forecasting

MDPI

Article

Side-Length-Independent Motif (*SLIM*): Motif Discovery and Volatility Analysis in Time Series—*SAX*, *MDL* and the Matrix Profile †

Eoin Cartwright [1,*,‡], Martin Crane [2,‡] and Heather J. Ruskin [1,‡]

1 Modelling & Scientific Computing Group (ModSci), School of Computing, Dublin City University, D09Y074 Dublin, Ireland; heather.ruskin@dcu.ie
2 ADAPT Centre, School of Computing, Dublin City University, D09Y074 Dublin, Ireland; martin.crane@dcu.ie
* Correspondence: eoin.cartwright3@mail.dcu.ie
† Special Issue "New Challenges in Energy and Finance Forecasting in the Era of Big Data".
‡ These authors contributed equally to this work.

Abstract: As the availability of big data-sets becomes more widespread so the importance of motif (or repeated pattern) identification and analysis increases. To date, the majority of motif identification algorithms that permit flexibility of sub-sequence length do so over a given range, with the restriction that both sides of an identified sub-sequence pair are of equal length. In this article, motivated by a better localised representation of variations in time series, a novel approach to the identification of motifs is discussed, which allows for some flexibility in side-length. The advantages of this flexibility include improved recognition of localised similar behaviour (manifested as *motif shape*) over varying timescales. As well as facilitating improved interpretation of localised volatility patterns and a visual comparison of relative volatility levels of series at a globalised level. The process described extends and modifies established techniques, namely *SAX*, *MDL* and the Matrix Profile, allowing advantageous properties of leading algorithms for data analysis and dimensionality reduction to be incorporated and future-proofed. Although this technique is potentially applicable to any time series analysis, the focus here is financial and energy sector applications where real-world examples examining *S&P500* and *Open Power System Data* are also provided for illustration.

Keywords: financial time series; matrix profile; symbolic aggregate approximation (*SAX*); minimum description length (*MDL*); time series motifs

Citation: Cartwright, E.; Crane, M.; Ruskin, H.J. Side-Length-Independent Motif (*SLIM*): Motif Discovery and Volatility Analysis in Time Series—*SAX*, *MDL* and the Matrix Profile. *Forecasting* **2022**, *4*, 219–237. https://doi.org/10.3390/forecast4010013

Academic Editors: Mariangela Guidolin and Luigi Grossi

Received: 8 December 2021
Accepted: 27 January 2022
Published: 4 February 2022

1. Introduction

A motif [1,2] is a repeated matched (or *partially* matched) sub-sequence taken from a larger parent time series (or set of time series). Given the ever-increasing prevalence of large, '*Big Data*' sets, commonly seen now in the *Energy* and *Financial* sectors, for example, the importance of motif analysis to facilitate interpretation of underlying series behaviour and prediction of future trends is increasing [3]. Here we apply a combination of existing algorithms and principles, namely *SAX*, *MDL* and the *Matrix Profile*, in order to improve the identification of repeated behaviour occurring within a series while allowing for flexibility of sub-sequence length.

The approach (designated **S**ide-**L**ength-**I**ndependent **M**otif or *SLIM*) is distinct from that of other motif search algorithms, which permit a user-defined length range with motif *side-length* of *A* and *B* to be equal, whereas our method permits motif sides to be of different lengths, extending pattern recognition potential for the series. Additionally, the details recorded during this process (described in Section 2.2) provide insight into series *volatility* at a local level but also facilitate the comparison of overall *volatility* between series.

This technique combination (Algorithm in Section 2.2.4), while developed for financial series initially, yields tangible results in more application areas than existing methods. For example, within the *energy sector*, the identification of motifs in power consumption data can represent patterns in user behaviour, improving forecasting of energy demand. In *finance*, these patterns or *motifs* represent repeated behaviour of a given series, such as a sharp rise in *share* or *market-rate* value with a slow decline or a gradual rise over a longer time period. Often these can take the form of commonly recognised '*shapes*' (or *behaviours*) in financial series, such as *Head and Shoulders*, for example [4].

1.1. Literature Review

Numerous approaches to time series analysis and forecasting appear in the literature [5]. For example [6], where several forecasting models, such as *Simple Exponential Smoothing* (*SES*) and *Autoregressive Integrated Moving Average* (*ARIMA*), are compared against a machine learning *Support Vector Regression* (*SVR*) model using weekly crude oil price data from 2009–2017. Additionally, in [7], local *Hurst Exponent* signals were used to investigate an anti-correlation signature in the share price evolution of the Warsaw stock exchange (*WIG20*) index, which occurs around the maximum share value.

ARIMA and *SES* use all data, whereas *Hurst* (and *motifs*) are more local in their focus. Here we are concerned with the use of *motifs* in time series analysis, which facilitate the examination of underlying trends and processes [3].

Focus on motif discovery in time series has intensified since the early 2000s, leading to significant algorithmic improvements in terms of speed and efficiency. Applications include *finance* [8], *health* [9] and *music* [10], amongst others. Additionally, motif discovery algorithms are used as subroutines in many time-series data mining tasks, such as rule mining, clustering and classification [11–13].

In general, motif discovery algorithms can be divided into two groups, categorised by *fixed* and *variable lengths* with a further distinction made between the use of *approximate* and *exact* approaches (Figure 1).

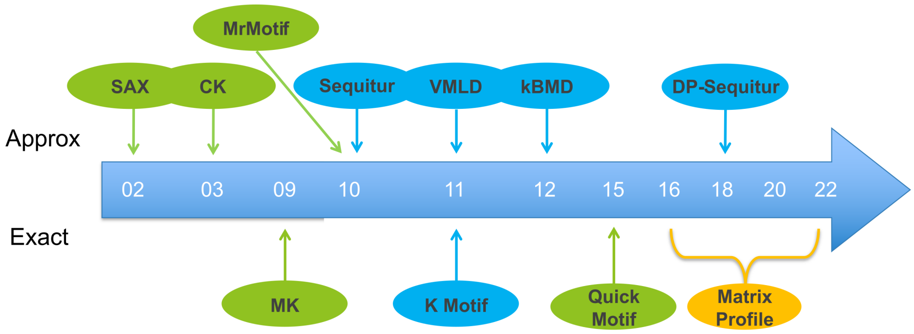


Figure 1. Motif discovery algorithm highlights timeline. *Green* indicates *fixed-length* algorithms while *blue* represents *variable length*.

Approximate fixed-length motif discovery is largely based upon *random projection* (*CK* Algorithm [14]) and *Symbolic Aggregate Approximation* or *SAX* [2,15] techniques (discussed further in Section 2.1.1). Of note is the use of *iSAX* in the *MrMotif* [16,17] algorithm that derives a set of top-K motifs for a fixed length through increasing *SAX* resolutions.

Exact approaches initially concentrated on the use of early abandonment or *Smart Brute Force* (*SBF*) in the *MK* [1] algorithm, with further speed efficiencies gained by *Quick Motif* [18] for example. More recently, the use of the *Matrix Profile* (*MP*) [19,20], a highly efficient *Euclidean Distance* similarity search algorithm, has predominated as state-of-the-art for fixed-length motif discovery.

Variable-length approximate algorithms include grammar induction-based approaches, such as *Sequitur* [21] and *DP-Sequitur* [11], along with *VMLD* [22] and *kBMD* [23], which eliminate the requirement for a predefined sliding window length parameter. Exact algorithms that allow for variable-length include *K-Motif* [24] and *VALMOD* [25], along with *SKIMP* [26]. *SKIMP* is the first practical technique to find motifs and discords for all lengths through the creation of a *Pan Matrix Profile*(*PMP*), which can also be easily visualised as a heatmap (see Figure 10d). Both *VALMOD* and *SKIMP* are part of a body of recent *MP* papers [20], providing an important resource for time series analysis.

1.2. Contribution

Here we propose a novel combination of several methods in order to investigate high-frequency changes (i.e., the *volatility* in *financial* and *energy* markets). Similarly, our approach offers increased flexibility in the identification of motif characteristics, such as *side length* and *shape*.

The main contribution of *SLIM* is the introduction of flexibility of *motif side-length* within an individual motif sub-sequence pair, allowing similar behaviour occurring over differing lengths to be identified and *directly* compared. This is demonstrated in Figure 2 with two identified motif pairs planted in a synthetic sample series, one of equal *motif side-length* (Figure 2b) and the other unequal (Figure 2c).

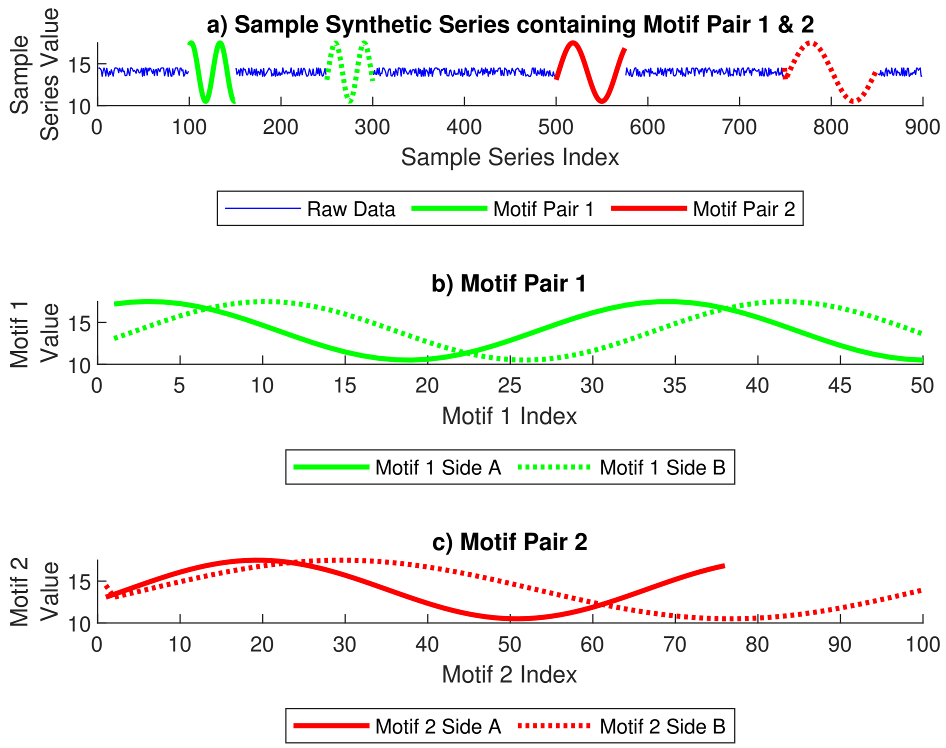


Figure 2. Sample synthetic time series showing two motif pair locations (**a**). *Motif Pair 1* (**b**), has equal length for Sides A and B, while *Motif Pair 2* (**c**) has differing motif side lengths.

