-24}$ s to illustrate the sensitivity of Equation (7) in its ability to estimate the energy dependence of R . An alternative way to extract R dependence on the energy is to use the BEC results of the boson pairs produced in $Pb-Pb$ collisions.

A clear evidence for the dependence of R on the mass of the BEC boson pairs is seen in Figure 2 that was obtained by the WA98 collaboration [8]. In Figure 2 are plotted the BEC dimension deduced from identical correlated boson pairs, including the deutron pairs, produced in $Pb-Pb$ collisions at the nucleon-nucleon center of mass energy of 158 GeV/A. As can be seen, apart from the proton pair result, the R dependence on the mass value is very well described by $A/\sqrt{m}$, with the fitted value of $A = (2.75 \pm 0.04)$ fm GeV$^{1/2}$. According to Equation (7) one finds that $A = c\sqrt{\hbar \Delta t}$ so that in the $Pb-Pb$ collisions case $\Delta t = (1.28 \pm 0.04) \times 10^{-22}$ s. Taking for prompt pp collision the representing strong interaction value of $\Delta t = 10^{-24}$ s one obtains for R versus the mass, in GeV units, the relation

$$R \;=\; \frac{0.244 \pm 0.005}{\sqrt{m(\mathrm{GeV})}} \;\; \mathrm{fm} \,, \tag{8}$$

which is shown by a ± 1 s.d. band in Figure 3 which is of the same value as the LEP data-based result.

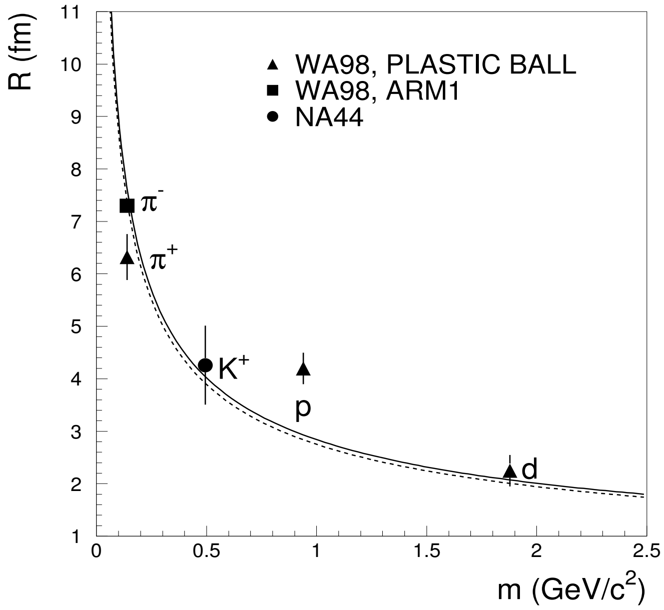

Figure 2. BEC analyses of hadron pairs emerging from central $Pb-Pb$ collisions at 158/A GeV [8]. The continuous line represents a fit of Equation (7) to the data of the Plastic Ball detector. The result of the fit where the WA98 data and the Kaon-pair R value from the NA44 collaboration were also included is shown by the dashed line.

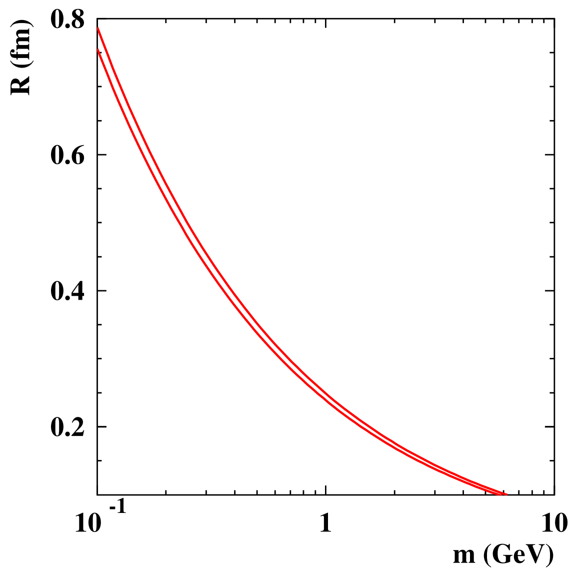

Figure 3. The ± 1 s.d. R band versus the mass energy of the boson pairs produced at a time delay of $\Delta t = 10^{-24}$ s as deduced from BEC of hadron pairs produced in $Pb-Pb$ collisions at 158 GeV/A [8] with a time delay of $\Delta t = (1.28 \pm 0.04) \times 10^{-22}$ s (see text). Here $\hbar = c = 1$ is used.

