Supplementary Materials: The Missing Indicator Approach For Accelerated Failure Time Model with Covariates Subject to Limits of Detection

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S1. Additional Simulation Results to Section 4 of the Main Article

S1.1. Missing Due to Upper and Interval LOD when covariates are independent

The simulation results for the accelerated failure time (AFT) model under the independent covariates case and when $X^*_{ij}, j = 1, 2$ are subject to upper and interval limit of detection (LOD), are shown below. Figures S1 and S2 represent the bias associated with the estimates of β_1, and β_2 in the AFT model when X^*_{ij} is subjected to upper and interval LOD, respectively. Tables S1 and S3 summarize the associated absolute value of the average biases (AAB), while tables S2 and S4 summarize the corresponded mean squared error (MSE) for the AFT model coefficients β_1, β_2, and γ_1. The findings from the upper and the interval LOD are generally compatible with the findings from the lower LOD in the main article.

S1.2. Missing Due to Upper and Interval LOD when covariates are correlated

Figures S3 and S4 show the bias associated with the estimates of the regression coefficients β_1 and β_2 in the AFT model when X^*_{ij} are subjected to upper LOD and interval LOD, respectively. Tables S5 and S6 and Tables S7, and S8 summarize the corresponded AAB and the MSE when X^*_{ij} are subjected to upper LOD and interval LOD, respectively. When the covariates are correlated, the missing indicator (MDI) approaches (M7 and M8) become more compatible to the complete-case (CC) analysis (M1) in the bases of the AAB and MSE. Overall, the findings from the AFT model with correlated covariates are generally compatible with the findings from the AFT model when the covariates are independent.

S1.3. Convergence rate for the complete-cases analysis

The convergence rate for the CC analysis under the simulation settings presented in the main document are summarized in Table S9, which shows fewer converged replications when the sample size is small (e.g., $n = 50$) or the missing proportions are high (e.g., $m_1 = 60\%$ or $m_2 = 60\%$).

S1.4. Large-Sample Simulation

Simulation results with $n = 500$ are presented in Tables S10 and S11. The CC analysis (M1) yielded a small AAB demonstrating the consistency of the M1 approach. Under the light missing proportion, M1 was comparable to the MDI approach (M7 and M8) and the embedded approach M12 in bases of bias and MSE. However, as the missing proportions (m_1 and m_2) increases the MSE of M1 became larger than the MSE of other approaches. As we noticed for the small samples and across all scenarios considered, M7 and M8 have among the lowest AAB and MSE.

S2. Extending Missing Indicator Approaches to Different Models

S2.1. Linear regression model

The AFT model reduces to a linear regression model when the response variable is completely observed. We apply the methods outlined in the main document in the linear regression setting. The response variable is generated from the following linear regression model.

$$Y_i = \beta_0 + \beta_1 X^*_{i1} + \beta_2 X^*_{i2} + \gamma_1 Z_i + \epsilon_i, \tag{S1}$$

where ϵ_i followed a standard normal distribution. We considered the case when the covariates are independent and kept the same specifications of the censored covariates, X^*_{i1} and X^*_{i2}, the fully observed covariate, Z_i, and the missing rates defined in the main document.

Figure S5 represents the bias associated with the least-squares estimates of β_1 and β_2 in (S1) when X^*_{ij} is subjected to lower LOD for $j = 1, 2$. Tables S12 and S13 summarize the AAB and the MSE correspond to β_1, β_2, and γ_1. For the upper and interval LOD, summary of the bias and MSE estimates are presented in Figures S6, and S7, Tables S14, S15, S16, and S17. The findings from the linear regression model are generally compatible with the findings from the AFT model.

S2.2. Cox proportional hazard Model

We conducted additional simulations to evaluate the effeteness of the MDI approach when applied to a Cox proportional hazard (PH) model. For the covariates X^*_{i1}, X^*_{i2}, Z_i defined as in the main paper, the Cox PH model imposes the hazard function for T_i

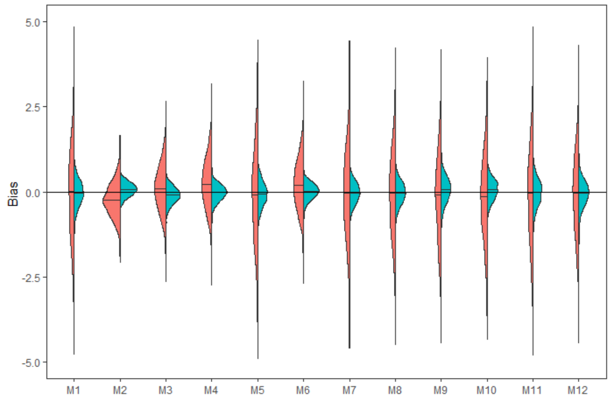
$$\lambda(t|X_{i1}, X_{i2}, Z_i) = \lambda_0(t) \exp(\beta_1 X^*_{i1} + \beta_2 X^*_{i2} + \gamma_1 Z_i), \tag{S2}$$

where $\lambda_0(t)$ is the baseline hazard function for T_i. We set $\lambda_0(t) = 2t$ and generated the censoring time from an independent uniform distribution over $[0, 3]$, yielding a 30% censoring rate on T_i.