Existing variable-length motif discovery algorithms produce results over a range with motif pair sides equal. Thus, extra processing is required in order to compare behaviour over two different length values *directly*, whereas *SLIM* can produce *individual* motifs with *variable side lengths*, obtained through the temporary compression of similar sub-sequence values before close matches are identified.

Additional benefits of *SLIM* include allowing a visual comparison of relative volatility levels *within* and *between* series (an important consideration, especially in finance as volatility furnishes key aspects, such as return on investments and effective hedging [27]). The identification of localised sub-sequences distinguishing volatility related to sudden large events from a consistent increase, for example, is also facilitated by *SLIM*.

The basis for the *SLIM* approach utilises established techniques, such as *Symbolic Aggregate Approximation* or *SAX* [2,15,28], the *Matrix Profile* (*MP*) algorithm [19,20] and the principle of *Minimum Description Length* (*MDL*). A brief outline of the algorithms employed is provided in Section 2, with a detailed explanation of the new technique of applying *MDL* to *SAX* strings in Section 2.2. Finally, real-world examples are used to illustrate technique application in Section 3. Future scope for improvements, such as more rigorous dimensionality reduction (offered by *SAX* for examination of the '*Big Data*' sets that are becoming more prevalent), are also discussed in Section 4.

2. Methodology

2.1. Underlying Algorithms

The three main algorithms of interest are:

2.1.1. Symbolic Aggregate Approximation (*SAX*)

Symbolic Aggregate Approximation or *SAX* [2,15,28] is used to discretise time-series data into a symbolic string that reduces dimensionality while indexed by a lower-bounding distance measure. It has proven to be a particularly effective tool for motif discovery, underpinning many string analysis techniques for motif detection borrowed from the study of DNA sequences. *SAX* relies upon the following definitions:

Definition 1. *A time series T, is a sequence of real-valued numbers $T = t_1, t_2, ..., t_n$ where n is the length of T [2].*

Definition 2. *A SAX string $\hat{C}$, is a symbolic representation of a time series $\hat{C} = \hat{C}_1, \hat{C}_2, \ldots, \hat{C}_w$ assigned to a Piecewise Aggregate Approximation reduction of T, from n to w dimensions $\overline{C} = \overline{C_1}, \overline{C_2}, \ldots, \overline{C_w}$ (adapted from [2]). w is the length of symbolic representation (i.e., no. of piecewise segments) where $w << n$.*

Definition 3. *Breakpoints are a sorted list of numbers $B = \beta_1, \ldots, \beta_{a-1}$ such that the area under a $N(0,1)$ Gaussian curve from β_i to $\beta_{i+1} = \frac{1}{a}$ (β_0 and β_a are defined as $-\infty$ and ∞, respectively), while a is the alphabet size [2].*

In summary, an input series is first normalised and broken into a user-provided number of horizontal segments. An average of the series values within each segment is taken, and a symbol is then assigned, depending upon the value range that contains the average. The symbol value intervals (i.e., vertical segment size or *breakpoints*) are calculated according to the equal assignment of area under a Gaussian curve, which, in turn, is dependent upon the alphabet size provided. Figure 3 illustrates this transformation process. Note, symbols can be alphanumeric, producing *SAX* strings, such as **bacbc**, for example, or **21323**, as shown here.

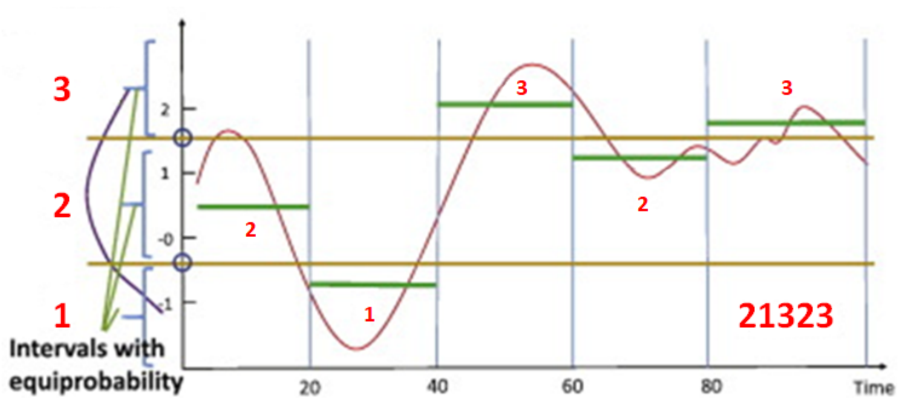


Figure 3. Translation of a sample raw series into a symbolic string using Symbolic Aggregate Approximation (*SAX*), adapted from [29]. Breakpoints for symbol designation are shown as brown horizontal lines. Green lines indicate average series values within a series segment (split by black vertical lines).

Numerous versions of *SAX* have evolved that improve and tailor the technique to particular applications. Examples include *iSAX*, which applies an indexing technique that is fast and scalable [30] and *Symbolic Fourier Approximation* (*SFA*) that introduces a symbolic representation based on the frequency domain, allowing for indexing of high-dimensional data-sets [31]. Here we are concerned with obtaining *SAX* strings from sample time series to serve as the basis for further pattern analysis.

2.1.2. Minimum Description Length (*MDL*)

The basic principle of *Minimum Description Length* (*MDL*) is that, given a limited set of observed data, the best description is one that permits the *greatest compression*. *MDL* is used in a wide range of disciplines, such as machine learning, data mining, biology and econometrics [32,33].

The *MDL* principle is applied here to *SAX* strings obtained from a sample series (see Figure 8, Section 3.1.1 for an illustration). Using *MDL*, a *SAX* series representation can be refined to highlight regions of stability/volatility, as well as to allow a length flexibility when identifying pattern repeats (*motifs*).

2.1.3. Matrix Profile (*MP*)

The *Matrix Profile* (*MP*), a novel algorithm due to [20], has already demonstrated considerable potential for numerous data mining and time series analysis tasks. It has been found to be highly scalable for time-series sub-sequence *all-pairs-similarity* search [19] and also efficiently identifies time series motifs and discords (i.e., mismatches). For a more comprehensive summary and further information on the *MP* (including extensions) see [20].

The *MP* can be represented as a *pseudo-series* where low *MP* values are indicative of close matches (in terms of *Euclidean* distance) to some other point within the examined series. The start location of the identified matching sub-sequence can be obtained from an associated *Matrix Profile Index* (*MPI*) value. When examining *MP* plots, low-value regions indicate matches or *motifs*, while high values illustrate mismatch regions, or *discords* (Figure 4).

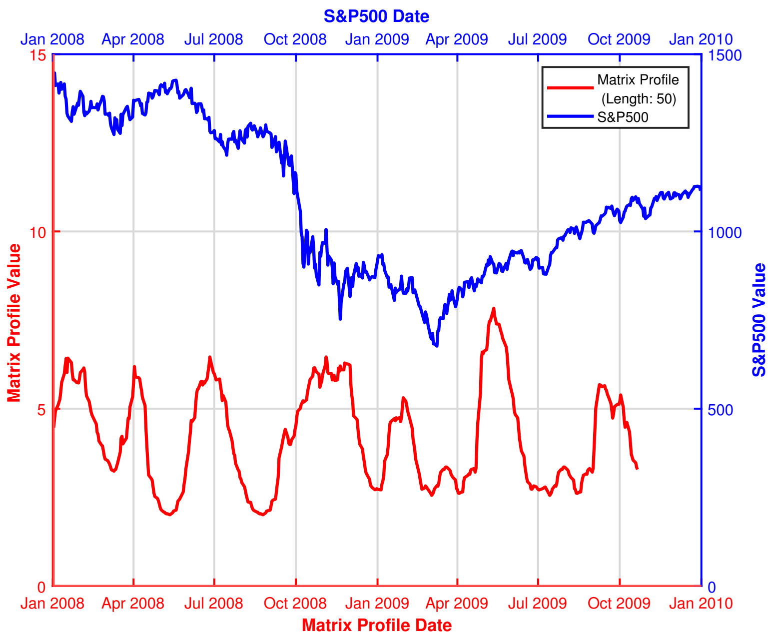


Figure 4. Sample matrix profile representation (*red series*) of the *S&P500* (*blue*) series between January 2008 and January 2010. Here low *MP* values indicate a close match with another sub-sequence (of *MP* length) at some other point within the *S&P500*. This point is given by the associated *Matrix Profile Index* (*MPI*) value.