3. The Strong Coupling Constant and the BEC

The short-range interactions between two hadrons can be described in terms of the constituent quark model. This idea dates to [9] (see also [10]) and was applied to the BEC in [3]. Namely, the short-range interaction between hadrons can be described by means of the quark-quark interaction potential [11]

$$V(r) = -(4/3)\alpha_s \hbar c/r + \kappa r\ . \tag{9}$$

The coupling constant α_s is usually taken as a parameter to be fitted, while the constant κ, that corresponds to the confinement part of the interaction, is of order of 0.9 GeV/fm [12], while r is the distance between the two hadrons.

We now make use of the virial theorem for the two-hadron system, which has the form [13]

$$\langle 2T \rangle = < \vec{r} \cdot \vec{\nabla} V(r) >\ , \tag{10}$$

where $\langle T \rangle$ is the average kinetic energy of the hadrons.

Note now that in our approximation $2T = p^2/m$, where p is the momentum. For momentum, using Equations (6) and (7) to exclude Δt, we immediately obtain $p^2 = (\hbar c)^2/r^2 = 2Tm$. Then, taking into account the spherical symmetry of the potential one obtains

$$\frac{(\hbar c)^2}{m} = r^3 \frac{dV}{dr}\ . \tag{11}$$

This yields straightaway an expression for the strong coupling constant, namely

$$r^3 \left(\kappa + \frac{4}{3} \frac{\alpha_s \hbar c}{r^2} \right) - \frac{(\hbar c)^2}{m} = 0\ , \tag{12}$$

from which one has that α_s is equal to

$$\alpha_s \ = \ \frac{3}{4} \frac{(\hbar c)^2 - r^3 \kappa m}{m r \hbar c}\ . \tag{13}$$

Inserting $\hbar c = 0.1973$ GeV fm one obtains

$$\alpha_s = \frac{1.267(0.1168 - 3r^3 \kappa m)}{mr}\ . \tag{14}$$

To evaluate α_s we use for the parameter κ the value of 0.18 GeV^2/0.9 GeV/fm [12] corresponding to the meson Regge trajectory. The variable r and its mass dependence are taken to be identical to the R dimension given by Equation (8) which was determined from the analyses of the BEC and FDC deduced from identical hadron pairs (see also [3]).

4. Conclusions

Our main results are shown in Figure 4. Since our system in the center of mass energy is nonrelativistic, the α_s that we determine corresponds to an energy scale of $E \sim m$, where m is the hadron mass and E is the hadron energy in the di-hadron pair in its center of mass reference frame (i.e., approximately half of the energy of the Bose-Einstein pair).

The non-perturbative α_s is calculated via Equation (14) and is shown in Figure 4 as a function of energy by the solid line. The accompanying dotted lines represent the ± 1 s.d. limits of the band. For comparison we also show the perturbative α_s curve, labeled by pQCD, which essentially overlaps with the non-perturbative strong coupling in the region of about 2 to 11 GeV. Using our low energy non-perturbative strong coupling we obtain for example α_s at the mass energy of the K-meson and the Λ-hyperon respectively the following values:

$$\alpha_s(0.494\ \text{GeV}) = 0.451 \pm 0.035 \quad \text{and} \quad \alpha_s(1.115\ \text{GeV}) = 0.392 \pm 0.019.$$

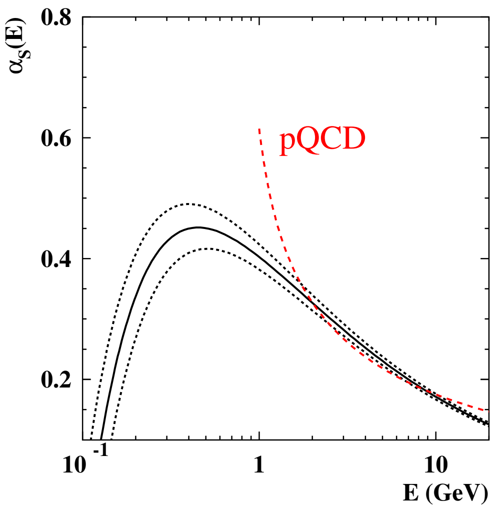

Figure 4. α_s as a function of energy below 20 GeV. The solid line represents the non-perturbative $\alpha_s(E)$ as a function of energy E of order m calculated from Equation (14). The dotted lines are defining the ± 1 s.d. band. The curved dashed line labeled as pQCD is the perturbative α_s calculated using reference [14] with the number of flavors N_f = 3 and setting $\Lambda_{\overline{MS}} = 200$ MeV [15]. Here $\hbar = c = 1$ is used.

