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Communication

Quantum-like Data Modeling in Applied Sciences: Review

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Abstract: This work presents a brief review on the modern approaches to data modeling by the methods developed in the quantum physics during the last one hundred years. Quantum computers and computations have already been widely investigated theoretically and attempted in some practical implementations, but methods of quantum data modeling are not yet sufficiently established. A vast range of concepts and methods of quantum mechanics have been tried in many fields of information and behavior sciences, including communications and artificial intelligence, cognition and decision making, sociology and psychology, biology and economics, financial and political studies. The application of quantum methods in areas other than physics is called the quantum-like paradigm, meaning that such approaches may not be related to the physical processes but rather correspond to data modeling by the methods designed for operating in conditions of uncertainty. This review aims to attract attention to the possibilities of these methods of data modeling that can enrich theoretical consideration and be useful for practical purposes in various sciences and applications.

Keywords: quantum-like paradigm; probability; mixed states; machine learning; cognitive science; decision making

MSC: 60-08; 60A05; 91-10; 91B80; 91C20; 91E10

1. Introduction

This review describes the so-called quantum-like paradigm approaches recently developed and implemented in various modern research studies performed with data modeling by the methods adopted from quantum physics. A wide spectrum of ideas and techniques of physics has been tried in various fields of information and behavior sciences, covering communications and artificial intelligence, cognition and decision making, sociology and psychology, biology and health studies, economics and political sciences, and more. These efforts have already created such new fields of science as sociophysics, sociodynamics, econophysics, and mediaphysics [1–5]. Many terms and methods well-known in statistics and applied mathematics actually came from other fields of knowledge, for instance: the Gibbs sampler used in the Markov Chain Monte Carlo technique for Bayesian estimations—from statistical physics; simulated annealing for optimization—from technological processes; or genetic and swarm algorithms—from biology.

Nowadays, new studies incorporating ideas from one area to others are continuing. The quantum computers and quantum computation algorithms have already been investigated theoretically and implemented practically in different aims and fields. The ideas of quantum information science have been developed for communications and artificial intelligence [6–10] in robotics and engineering [11–13]. The U.S. government together with numerous private big tech companies, such as IBM, Google, Intel, and many others, plan to invest about one billion USD from 2020 over the next five years for establishing multiple quantum information and artificial intelligence research institutes in universities and national laboratories [14]. Similar expenditures and research efforts are going on in all technically advanced countries around the world.

With the contemporary progress in artificial intelligence and quantum information science, the ideas of quantum-like description have been tried for enhancing various

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methods of statistical data analysis, as well as for modeling in different technical, natural, and human sciences, including psychology, decision making, human judgements, and social relations. For example, lasers are studied both in technical and social meaning [15–18]. A wide area of applications of quantum and quantum-like descriptions and tools can be found in biological sciences from general and evolutionary problems [19,20] to computational biology [21–24], system biology [25–28], molecular biology [29,30], and health studies [31]. In research on human cognition and behavior, the quantum approaches and methods correspond to explanations of complex objects and processes in conditions of a deep uncertainty produced by the unclear nature of the phenomena and lack of knowledge on them [32–36].

The current review describes several main directions of the quantum-like paradigm developments in applied data modeling. The considered areas, topics, and techniques are so vast that just several recent sources are taken as illustrations in each case. Section 2 describes the quantum-like methods related to statistical tools, and Section 3 considers the quantum-like approaches to cognitive and decision-making problems. Section 4 describes several examples of applications, and Section 5 summarizes.

2. Quantum Ideas and Applications in Statistical Tools

Probability in quantum description corresponds to the interpretation by Max Born, one of the founders of quantum mechanics: the squared module of a complex wave function describes a state or a superposition of states, and it can be used to define the probability of belonging to each state. Quantum-like observables and states can be related to responses and beliefs of respondents, and quantum formalism can be used as a tool to calculate probabilities. Two pure states can be mixed in the superposition of the quantum bit or qubit in terms of the quantum information theory, and that can be extended to n-qubit mixed states. Quantum logic can differ from the classical logic and Boolean laws because it operates with probabilities of pure states and mixed or entangled states.