Here the focus is on the interpretation of *MP* plots in *financial* and *energy* sector time series, where low *MP* plot values (based upon *SAX* series representations) highlight match regions or *motifs*.

2.2. Combined Methodology

There are two parts to our approach: Firstly, the *MDL* principle is applied to *SAX* strings, followed by the application of the *Matrix Profile* to an *MDL-SAX* string in order to identify match regions (or *motifs*).

As a motif is a pair of similar sub-sequences (or *segments*) of a larger time series, there must be a minimum of two sides or parts to the match. For clarity, the sides of a motif pair are designated as *Sides A* and *B*, where *Side A* is considered as the initial candidate segment, and *Side B* is the segment identified as a match of *Side A*.

2.2.1. Application of *MDL* to *SAX* strings

Series values are parsed into *SAX* strings, which are then compressed, where adjacent repeat elements are collapsed into one element, with the superscript value indicating the number of repeated elements, Figure 5.

Definition 4. *An MDL-SAX string $\hat{M}$, is a compressed representation of a SAX string $\hat{M} = \hat{M}_1, \hat{M}_2, ..., \hat{M}_z$ obtained by the application of the Minimum Description Length principle, such that adjacent equal values $\hat{M}_i, ..., \hat{M}_n$ are represented by $\hat{M}_i^x$ where $x = |\hat{M}_i, ..., \hat{M}_n|$ (i.e., the number of compressed elements).*

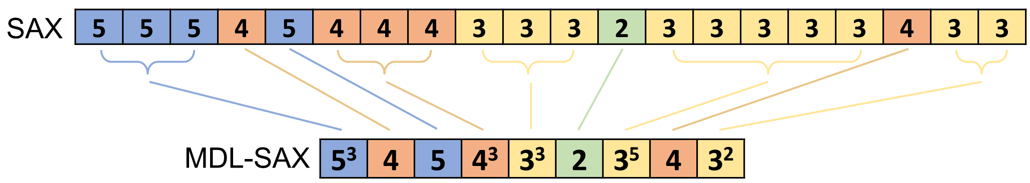


Figure 5. Illustration of *MDL-SAX* string created from a *SAX* time series representation. Consecutive *equal SAX* elements are collapsed to a single *MDL-SAX* element, with the superscript value indicating the number of repeats.

Thus, a sample *SAX* string of **55545444333233333433** becomes $\mathbf{5^3 4 5 4^3 3^3 2 3^5 4 3^2}$. The process is demonstrated for a real series (*S&P500*) in Figure 8, Section 3.1.1, for an initial time window of January 2008 to January 2010, with the resulting *MDL-SAX* value string displayed in Figure 8d.

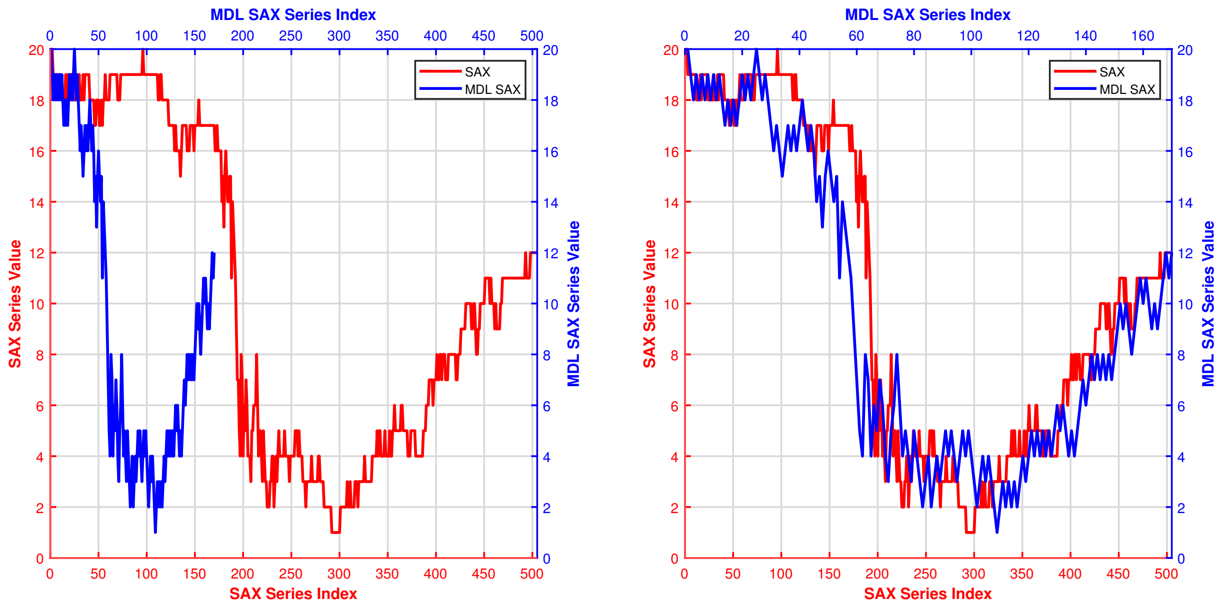
Additional detail retained for further analysis includes the number of consecutive *SAX* string elements combined, the value difference between adjacent *MDL-SAX* string elements, along with *SAX/MDL-SAX* string and raw series indexes (Table 1). Note the length reduction between the *SAX*(c) and *MDL-SAX*(d) representations in Figure 8.

Thus, an *MDL-SAX* string is a compression of the original *SAX* string representation of the raw time-series data. For time series, *MDL-SAX* compression of the original *SAX* string has the net effect of '*removing*' periods of stability while *retaining* the volatility profile (Figure 6a).

Table 1. Sample additional detail recorded during the construction of an *MDL-SAX*string. **SAXVal** is the value of the *SAX* series at a given index, **SAXValDiff** is the difference between the current and previously recorded *SAX* value, while **SymJoinNum** is the number of consecutive *SAX* series values combined through the application of *MDL*.

Raw Series Date	SAX Series Index	SAXVal	SAXValDiff	SymJoinNum	RawSeries Index
2 January 2009	97	5	1	3	254
7 January 2009	98	4	−1	1	257
...	...	...	...	...	...
28 January 2009	104	4	1	1	271
29 January 2009	105	3	−1	2	272

Figure 6 illustrates a comparison between *SAX* and *MDL-SAX* representations of the *S&P500* from January 2008 to January 2010, a window chosen for the volatility that reflects the considerable stress experienced in the global marketplace at this time [34,35] and extending previous work [36,37].



(**a**) Non-adjusted scale to illustrate length compression. (**b**) Scale adjusted to illustrate relative shape.

Figure 6. *SAX* and *MDL-SAX* representations of the *S&P500* from January 2008 to January 2010. *SAX* alphabet size = 20, Num of SAX segments = series length.

In the illustrated example, the *MDL-SAX* and *SAX* strings are plotted using both non-adjusted (Figure 6a) and adjusted (Figure 6b) scales to highlight the features of the *SAX* string captured by *MDL-SAX*. The overall shape and dynamics are well preserved, as the series examined has relatively few periods of stability. However, if an alternative series with low volatility is chosen, then a higher rate of compression would be observed, affecting the profile of the *MDL-SAX* string relative to the *SAX* string values.

2.2.2. Hyperparameter Selection: Influence of Alphabet Size Choice upon *MDL* Compression Rate

The choice of *alphabet size* used when creating the initial *SAX* string from the raw series will influence the *compression rate* when *MDL* is applied. Thus, while *MDL* compression is dependent upon the volatility level of the series in question, an increase in alphabet size chosen will reduce the overall compression rate.

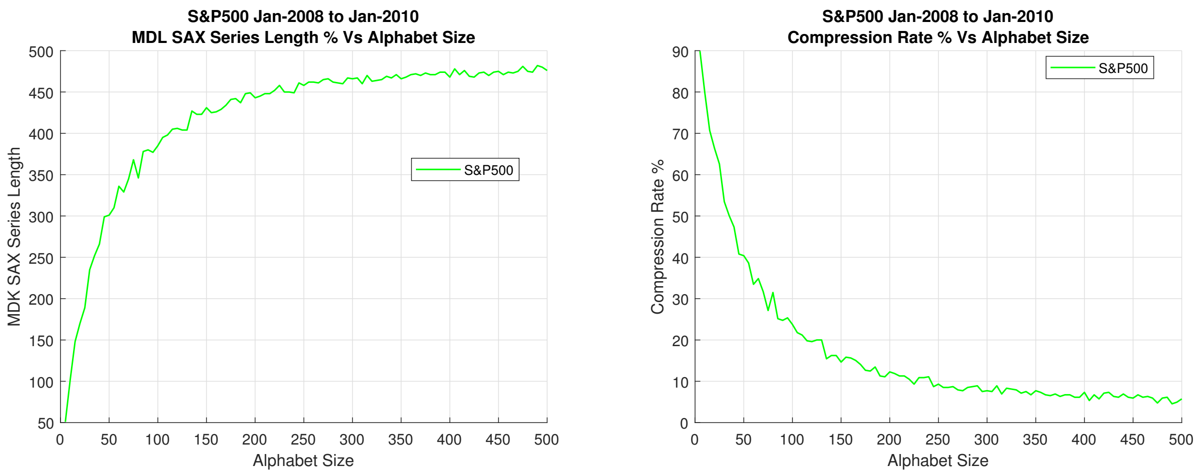
This result is intuitive, of course, as a larger alphabet size requires a corresponding increase in *SAX* breakpoints, which in turn leads to an increased *resolution* of *SAX* values. As a given series segment is then represented by an *increased* range of *SAX* values, overall *MDL* compression is reduced. Figure 7b illustrates this for the *S&P500* from January 2008 to January 2010, where the compression rate *CR%* is given as:

$$CR\% = \left(\frac{L_{SAX} - L_{MDLSAX}}{L_{SAX}} \right) \times 100 \tag{1}$$

where L_{SAX} is the length of the *SAX* series and L_{MDLSAX} is the length of the *SAX* series after *MDL* has been applied.