The non-perturbative α_s determined here as a function of energy agrees well with the results obtained in [16,17], where effective α_s was obtained by solving the Bethe-Saltpeter equation for quarkonium. As a result, the strong coupling constant α_s determined here agrees well with the one-loop Analytic Perturbative Theory (APT) approach [18]). In particular we have good agreement with the so called "massive" variation of the APT prescription [18,19]. The latter one approach coincides with the standard APT approach for energy scales of $E > 200$ MeV [20], i.e., above the pion mass. However, for small E the strong coupling constant goes down to zero [18,19], exactly as we have in Figure 4. It is worthwhile to note however that although our curve is in good agreement with the "massive" APT prescription at small E of order of the pion mass, strictly speaking it is questionable if we can apply our estimates at the pion scale, based on a simple nonrelativistic quark model. Thus, "reasonable" agreement of our result for the pion mass with a particular version of APT prescription deserves further study.

Please note that recently additional approaches to the determination of α_s behavior in the low transverse momenta region were discussed, see [21–23]. The approach in [21] is based on one-loop renormalization loop calculations, while in [23] on combining dispersion relations with lattice simulations results and experimental data on e^+e^- annihilation. It is quite amazing that all the approaches lead to the same qualitative form of α_s as a function of energy/transverse momentum scale.

On the other hand, it is clear that all theoretical descriptions of the infrared dynamics currently are model-dependent and based, from field theoretical point of view , on different ansats for field theoretical resummation of corrections. In particular, there arises a question of connection between strong coupling constant in nonrelativistic quark model used in this paper and α_s defined in field theoretical schemes for infrared QCD dynamics. This question was discussed in detail in [16–19] where it was shown that α_s defined in particular renormalization/resummation scheme in these references, can indeed be identified with (up to short-range corrections) with the running coupling constant that enters the potential for nonrelativistic bound states.

In conclusion, it is shown that the strong coupling constant $\alpha_s(E)$ can be evaluated in the non-perturbative region using the Bose-Einstein and Fermi-Dirac correlations dimension results. The resulting $\alpha_s(E)$ is in good agreement with the so called APT "massive" prescription [18,19] and extrapolates well at the higher energies to the conventional perturbative $\alpha_s(E)$. Our results indicate that the BEC/FDC correlations both for baryons and mesons, correspond to a picture where the two participating hadrons strongly overlap, and the R radius, that conventionally characterizes the scale of the BEC/FDC, corresponds to the distance r between the centers of these two correlated particles. Thus, our results indicate that these correlations may well serve as an alternative approach for the study of short-range correlations between hadrons [24].

Finally, let us note that in our simulations we used the LEP and RHIC results. Recently high accuracy results for BEC were also obtained at LHC [25–27]. The use of their data does not change the results obtained in the current letter.

Author Contributions: The authors equally contributed to the text at all stages.

Funding: The research of B.B. was funded by ISF grant 2025311.