The quantum probability theory as a generalization of Bayesian probability theory is based on a set of von Neumann axioms relaxing some of the classic Kolmogorov axioms. For example, due to the communicative property of two events A and B in the classical theory, the joint probability can be expressed via the conditional probabilities $p(A \cap B) = p(A)p(B \mid A) = p(B)p(A \mid B)$ that lead to the Bayesian conditional probability $p(B \mid A) = p(A \cap B)/p(A)$. However, it does not hold in a quantum description with non-communicative properties, $p(A)p(B \mid A) \neq p(B)p(A \mid B)$, where the conditional probability depends on the events' order [37]. The laws of total probability in Kolmogorov's set theory and the Bayesian update of respondents' preferences can be violated when the choice probability is defined with additional items of interference between the states where the interference magnitude parameters are estimated from the data [38–40].

Quantum machine learning incorporates and implements quantum algorithms and software [41–43] that correspond to the hardware of quantum computers that would create much faster and more powerful computing, although the challenges still remain substantial. Among the open problems of quantum machine learning there is the amount of input data required to run the computation, with the complexity of the data encoding which can overshadow the gained speed up; the algorithms for the data processing, which should produce the needed result with the highest probability; the output when the problems with limited numbers of values have more feasible solutions; and the destruction problem, when a model can be queried only once because of the measurement collapse. Successful applications of quantum-inspired machine learning algorithms have been found in chemistry and material sciences, cryptography and optimization, in data clustering, classification, and pattern recognition by the nearest centroid, and other classifier techniques [43–48].

The problems of optimizing artificial neural and Bayesian networks are closely related to the pattern recognition algorithms operating with discrete or quantified variables. Modeling the processes of people reasoning about causes and effects can also be performed by quantum generalization of Bayesian causal networks where classical probability is substituted by probability assessed via the superposition of quantum amplitudes, and these possibilities have been tried in many approaches [49–54].

The quantum language of probabilities defined via the aggregates of pure states' amplitudes closely corresponds to the game theory with mix strategy used to assign probability to each pure strategy. A quantum approach to the mix strategies helps to consider the antagonistic games as well as cooperative and bargaining games, for instance, those proposed by the Nobel Prize 1994 winner in economics J. Nash, whose works on the equilibrium theory are applicable to mechanisms of economics, auctions, and markets [55,56]. Another well-known tool of the cooperative game theory is the Shapley value, which was suggested by the Nobel Prize winner in economics 2012, L. Shapley. It has been widely used in multiple real projects, including regression modeling, key driver analysis, total unreplicated reach and frequency (TURF) estimations, machine learning, and many other applications, particularly in marketing research, described in dozens of real examples in [57–59]. Finding the Shapley values could present a difficult computational combinatorial problem; however, a recent work [60] proposed a novel quantum algorithm reducing this problem to estimation via averaging a binomial distribution which can be performed within a polynomial time.

Quantum stochastic calculus and differential equations have been developed for the non-commuting variables, and various stochastic processes have been studied in different applications [39,61,62].

3. Quantum Description of Cognition and Decision Making

Cognitive processes in judgement and decision making can be adequately modeled by unifying quantum information and probability approaches. Interference of a person's states of mind can be described as interference of waves in the optics, and the so-called quantum superposition means that the respondent has no definite opinion until they are asked about it. Human perception depends on the experimental settings and could demonstrate violations of classical probability laws of additivity. Many works on quantum cognition and contextuality in psychology and economics can be found in the two special issues [63,64].

A non-classical probability can oscillate in time and by other characteristics corresponding to the spontaneous decisions of respondents based on their previous sets of views and attitudes to the issues. Some recent works showed that the human mind operates with concepts not by rules of classical logic, even when those are simple conjunctions or disjunctions, but rather by the context of a situation. In cognitive modeling the superposition of concepts reveals a state of mind with interference and entanglement in concept formation [65–68]. In linguistic studies, quantum-like approaches have been successfully tried as well; particularly, they can be employed as search tools on the internet [69–73].