An increased alphabet may be required for a stable series in order to obtain a similar compression rate to that of a more volatile series, i.e., in order to compensate for slow increases or decreases represented by the same *SAX* value.

Thus, *Compression Rate* % vs. *SAX Alphabet Size* plots, as shown in Figure 7b, allow a choice of a suitable *alphabet size* value to be made in order to achieve a desired *compression rate* when obtaining an initial *MDL-SAX* string from the raw series. Additionally, when used in conjunction with an examination of raw series *motifs* (as discussed in Section 2.2.4), these plots facilitate a choice based on the amount of *compression* required.



(**a**) *MDL-SAX* series length for increasing alphabet size. (**b**) Compression Rate % for increasing alphabet size.

Figure 7. Effect of *MDL* application to a *SAX S&P500* series from January 2008 to January 2010. Accordingly, as the alphabet size used to create the *SAX* representation of the *S&P500* series increases, the length of the resulting *MDL-SAX* series also increases (**a**) while the compression rate is reduced (**b**). A large alphabet size range is utilised here to illustrate the stability of the compression rate at higher alphabet values.

Of further note is the choice of the *alphabet* and *segment* size (i.e., the length of raw series that each *SAX* symbol represents) to provide a suitable *compression rate* (as here) when *MDL* is applied, as opposed to preservation of raw series features. The original *SAX* technique objective is dimensionality reduction [15], where the choice of *alphabet size* and *segment number* determines the maximum reduction in data while preserving features of an input series with a lower bound.

However, since the primary concern here is a desired compression rate upon the application of *MDL*. *SAX* series breakpoints may be, in consequence, more frequent in relation to the original raw series than the volatility demands. Of course, extremely large series (such as those for *high-frequency* financial trading) may require initial data reduction from the *SAX* representation, achieved by a suitable choice of *segment size*.

2.2.3. Motif Discovery

It should be noted that *SAX* or *MDL-SAX* representations of an original series can also be used as input to the *MP* algorithm allowing match regions (or *motifs*) to be identified (within the *SAX* or *MDL-SAX* strings), as indicated by low *MP* values.

This raises the question of why we choose to use the *MP* of a *SAX* string for motif detection at all, as opposed to available alternatives, such as comparing *SAX* string values directly, using, for example, some form of a sliding window. While this was investigated and is relatively simple, the number of trivial matches obtained is very large. Further, using a 1:1 ratio between *SAX* values and raw data means that searching is limited to finding an exact match (or ideal *motif*), which is a rare event [38].

The *MP* algorithm is more efficient as a motif candidate is obtained even for non-exact matches (in terms of *Euclidean* distance). It also incorporates an in-built exclusion zone principle that rejects trivial (or same position) matches. Finally, the efficiency of the *MP* is $O(nlogn)$ (where n is the length of the input time series) [19], which is an improvement on a brute-force comparative approach based upon *SAX* strings.

2.2.4. Independent Side-Length Motif Discovery Process

Clearly, an *MP* can be obtained directly from raw data input with less effort than by first creating a *SAX* representation of the series. However, the use of *SAX* presents further in-depth opportunities for analysis through the application of *MDL* to the *SAX* string before the *MP* plot is generated. This allows motifs to be identified after the *MDL* compression has occurred, capturing higher-order match regions of similar behaviour.

Additionally, when returning to either the original *SAX* string or raw data without *MDL* applied (using values stored in Table 1, for example), variable-length introduction is possible without reference to the side-length of the motif pair as it relies solely on the compression that occurs within the *individual* segment of the *MDL-SAX* string that is identified as a close match. Thus, *side-length-independent motifs* can be obtained from the raw data where the length of *Side A* does not necessarily match that of *Side B*.

The process by which this is achieved is as follows Algorithm 1:

Algorithm 1: Side-**L**ength-**I**ndependent **M**otif (*SLIM*) pseudo-code.

Data: Input raw time series
Result: Candidate motif locations with variable side-length
Step A: Transform raw input series into a suitable *SAX* representation
Step B: Compress the *SAX* series using *MDL* to create an *MDL-SAX* series
Step C: *MDL-SAX* series serves as input to the *MP* algorithm creating an *MDL-SAX-MP* series
while *examining MDL-SAX-MP series* **do**
Identify low *MDL-SAX-MP* series values indicating close match, i.e., *motifs*
- Sub-sequence length = length parameter input to the *MP* algorithm
- Location of matching side of the *motif* pair is obtained from *MPI* value (i.e., motif sides A and B as described earlier)

Obtain end point of *MDL-SAX* motif segment for each side of the motif pair
- Allowing *MDL* compression to be removed when plotting corresponding *SAX* and raw series segments

end

2.2.5. Advantages

Our approach, designated the **S**ide-**L**ength-**I**ndependent **M**otif (*SLIM*), offers several advantages over the reported methods to date:

- Permits identification of motif pairs in which the length of each side is independent.
- Properties of the underlying algorithms are inherited.
  - Dimensionality reduction of *SAX* (if required).
  - Efficiency and scalability of the *MP*.

- Is independent of *SAX* and *MP* versions used and so can take advantage of further improvements to these algorithms.

3. Results and Discussions

In the following section we highlight potential applications and advantages of *SLIM* (**S**ide-**L**ength-**I**ndependent **M**otif identification) in the *financial* and *energy* sectors, two areas generating increasingly large data-sets.

In the *energy sector*, a set of hourly *Open Power System Data* (*OPSD*) relevant for power system modelling within the EU and neighbouring countries was considered [39]. For the *financial* illustration, we build upon previous work [36,37] and continue with *S&P500* data, as it is widely regarded as the best single gauge of U.S. equities and serves as the foundation for a wide range of investment products [40].

For the *S&P500*, localised aspects of volatility are highlighted, while *compression rate* % vs. *alphabet size* plots are also used to compare volatility levels for power usage across several European countries.

3.1. Finance

3.1.1. Side-Length-Independent Motif Discovery

To demonstrate the steps outlined in Section 2.2.4, *S&P500* index data between January 2008 and 2010 (Figure 8a) is converted into a *SAX* representation in Figure 8b. Conversion of these *SAX* values to an *MDL-SAX* series is shown in Figure 8c,d (using a single month for clarity).

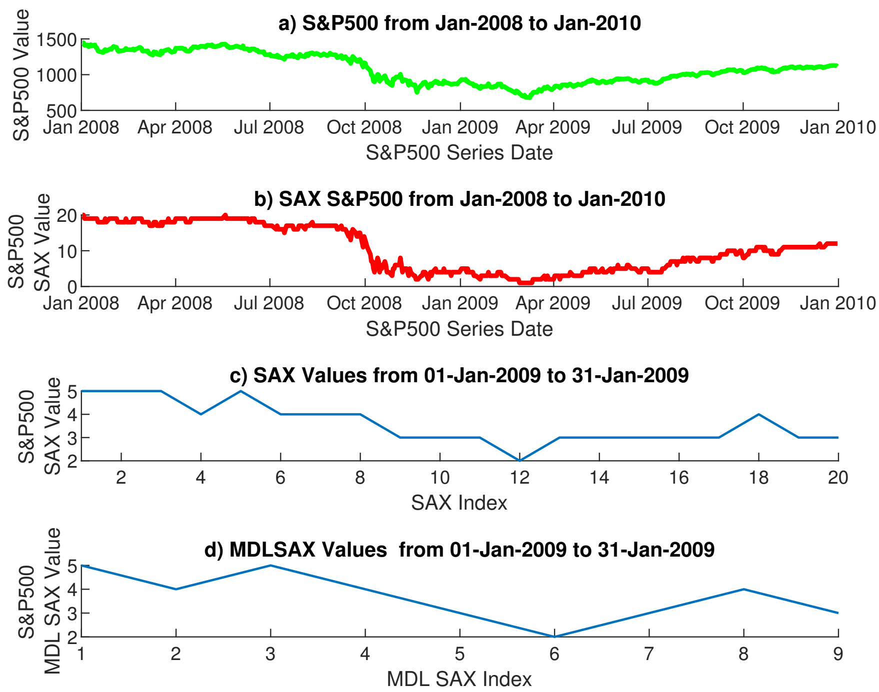


Figure 8. Sample *S&P500* raw series input (**a**) to *SAX* string transformation (**b**). Note the length reduction between the *SAX* (**c**) and *MDL-SAX* (**d**) representations for January *S&P500* values.

An *MDL-SAX-MP* plot for the full time window was created (Figure 9a). Close matches, as indicated by low *MP* values within the *MDL-SAX-MP* series, are highlighted as points **X**, **Y** and **Z**, while corresponding *motif pairs* within the *MDL-SAX* series are shown in the top plots of Figure 9b–d.

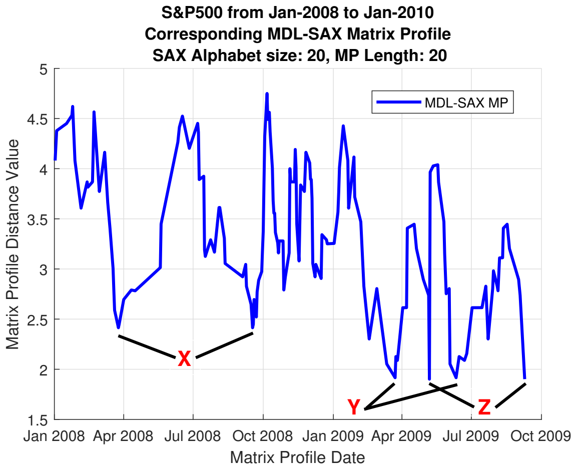
The *MDL* criterion is then removed by returning to the start and end points of the *SAX* series that the *MDL-SAX* motif represents. The length of each side of the motif pair will vary independently as a result of the level of compression that occurred in the particular section of the *MDL-SAX* series, as shown in Figure 8c,d.