Acknowledgments: One of us (B.B.) would like to thank M.S. for useful discussions.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kittel, W.; de Wolf, E.A. *Soft Multihadron Dynamics*; World Scientific: Hackensack, NJ, USA, 2005; 652p.
2. Alexander, G. Bose–Einstein and Fermi-Dirac interferometry in particle physics. *Rep. Prog. Phys.* **2003**, *66*, 481. [CrossRef]
3. Alexander, G.; Cohen, I.; Levin, E. The dependence of the emission size on the hadron mass. *Phys. Lett. B* **1999**, *452*, 159–166. [CrossRef]
4. Goldhaber, G.; Fowler, W.B.; Goldhaber, S.; Hoang, T.F.; Kalogeropoulos, T.E.; Powell, W.M. Pion-Pion Correlations in Antiproton Annihilation Events. *Phys. Rev. Lett.* **1959**, 3, 181. [CrossRef]
5. Goldhaber, G.; Goldhaber, S.; Lee, W.; Pais, A. Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process. *Phys. Rev.* **1960**, *120*, 300. [CrossRef]
6. Kucharczyk, M. Fermi-Dirac correlations in Z0 to ppX. In Proceedings of the 39th Rencontres de Moriond on QCD and High-Energy Hadronic Interactions, La Thuile, Italy 28 March–4 April 2004; hep-ex/0405057.
7. Alexander, G.; Reinherz-Aronis, E. On the role of the time scale Δt in Bose–Einstein correlations. *Phys. Lett. B* **2010**, *687*, 310–313. [CrossRef]
8. WA98 Collaboration; Aggarwal, M.M.; Ahammed, Z.; Angelis, A.L.S.; Antonenko, V.; Arefiev, V.; Astakhov, V.; Avdeitchikov, V.; Awes, T.C.; Baba, P.V.K.S.; et al. Source radii at target rapidity from two-proton and two- deutron correlations in central $Pb - Pb$ collisions at 158 A GeV. *arXiv* **2007**, arXiv:0709.2477.
9. Liberman, D.A. Short-range part of the nuclear force. *Phys. Rev. D* **1977**, *16*, 1542. [CrossRef]
10. Frankfurt, L.; Strikman, M. *Nucleon Structure and Small Distances in Nuclei*; LNPI Preprint 173; LNPI: Évreux, France, 1975.
11. Eichten, E.; Gottfried, K.; Kinoshita, T.; Kogut, J.; Lane, K.D.; Yan, T.-M. Spectrum of Charmed Quark-Antiquark Bound States. *Phys. Rev. Lett.* **1975**, *34*, 369. [CrossRef]
12. Mendel, R.R.; Trottier, H.D. Connection between the string tension, light-meson Regge trajectories, and the QCD scale parameter. *Phys. Rev. D* **1990**, *42*, 911, [CrossRef]
13. Cohen-Tannoudji, C.; Diu, B.; Laloe, F. *Quantum Mechanics, v. 1*; John Willey and Sons: Hoboken, NJ, USA, 1977; p. 344.
14. Bethke, S. Experimental Tests of Asymptotic Freedom. *Prog. Part. Nucl. Phys.* **2007**, *58*, 351–386. [CrossRef]
15. Tanabashi, M.; Hagiwara K.; Hikasa, K.; Nakamura, K.; Sumino, Y.; Takahashi, F.; Tanaka, J.; Agashe, K.; Aielli, G.; Amsler, C.; et al. [Particle Data Group]. Review of Particle Physics. *Phys. Rev. D* **2018**, *98*, 030001. [CrossRef]
16. Prosperi, G.M.; Raciti, M.; Simolo, C. On the running coupling constant in QCD. *Prog. Part. Nucl. Phys.* **2007**, *58*, 387–438. [CrossRef]
17. Simolo, C. Extraction of characteristic constants in QCD with perturbative and nonperturbative methods. *arXiv* **2008**, arXiv:0807.1501.
18. Baldicchi, M.; Nesterenko, A.V.; Prosperi, G.M.; Shirkov, V.; Simolo, C. Bound-State Approach to the QCD Coupling Constant at Low-Energy Scales. *Phys. Rev. Lett.* **2007**, *99*, 242001. [CrossRef]
19. Baldicchi, M.; Nesterenko, A.; Prosperi, G.; Simolo, C. QCD coupling below 1 GeV from quarkonium spectrum. *Phys. Rev. D* **2008**, *77*, 034013. [CrossRef]

20. Solovtsov, I.L.; Shirkov, D.V. Analytic approach in quantum chromodynamics. *Teor. Mat. Fiz.* **1999**, *120*, 482–510; Reprinted in *Theor. Math. Phys.* **1999**, *120*, 1220–1224. [CrossRef]
21. Siringo, F. Analytical study of Yang–Mills theory in the infrared from first principles. *Nucl. Phys. B* **2016**, *907*, 572–596. [CrossRef]
22. Siringo, F. Analytic structure of QCD propagators in Minkowski space. *Phys. Rev. D* **2016**, *94*, 114036. [CrossRef]
23. Ayala, C.; Cvetic, G.; Kogerler, R.; Kondrashuk, I. Nearly perturbative lattice-motivated QCD coupling with zero IR limit. *J. Phys. G* **2018**, *45*, 035001. [CrossRef]
24. Frankfurt, L.; Strikman, M. Hard nuclear processes and microscopic nuclear structure. *Phys. Rep.* **1988**, *160*, 235–427. [CrossRef]
25. Aad, G.; Abbot, B.; Abdallah, J.; Abdel Khalek, S.; Abdinov, O.; Aben, R.; Abi, B.; Abolins, M.; AbouZeid, O.S.; Abramowicz, H.; et al. [Atlas Collaboration]. Two-particle Bose?Einstein correlations in pp collisions at $\sqrt{\mathbf{s}} = 0.9$ and 7 TeV measured with the ATLAS detector. *Eur. Phys. J. C* **2015**, *75*, 466. [CrossRef]
26. Sirunyan, A.M.; Tumasyan, A.; Adam, W.; Ambrogi, F.; Asilar, E.; Bergauer, T.; Brandstetter, J.; Brondolin, E.; Dragicevic, M.; Erö, J.; et al. [CMS Collaboration] Bose-Einstein correlations in pp, pPb, and PbPb collisions at $\sqrt{s_{NN}} = 0.9 - 7$ TeV. *Phys. Rev. C* **2018**, *97*, 064912. [CrossRef]
27. Aaij, R.R.; Adeva, B.; Adinolfi, M.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; Alfonso Albero, A.; Ali, S.; et al. [LHCb Collaboration]. Bose-Einstein correlations of same-sign charged pions in the forward region in pp collisions at $\sqrt{s}$ = 7 TeV. *JHEP* **2017**, *1712*, 025. [CrossRef]