Preferences in human choice can be described more adequately by a decision model based on a quantum approach with solutions applicable in cognitive psychology and decision making, economics and finance, and social science and politics [74–79]. Quantum descriptions of amplitudes needed for probability estimations are given in [80,81]. Consideration of many other problems in quantum-like approaches, including finance, culture, metaphysics, philosophy, and moral choice can be seen in [82–86].

Within sixteen papers collected in the recent handbook on quantum models in social sciences, we find not just works on mathematical formalism, but also quantum description and applications to financial and economic problems, game theory, decision making, cognition modeling, adaptive dynamics, neural oscillators, strategic choices, voters' preferences, quantum prospect theory, human causal reasoning, and other problems [87,88].

4. Examples of Application

Let us consider several examples of employing some ideas from physics in applied marketing and advertising research where these methods have been successfully tried in real projects. One approach is related to the so-called supercritical pitchfork bifurcation, which describes a tri-critical point of phase transitions in physics where it corresponds to pressure–temperature diagrams of gas, liquid, and solid states. Besides applications for transition modeling in social dynamics, in biology for describing ant colonies' complicated behavior, in fuzzy decision making, and in the design of autonomous robotic problems, this approach has also been tried for implicit regression modeling of data split at some point continuing into several dependencies, where the market mix modeling for advertising in the car industry was studied [89].

Human decisions often demonstrate violations of probability theory and Boolean logic, and the prospect theory of Tversky and Kahneman explains some observations in human behavior better than classical probability theory [90–93]. In research by R. Thaler, the Nobel Prize winner in economics 2017, it is shown that people in their behavior are prone to errors and are not always rational, and that has impact on markets which are otherwise expected to be efficient [94]. D. Kahneman, the Nobel Prize winner in economics 2002, describes two kinds of thinking: the so-called System 1 as a spontaneous, emotional, intuitive, and unconscious way of thinking and its opposite, System 2, as a meticulous, controlled, deliberated thinking [95]. An example of System 1 application to discrete choice modeling for prioritization via the best–worst scaling is given in [96].

Another interesting approach was developed in works [97,98] for the analysis of data elicited on the Likert scales, which are considered as discrete quantum states of multipoles. This approach was found to be useful in cases of the so-called high and low raters which could lead to distortion and inconsistency in data because of differences in cultures and languages, the survey methodology, and the type of market. Multipole presentation of the Likert scales leads to decreased respondent heterogeneity in the clustering that was used for data smoothing via dipole adjustment and noticeably improved the segmentation results [99].

The problem of building the positive or nonnegative loading parameters in the principal component analysis and in the singular value decomposition was considered in the work [100] based on the exponential, logistic, and multinomial-logit parameterization on one hand; and on the symmetrization and antisymmetrization of the eigenvectors for close eigenvalues due to the Perron–Frobenius theory on the other hand. Using such paired "plus" and "minus" symmetrized and antisymmetrized eigenvectors was inspired by the similar technique known in atomic physics for finding complicated states' wave functions based on the simpler main functions [101].

As an example of application of the wave functions, consider a known quantum mechanics problem described in the classic monograph [102] about a wave propagation with the amplitude B and its reflection at the potential barrier with the amplitude $A = -iB exp(-z/2)$, where the imaginary unit is $i = \sqrt{-1}$, and z corresponds to an aggregate of the parameters describing the process. Squaring modules of the amplitudes in this relation yields another relation $|A|^2 = |B|^2 exp(-z)$. The total of the probability $|B|^2$ of penetration and probability $|A|^2$ of reflection equals one, $|A|^2 + |B|^2 = 1$, and from the last two equations, it is easy to obtain the relation $|B|^2(1 + exp(-z)) = 1$. This equation yields the probability of penetration as the squared module of its amplitude, which equals $|B|^2 = 1/(1 + exp(-z))$, or it can be represented as $|B|^2 = exp(z)/(1 + exp(z))$. The probability of the dual event of the reflection is defined by the complementary probability $|A|^2 = exp(-z)/(1 + exp(-z))$, which can also be represented as $|A|^2 = 1/(1 + exp(z))$. This model is analogous to the choice between two alternatives. Thus, the simple consideration produces the logistic model with a binary outcome, widely applied in statistical estimations for finding probability of the event in two options. It may be the shortest way of derivation of the formula for the logit model.