The resulting change in *motif side-length* of the *SAX* series is shown in the middle plots of Figure 9b–d, while the equivalent raw series segments are also included in the bottom plots for comparison.

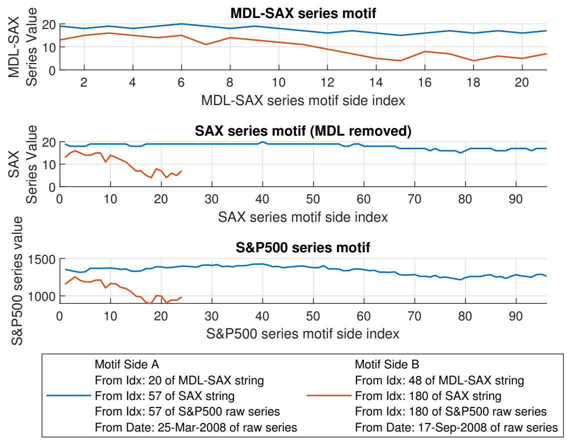
Figure 9b shows the most compression and gives a greater length differential between the sides of the motif pair when *MDL* is removed. Since *MDL* compression is obtained from the combination of successive *equal SAX* values, its removal leads to flatter plots, as indicated by the variance in *SAX* value range between *Sides A* and *B* in the middle plot of Figure 9b.

Overall the behaviour of the *SAX* and *raw data* series correspond quite well, as shown in the middle and bottom plots of Figure 9b–d. Additionally, even though no normalisation has been applied to the raw series data, a close match between each side of the motif pair is still observed, particularly in Figure 9c,d.

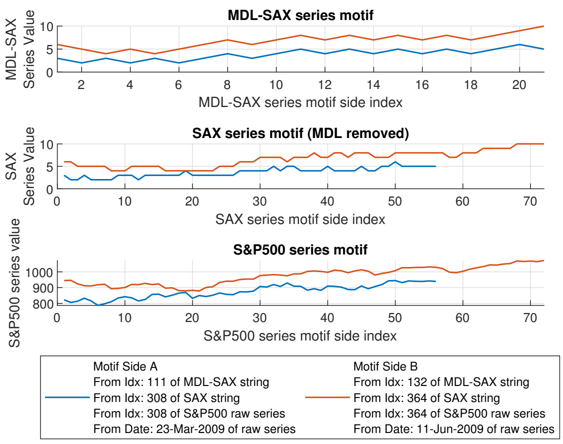
This identification of similar behaviour (characterised by *motif shape*) within financial time series may be used to identify potential investment opportunities through identification of pattern repeats now flexibly interpreted with respect to match length.



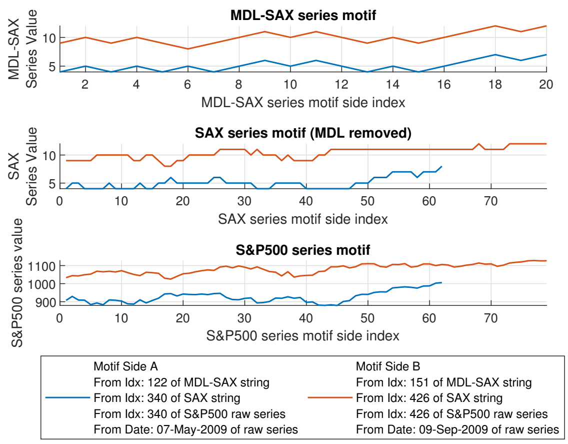
(**a**) *S&P500 SAX MDL-SAX-MP* from January 2008 to January 2010. Motif pairs highlighted at points **X**, **Y** and **Z**.



(**b**) *Motif* pair identified by low *MP* values at point **X**, index 20 and 48 from *MDL-SAX-MP*.



(**c**) *Motif* pair identified by low *MP* values at point **Y**, index 111 and 132 from *MDL-SAX-MP*.



(**d**) *Motif* pair identified by low *MP* values at point **Z**, index 122 and 151 from *MDL-SAX-MP*.

Figure 9. Matrix profile of *SAX* representation of the *S&P500* from January 2008 to January 2010, after application of *MDL*. Motif pairs identified by *MP* minima indexes highlighted at points **X**, **Y** and **Z** in (**a**) are illustrated in (**b**–**d**) with *variable lengths* per *motif side* illustrated in both the *SAX* (middle) and *raw data* (bottom) sub-sequences.

3.1.2. Alternative Motif Identification Algorithms Comparison

To aid comparison to other motif identification methods, Figure 10 was constructed with *SLIM* contrasted to the *MP*, *MrMotif* and *SKIMP*. The choice of *MP* for the initial analysis (Figure 10a,b) was based on its current perception in the literature as state-of-the-art. The equivalent index from previously identified low *MP* values of the *MDL-SAX-MP* (i.e., start indexes of *Side A* in Figure 9c,d) was translated to an *MP* based upon *S&P500* series data and matching *raw series sub-sequences* identified.

Overall, the *behaviour* (i.e., *motif shape*) is in good agreement between the original *fixed-length MP* (top) and *variable-length SLIM* motif plots (bottom) in Figure 10a,b. Along with an increase in sub-sequence length, even for the shorter *side-length SLIM* motif. We note that the index of the *raw series MP* plot used may not necessarily be a low *MP* value (in the context of the overall *raw series MP*). As in this case, we use a fixed index as a starting point and corresponding *MPI* to illustrate a match, rather than consulting the entire *raw series MP* for a global minimum (indicating the point of best match obtained).

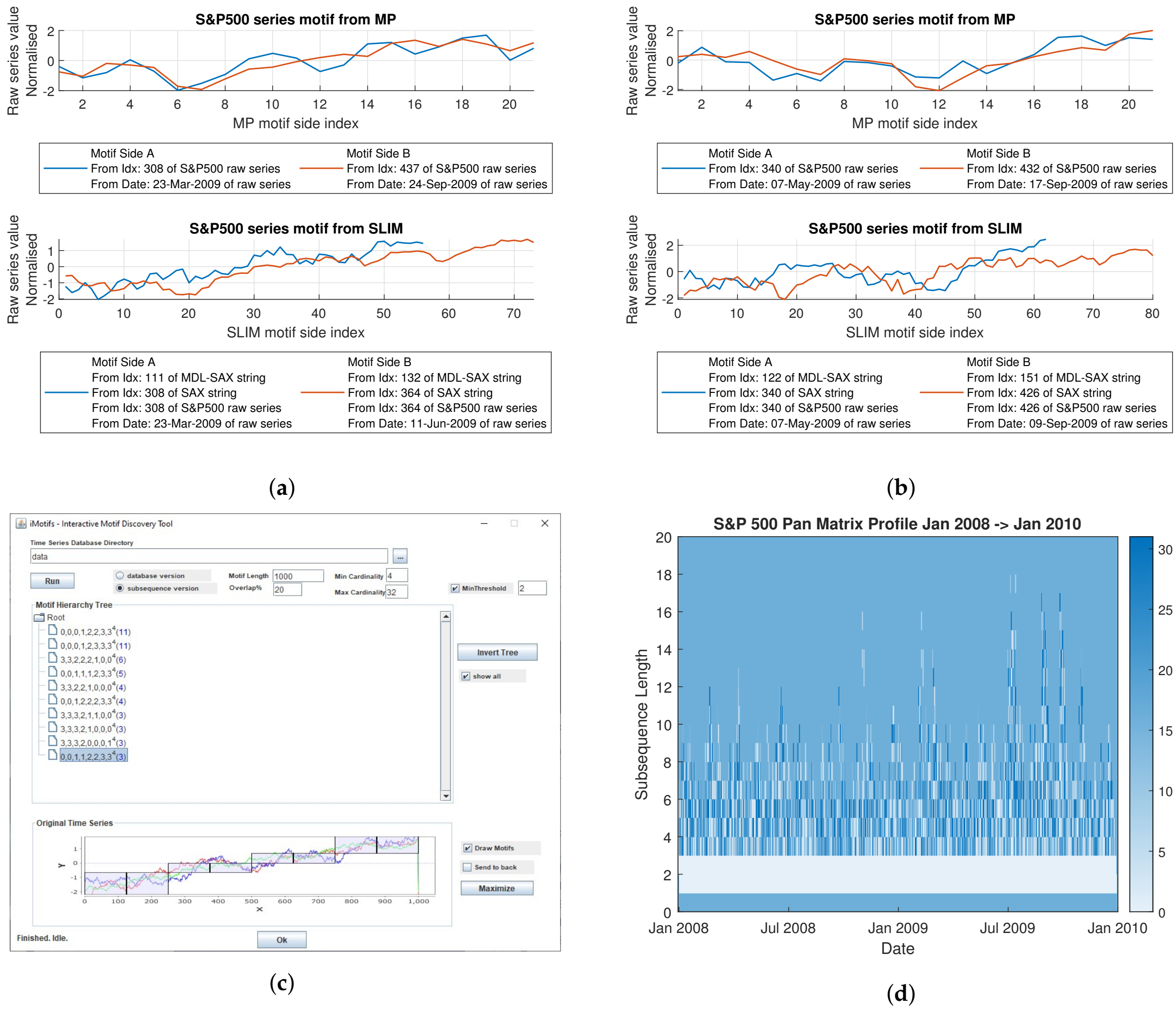


Figure 10. Comparison of motifs obtained from *MP* and *SLIM*. (**a**) *SLIM* vs. *MP* from Figure 9c. (**b**) *SLIM* vs. *MP* from Figure 9d. (**c**) *MrMotif* [16,17] front end. (**d**) *Pan Matrix Profile*(*PMP*) heatmap created by the *SKIMP* [26] algorithm.

Figure 10c contains the results for the *MrMotif* [16,17] algorithm (based upon author-provided sample data), illustrating a set of *fixed-length* motifs obtained at increasing *SAX* resolutions. While in Figure 10d, a heatmap of a *Pan Matrix Profile* (*PMP*) created by the *SKIMP* [26] algorithm is shown. This indicates the location and lengths identified within the same *S&P500* data-set used previously from January 2008 to January 2010. Of the algorithms examined, only *SLIM* returns a direct comparison of sub-sequences of differing lengths, without the need for an extra processing step.

3.1.3. Localised Volatility Analysis

The additional details recorded when *MDL* is applied to a *SAX* string (Table 1) also permit volatility analysis at the local level, with *segments* (or *sub-sequences*) of an overall series identifiable in terms of *volatility* match, for example (i.e., *max*, or *min*, as shown here). In finance, volatility is an important measure that represents the dispersion of returns for a given security or index and essentially measures risk [41].

Key segments are identified by the creation of a sliding window (of user-specified segment size) parsing through the previously created *MDL-SAX* combination table (Table 1). Values such as the sum of the absolute values of *SAX* value differences column (*SAXValDiff*), symbol join number (*SymJoinNum*) column and *amplitude* (calculated as the difference between the minimum and maximum *SAX* values within the sliding window) are recorded in a volatility summary table along with the *MDL-SAX* series index of the current location of the sliding window (see Table 2).

Table 2. Sample extract of volatility table details from *MDL-SAX* table.

MDLSAXSeriesIdx	SAXValDiffTotal	SymJoinNumTotal	SAXValAmplitude
1	25	12	15
2	26	11	14
3	26	10	17
...	...	...	...

Table 2 now contains summary information on the original *MDL-SAX* table that can be used to identify volatility areas of interest in the original series. A large difference between *SAX* (or *MDL-SAX*) values (i.e., *SAXValDiff*) corresponds to a large shift in raw series value. Similarly, a high value of *SymJoinNum* (i.e., consecutive, unchanged *SAX* values) indicate series stability.

Overall levels of volatility in a segment can be ordered by *SAXValAmplitude* (maximum corresponding to highest volatility and vice versa), *SymJoinNum* (reflecting stability within the sliding window) or by *SAXValDiff* (indicating the number of changes within the sliding window).

Thus, the identification of segments is based upon a combination of values rather than a single *standard deviation* value, which is commonly used [41]. Additionally, the particular focus can be emphasised by the choice of primary column ordering. For example, prioritising *SAXValAmplitude* over *SAXValDiff* emphasises the significance of a single large event within the sliding window, as opposed to smaller but more numerous changes captured by *SAXValDiff*.

Figure 11 shows the *MDL-SAX* representation of the *S&P500* between January 2008 and January 2010 with identified high (*red*) and low (*green*) volatility segments. Additionally, identified individual raw series segments and locations within the original series are provided for clarity. The standard deviation was also examined, returning values of 13.02 and 93.97 for the isolated low and high segments, respectively (confirming very different volatility levels).

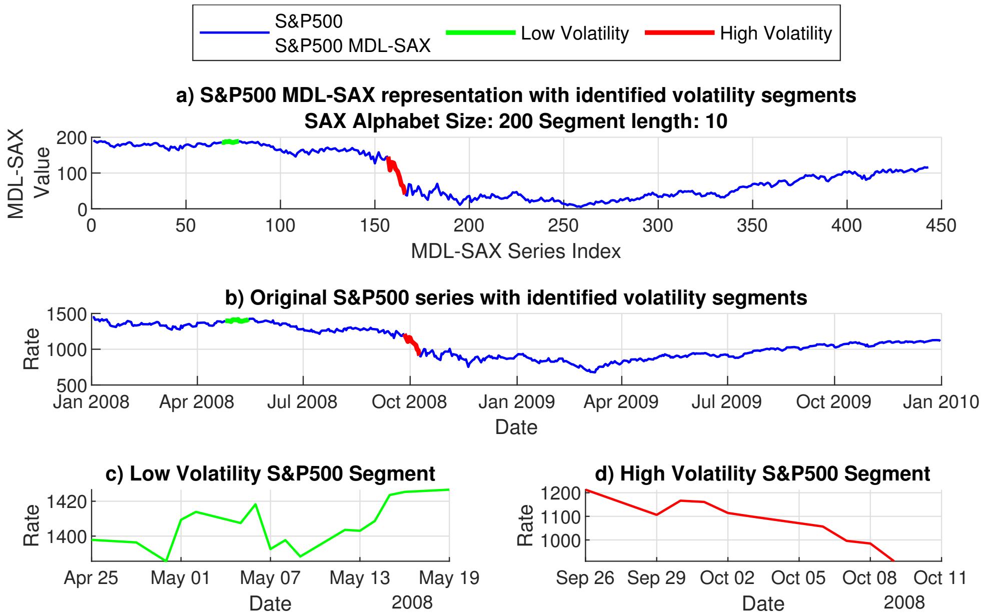


Figure 11. *S&P500 MDL-SAX* representation (**a**) with identified high/low volatility segments. For clarity, the original *S&P500* series from January 2008 to January 2010 is shown in (**b**), with isolated low (**c**) and high (**d**) volatility segments. *SAX* Alphabet size = 200, Num of SAX segments = series length, *MDL-SAX* segment length = 15.

Limitations to our *SLIM* approach mainly centre on the normalisation step applied during the application of *SAX* to the raw input series. In the initial step within the *SAX* algorithm, a normalisation (of *Gaussian* form) is applied to the input series, such that the range of original series values represented by each *SAX* symbol is larger for extreme raw series values than mid-range. This translates to increased sensitivity of *SAX* values to the raw series in the mid-range of the data (and corresponding higher *SAXValDiff* sum when applying a sliding window). This may result in the identification of more segments at the outer edges of the *MDL-SAX* series, particularly when looking for low volatility.

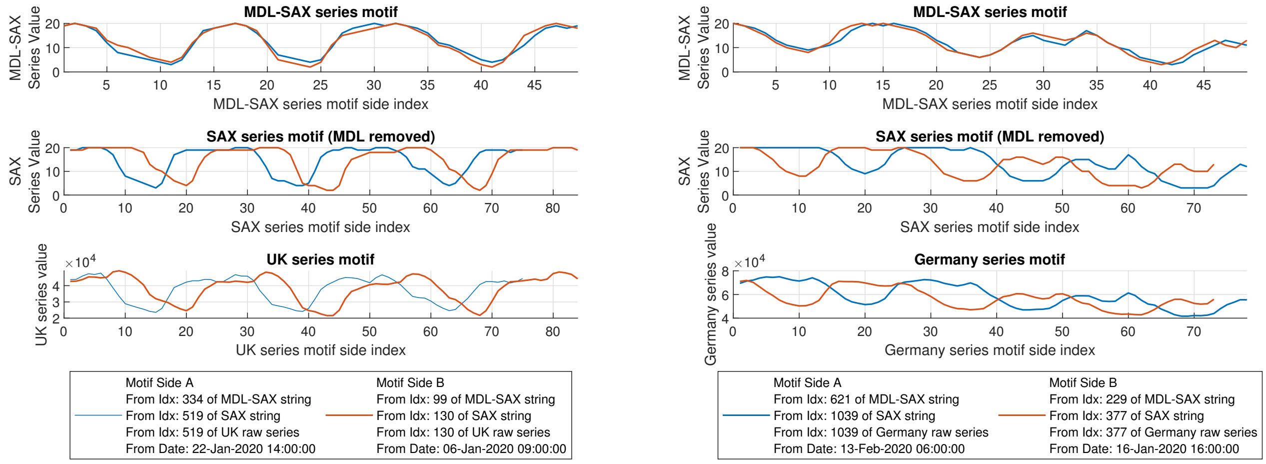
This effect can be mitigated somewhat through the choice of a large alphabet size, resulting in a reduced interval range when assigning *SAX* symbol values. However, even with small alphabets, the *SLIM* technique provides a good starting point for further analysis.

3.2. Energy Sector

3.2.1. Side-Length-Independent Motif Discovery

Application of *SLIM* to an energy sector example is illustrated for a set of 2020 *Open Power System*, hourly power consumption series for *Germany* and the *UK* [39], Figure 12. The same *MDL-SAX* process as outlined in Section 2.2.4 was followed. The sub-sequence length chosen for the *MP* algorithm was 48 (equivalent hours), permitting a search for similarities of 2 days in length within the compressed *MDL-SAX* string. An alphabet size of 20 was chosen in order to obtain a desired level of compression in the more volatile hourly data (see Figure 13 for relevant global *compression rate* % vs. *SAX alphabet size* plots).

Figure 12a shows motif segments identified from a sample low *MP* distance value occurring on 22 January 2020 (index *334* of the *MDL-SAX-MP* string) along with the matching segment identified (from the *MPI*), giving an index of 99. Overall a close correlation in behaviour between both motif sides is observed. When returning to non-compressed *SAX* and *raw* series from *MDL-SAX*, we observe that side length differs by 10 h, indicating similar power consumption behaviour occurring over a shorter timeframe.



(**a**) *UK* 2020 power consumption *motif*. Obtained from index 334 and 99 of a *UK MDL-SAX-MP* series.