Another example is based on the amplitudes of superposition of discrete states for practical application in marketing research problems. It employs building utility functions of trigonometric and multinomial-logit kinds that permit finding choice probabilities, as it was proposed in the work [103]. This approach yields the probabilities of discrete states themselves plus probabilities of different choices entangled. For example, in modeling choices among brands, finding an amplitude function and squaring its modulus yields the probability of discrete choices of each brand together with additional terms corresponding to the entangled choice of one or another alternative simultaneously, when it is not known beforehand what the actual choice would be. This technique was applied to the data on the top-of-mind awareness and total unaided awareness modeled by six hundred observations with two dozen predictors, as well as to the data elicited from three thousand respondents in the best–worst scaling approach to the prioritization of seventeen products. The quality of the results obtained by the quantum amplitude and probability modeling was estimated by various statistical criteria, and the results were very good.

In the simple case of two brands, for instance, Coke and Pepsi, the probability of choice of one or another brand can be modeled and defined by the regular logistic regression $p_1 = \frac{1}{1 + exp(z)}$ and $p_2 = \frac{exp(z)}{1 + exp(z)}$, respectively for each brand, where z is the aggregate of predictors, $z = a_{1 \times 1} + \ldots + a_n x_n$. In contrast to these classic probabilities, the quantum amplitudes yield the extended logit model with probabilities $p_1 = \frac{1}{1 + e^z + 2e^{0.5z}}$, $p_2 = \frac{e^z}{1 + e^z + 2e^{0.5z}}$, $p_{12} = \frac{2e^{0.5z}}{1 + e^z + 2e^{0.5z}}$, which can be seen as a Venn diagram for pure choices p_1 and p_2 with additional overlapping segment with probability p_{12} of two entangled brands. In marketing research terms, it could mean that there are respondents preferring only Coke or only Pepsi, but also respondents who do not care or have mixed feelings about these brands, and in an actual purchase process, their decision to pick one or another of them would depend on the current physical and emotional conditions, social environment, and other circumstances of influence. As it was proposed in [103], in a survey sampling on the preferences, the third option of "having no preference" should be accounted for the more adequate description of the possible choices.

5. Conclusions

The quantum-like paradigm has nowadays received a wide development and implementation of probability estimation and statistical modeling for complex processes and objects, including human cognition known by the immanent uncertainty and even irrationality. Quantum-like descriptions can have better performance and more sophisticated capabilities than conventional approaches. People's perceptions and decision making could depend on their demographic and cultural features, socio-economic situation, and changing physical and emotional state. Their behavior can demonstrate violations of classical probability laws and fit the framework of quantum description by interference and entanglement.

The work on development of new quantum-like tools is continuing in multiple directions of human interests in various fields. Those include, just for a few examples, investigations on the quantum interpretation of the world phenomena [104], applications of quantum ideology in management sciences [105], quantum computing in the arts and humanities [106], methods of quantum machine learning [107], and the general development of quantum-like techniques in cognitive and socio-economic sciences [108].

Numerous other studies on the quantum and quantum-like models can be found on the internet, and readers can receive more information in the references within the given sources. The described methods can facilitate a deeper understanding of numerous processes and phenomena and enrich their modeling. The innovative quantum and quantum-like approaches can be further extended to many other purposes and tools of statistical analysis, classification, modeling, and prediction in various fields of research, development, and implementation.

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