(**b**) *Germany* 2020 power consumption *motif*. Obtained from index 621 and 229 of a *German MDL-SAX-MP* series.

Figure 12. Sample *motifs* identified by *SLIM* in 2020 *UK* and *Germany* hourly power consumption data.

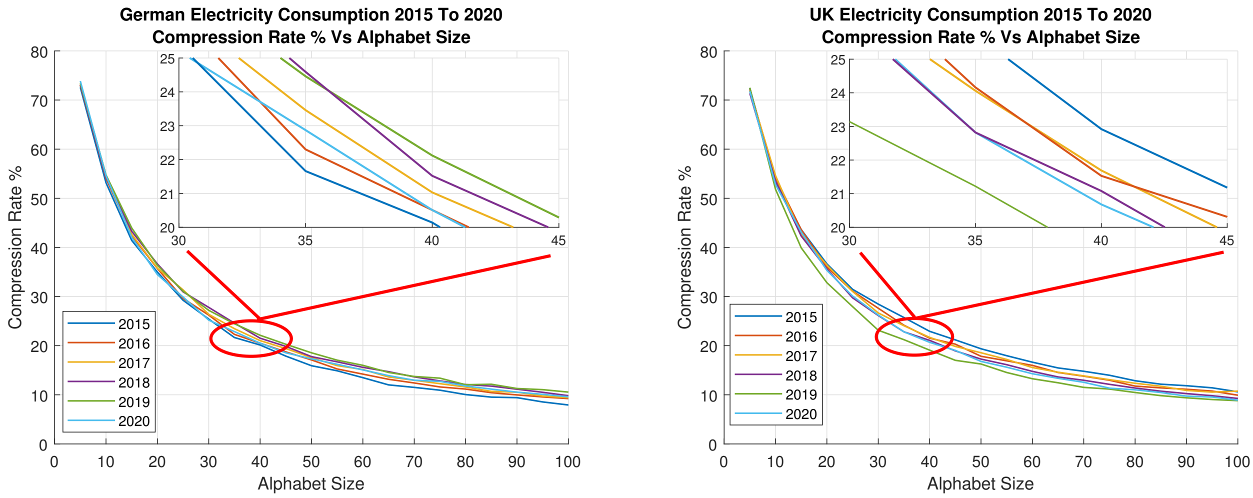
This is a lower level of compression than previously observed and is reflective of the challenges encountered when dealing with increased volatility in the *hourly* energy data when compared to the *daily* financial data previously examined. Although the *SAX* alphabet size can be reduced to counter increased volatility in the raw data, there are limits to the efficacy of this tuning (in terms of distinguishing differences between motif *Sides A* and *B* lengths).

In Figure 12b, the results of a different approach are illustrated. Rather than using a low *MP* distance value obtained from an *MP* plot of an *MDL-SAX* string as a starting point, the table obtained during the creation of the *MDL-SAX* string was consulted (equivalent to Table 1). A high *SymJoinNum* value was chosen as a starting point, providing an initial index of *621* in the *MDL-SAX* string (corresponding to 13 February 2020 in the raw series), with the alternative side of the match obtained from the *MPI* value at this point. A less accurate match may be obtained (an intuitive result as the *MP* distance value was greater in this case than that used in Figure 12a), but a length difference between motif sides is more likely with compression being removed. The approach is particularly useful where this is of importance, relative to the match accuracy from *MDL-SAX-MP*.

For highly volatile data, as here, less compression occurs in the creation of the *MDL-SAX* string, even for a small alphabet size in the initial *SAX* representation. Identifying different motif side-lengths is consequently more difficult when applying MDL. Figure 12 illustrates motifs of hourly power consumption with differing side lengths, representing potential patterns in user behaviour while allowing for flexibility in the match. Figure 12a shows overall matching behaviour while Figure 12b indicates a prolonged higher consumption level initially for *Side A* of the match, causing the remainder of the identified segment to be out of phase with that of *Side B* (a result of the initial choice of start location of *Side A* with high *SymJoinNum* value).

3.2.2. Globalised Volatility Analysis

Compression rate % vs. *alphabet size* plots also permit a visual analysis and comparison of relative volatility levels of series at a global level, an important feature as the availability of large data-sets increases. To demonstrate, Figure 13 shows relative volatility levels of *German* and *UK hourly* power consumption over a time span of 5 years. Here a lower overall compression rate is observed, indicating a higher level of volatility than previously observed for *daily S&P500* data (Figure 7b), where maximum compression is approx 90%, as opposed to 75% here.



(**a**) *Germany*. (**b**) *UK*.

Figure 13. Hourly power consumption compression rates 2015 to 2020.

Furthermore, also of note in Figure 13 is the consistency of volatility levels observed, even for 2020, where differences compared to previous years might be expected due to the *Covid-19* pandemic [42] triggering lock-downs in many countries and, by extension, alternative consumption patterns. Although a larger spread occurs for the *UK* in Figure 13b, the 2020 values still fall within the typical series distribution profile. In summary, despite different pandemic policies, neither Germany nor UK power consumption volatility appears to show a marked change from previous years.

4. Conclusions

In this work we have explored the novel use of a combination of several established data mining techniques for motif detection in time series. Specifically, these included *Symbolic Aggregate Approximation* (***SAX***), *Minimum Description Length* (***MDL***) and *Matrix Profile* (***MP***). Applications for finance and energy series are discussed.

The compression resulting from an application of *MDL* to *SAX* string representations of time series effectively removes periods of stability while retaining volatility. The compression rate achieved is a combination of the *alphabet* and *segment size* chosen during the creation of the initial *SAX* string and the volatility level of the series in question.

Construction of *MP* plots based on *MDL-SAX* representations permits the identification of motif pairs with an independent length per side. This is a highly useful feature for financial, as well as other series analysis, allowing similar behaviour (represented as *motif shape*) occurring over differing timescales to be identified. Example applications in the *energy* and *financial* domains are used in illustration with features, such as input tuning and motif side-length discussed in more detail.

Compression rate % vs. *alphabet size* plots provide a picture of the amount of compression obtained and act as an indicator of the overall volatility level within a given series or set of series. This technique can also be used for the identification and isolation of localised periods of high volatility or stability through the examination of additional detail on *MDL-SAX* representation.

Although *SAX* normalisation leads to some bias in terms of over-identification of significant matches at extremes of the data range, inputs may be tuned to optimise individual data-set analysis. Overall, the inherent algorithm properties are both flexible and highly scalable, with *MDL-SAX* independent of *SAX* and *MP* type, so that further potential for series analysis is considerable.

Future improvements include automation of low *MP* value selection and corresponding motif display. Additionally, given the dependence on the amount of compression within an individual segment of the *SAX* series representation, exact motif side-length values can not currently be determined in advance, and this might usefully be a target for more detailed quantification.

Further work is also needed to assess the impact of an initial data reduction when converting the raw series to a *SAX* string in order to facilitate analysis of the ever-increasing volume of data generated for finance and other applications.

Author Contributions: Conceptualisation, methodology, software, formal analysis, investigation, visualisation and writing—original draft, E.C.; writing—review and editing and validation, M.C. and H.J.R. All authors have read and agreed to the published version of the manuscript.

Funding: This research was in part supported by the Science Foundation Ireland under Grant Agreement No. 13/*RC*/2106_*P*2 at the ADAPT SFI Research Centre at DCU. ADAPT, the SFI Research Centre for AI-Driven Digital Content Technology, is funded by the Science Foundation Ireland through the SFI Research Centres Programme.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Financial Time-Series data available at: https://finance.yahoo.com/lookup (accessed on 6 December 2021). Open Power System Data available at: https://data.open-power-system-data.org//time_series/latest/ (accessed on 6 December 2021). *Matlab Matrix Profile* code available at: https://www.cs.ucr.edu/~eamonn/MatrixProfile.html (accessed on 6 December 2021). *Matlab SAX* code available at: https://www.cs.ucr.edu/~eamonn/SAX.htm (accessed on 6 December 2021). *TSLearn SAX* Python code available at: https://tslearn.readthedocs.io/en/stable/index.html (accessed on 6 December 2021). *SLIM* experiment code available at: https://github.com/eoincart/SLIM (accessed on 6 December 2021).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

SLIM	Side-Length-Independent Motif
SES	Simple Exponential Smoothing
ARIMA	Autoregressive Integrated Moving Average
SVR	Support Vector Regression
WIG20	Warsaw stock exchange index
SAX	Symbolic Aggregate Approximation
MDL	Minimum Description Length
MP	Matrix Profile
MPI	Matrix Profile Index
SFA	Symbolic Fourier Approximation
S&P500	Standard and Poor's 500

References

1. Mueen, A.; Keogh, E.; Zhu, Q.; Cash, S.; Westover, B. Exact Discovery of Time Series Motifs. In Proceedings of the SIAM International Conference on Data Mining, Sparks, NV, USA, 30 April–2 May 2009; pp. 35–53, 473–484. [CrossRef]
2. Lin, J.; Keogh, E.; Lonardi, S.; Patel, P. Finding motifs in timeseries. In Proceedings of the Second Workshop on Temporal Data Mining, (KDD 2002), Edmonton, AB, Canada, 23–26 July 2002.
3. Mueen, A. Time series motif discovery: Dimensions and applications. *Wiley Interdiscip. Rev. Data Min. Knowl. Discov.* **2014**, *4*, 152–159. [CrossRef]
4. Investopedia (a): Common Chart Pattern Definitions. Available online: https://www.investopedia.com/articles/technical/112601.asp (accessed on 26 January 2022).
5. Vivas, E.; Allende-Cid, H.; Salas, R.; Vivas, E. A Systematic Review of Statistical and Machine Learning Methods for Electrical Power Forecasting with Reported MAPE Score. *Entropy* **2020**, *22*, 1412. [CrossRef] [PubMed]
6. He, X.J. Crude Oil Prices Forecasting: Time Series vs. SVR Models. *Int. Inf. Manag. Assoc.* **2018**, *27*, 25. Available online: https://scholarworks.lib.csusb.edu/jitim/vol27/iss2/2 (accessed on 26 January 2022).
7. Domino, K. The use of the Hurst exponent to investigate the global maximum of the Warsaw Stock Exchange WIG20 index. *Phys. Stat. Mech. Its Appl.* **2012**, *391*, 156–169. [CrossRef]

8. Xiaoxi, D.; Ruoming, J.; Liang, D.; Lee, V.E.; Thornton, J.H. Migration Motif A Spatial Temporal Pattern Mining Approach for Financial Markets. In Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Paris, France, 28 June–1 July 2009; pp. 1135–1144. [CrossRef]
9. Elangovan, R.; Padmavathi, S. A Review on Time Series Motif Discovery Techniques an Application to ECG Signal Classification: ECG Signal Classification Using Time Series Motif Discovery Techniques. *Int. J. Artif. Intell. Mach. Learn. (IJAIML)* **2019**, *9*, 39–56. [CrossRef]
10. Silva, D.F.; Yeh, C.-C.M.; Zhu, Y.; Batista, G.E.A.P.A.; Keogh, E. Fast Similarity Matrix Profile for Music Analysis and Exploration. *IEEE Trans. Multimed.* **2019**, *21*, 29–38. [CrossRef]
11. Gao, Y.; Lin, J. Exploring variable-length time series motifs in one hundred million length scale. *Data Min. Knowl. Discov.* **2018**, *32*, 1200–1228. [CrossRef]
12. Torkamani, S.; Lohweg, V. Survey on time series motif discovery. *WIREs Data Min. Knowl. Discov.* **2017**, *7*, e1199. [CrossRef]
13. Fu, T.K. A review on time series data mining. *Eng. Appl. Artif. Intell.* **2011**, *32*, 164–181. [CrossRef]
14. Chiu, B.; Keogh, E.; Lonardi, S. Probabilistic discovery of time series motifs. In Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Washington, DC, USA, 24–27 August 2013; pp. 493–498. [CrossRef]
15. Lin, J.; Keogh, E.; Wei, L.; Lonardi, S. Experiencing SAX: A novel symbolic representation of time series. *Data Min. Knowl. Discov.* **2007**, *15*, 107–144. [CrossRef]
16. Castro, N.; Azevedo, P.J. Multiresolution Motif Discovery in Time Series. In Proceedings of the 10th SIAM International Conference on Data Mining (SDM2010), Columbus, ON, USA, 29 April–1 May 2021; pp. 665–676. [CrossRef]
17. Castro, N.; Azevedo, P.J. Time Series Motifs Statistical Significance. In Proceedings of the 11th SIAM International Conference on Data Mining (SDM2011), Mesa, AZ, USA, 28–30 April 2011; pp. 687–698. [CrossRef]
18. Li, Y.; Hou, U.; Yiu, M.L.; Gong, Z. Quick-motif: An efficient and scalable framework for exact motif discovery. In Proceedings of the IEEE 31st International Conference on Data Engineering (ICDE 2015), Seoul, Korea, 13–16 April 2015; pp. 579–590. [CrossRef]
19. Yeh, C.M.; Zhu, Y.; Ulanova, L.; Begum, N.; Ding, Y.; Dau, H.; Silva, D.F.; Mueen, A.; Keogh, E. Matrix Profile I: All pairs similarity joins for time series a unifying view that includes motifs discords and shapelets. In Proceedings of the IEEE ICDM, Barcelona, Spain, 1–15 December 2016; pp. 1317–1322. [CrossRef]
20. The University of California Riverside (*UCR*) *Matrix Profile*. Available online: https://www.cs.ucr.edu/~eamonn/MatrixProfile.html (accessed on 26 January 2022).
21. Yuan, L.; Lin, J. Approximate variable-length time series motif discovery using grammar inference. In Proceedings of the Tenth International Workshop on Multimedia Data Mining, Washington, DC, USA, 25 July 2010; pp. 1–9. [CrossRef]
22. Nunthanid, P.; Niennattrakul, V.; Ratanamahatana, C.A. Discovery of variable length time series motif. In Proceedings of the 8th Electrical Engineering/ Electronics, Computer, Telecommunications and Information Technology (ECTI-CON 2011), Khon Kaen, Thailand, 17–19 May 2011; pp. 472–475. [CrossRef]
23. Nunthanid, P.; Niennattrakul, V.; Ratanamahatana, C.A. Parameter-free motif discovery for time series data. In Proceedings of the 9th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON 2012), Hua Hin, Thailand, 16–18 May 2012; pp. 1–4. [CrossRef]
24. Lam, H.; Calders, T.; Pham, N. Online Discovery of Top-k Similar Motifs in Time Series Data Read. In Proceedings of the 2011 SIAM International Conference on Data Mining (SDM11), Mesa, AZ, USA, 28–30 April 2011; pp. 1004–1015, ISBN 978-0-898719-92-5.
25. Linardi, M.; Zhu, Y.; Palpanas, T.; Keogh, E. Matrix Profile X: VALMOD–Scalable Discovery of Variable-Length Motifs in Data Series. In Proceedings of the 2018 International Conference on Management of Data (SIGMOD18), Houston, TX, USA, 10–15 June 2018; pp. 1053–1066. [CrossRef]
26. Madrid, F.; Imani, S.; Mercer, R.; Zimmerman, Z.; Shakibay, N.; Mueen, A.; Keogh, E. Matrix Profile XX: Finding and Visualizing Time Series Motifs of All Lengths using the Matrix Profile. In Proceedings of the IEEE International Conference on Big Knowledge (ICBK), Beijing, China, 10–11 November 2019; Volume 1, pp. 175–182. [CrossRef]
27. Somarajan, S.; Shankar, M.; Sharma, T.; Jeyanthi, R. Modelling and Analysis of Volatility in Time Series Data. In *Soft Computing and Signal Processing (ICSCSP 2018). Part of the Advances in Intelligent Systems and Computing Book Series (AISC, Volume 898)*; Wang, J., Reddy, G., Prasad, V., Reddy, V., Eds.; Springer: Singapore, 2019; Volume 898, pp. 609–618. [CrossRef]
28. The University of California Riverside (*UCR*) *SAX*. Available online: https://www.cs.ucr.edu/~eamonn/SAX.htm (accessed on 26 January 2022).
29. Ruan, G.; Hanson, P.C.; Dugan, H.A.; Plale, B. Mining lake time series using symbolic representation. *Ecol. Inform.* **2017**, *39*, 10–22. [CrossRef]
30. Shieh, J.; Keogh, E. ISAX: Indexing and mining terabyte sized time series. In Proceedings of the ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Las Vegas, NV, USA, 24–27 August 2008; Volume 14, pp. 623–631. [CrossRef]
31. Schäfer, P.; Högqvist, M. SFA: A Symbolic Fourier Approximation and Index for Similarity Search in High Dimensional Datasets. In Proceedings of the 15th International Conference on Extending Database Technology (EDBT), Berlin, Germany, 26–30 March 2012; Volume 1, pp. 516–527. [CrossRef]

32. Amornbunchornvej, C.; Navaporn, S.; Anon, P.; Suttipong, T. Identifying Linear Models in Multi-Resolution Population Data Using Minimum Description Length Principle to Predict Household Income. *ACM Trans. Knowl. Discov. Data* **2021**, *15*, 1–30. [CrossRef]
33. Grünwald, P.D. *The Minimum Description Length Principle*; MIT Press: Massachusetts, MA, USA, 2007; ISBN 9780262072816. [CrossRef]
34. Meegan, A.; Corbet, S.; Larkin, C. Financial market spillovers during the quantitative easing programmes of the global financial crisis (2007–2009) and the European debt crisis. *J. Int. Financ. Mark. Institutions Money* **2018**, *56*, 128–148. [CrossRef]
35. Bracke, T.; Michael, F. The macro-financial factors behind the crisis: Global liquidity glut or global savings glut? *N. Am. J. Econ. Financ.* **2012**, *23*, 185–202. [CrossRef]
36. Cartwright, E.; Crane, M.; Ruskin, H.J. Financial Time Series: Motif Discovery and Analysis Using VALMOD. In Proceedings of the International Conference on Computational Science, Faro, Portugal, 12–14 June 2019; pp. 771–778. [CrossRef]
37. Cartwright, E.; Crane, M.; Ruskin, H.J. Financial Time Series: Market Analysis Techniques Based on Matrix Profiles. *Eng. Proc.* **2021**, *5*, 45. [CrossRef]
38. Ferreira, P.G.; Azevedo, P.J. Evaluating deterministic motif significance measures in protein databases. *Algorithms Mol. Biol.* **2007**, *2*, 16. [CrossRef] [PubMed]
39. Open Power System Data. 2020. Data Package Time Series. Version 2020-10-06: Primary Data from Various Sources, for a Complete List. Available online: https://data.open-power-system-data.org//time_series/latest/ (accessed on 26 January 2022).
40. Bloomberg S&P500 Index, Including Summary. Available online: https://www.bloomberg.com/quote/SPX:IND (accessed on 26 January 2022).
41. Investopedia (b): Volatility Summary. Available online: https://www.investopedia.com/terms/v/volatility.asp (accessed on 26 January 2022).
42. World Health Organisation Covid-19 Pandemic Timeline. Available online: https://www.who.int/news/item/29-06-2020-covidtimeline (accessed on 26 January 2022).