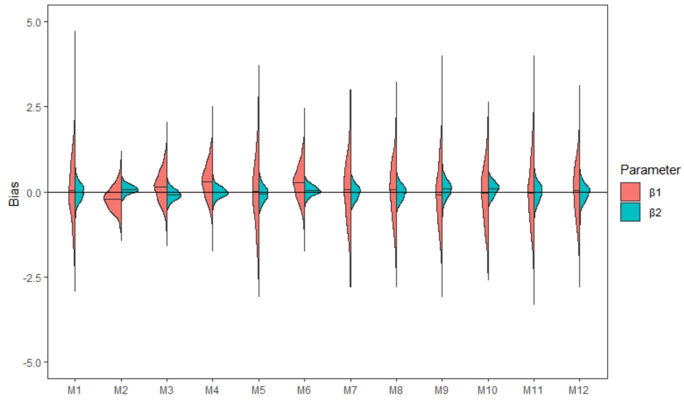
Figure S8 represents the bias associated with the estimates of the regression coefficients β_1, and β_2 in (S2) when X^*_{ij} is subjected to lower LOD for $j = 1, 2$. Tables S18 and S19 summarize the corresponded AAB and the MSE for the Cox PH model coefficients β_1, β_2, and γ_1. The findings from the Cox PH model are generally compatible with the findings from the AFT model. For the upper and interval LOD, a summarising of the bias and MSE estimates are presented in Figures S9 and S10, Tables S20, S21, S22, and S23. The findings from the missing due to the upper and interval LOD in the Cox PH model are generally compatible with the findings from the missing due to the lower LOD.

S3. Simulation Code

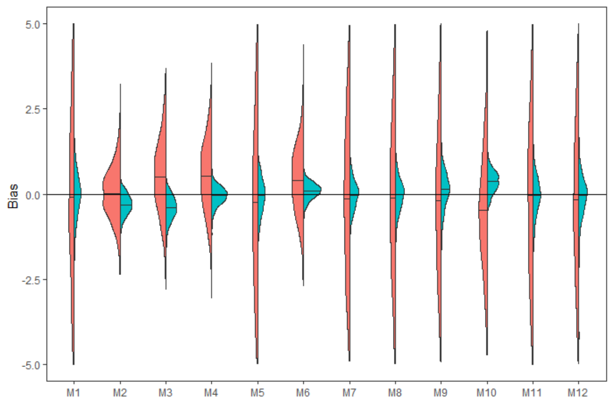
The simulation codes are available in https://github.com/NorahAlyabs/LOD.



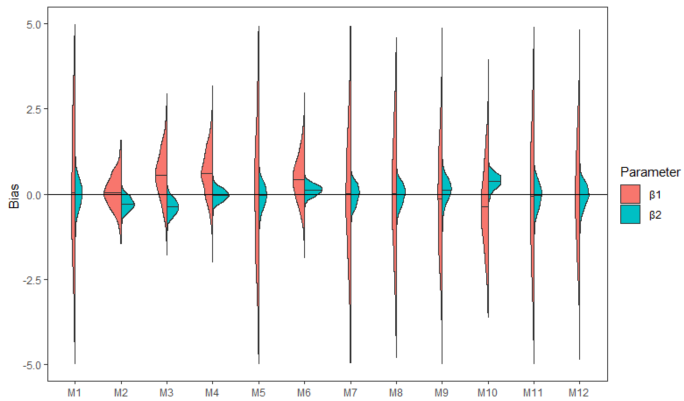
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



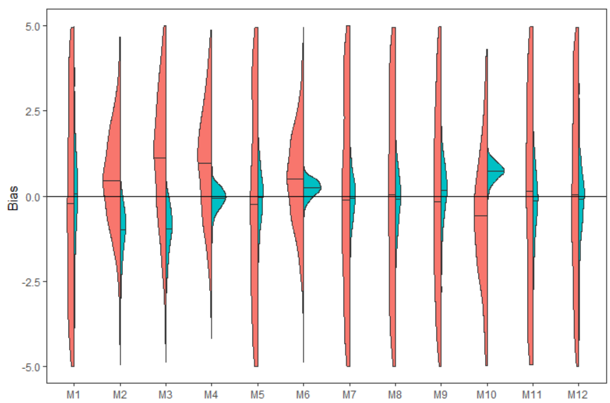
(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



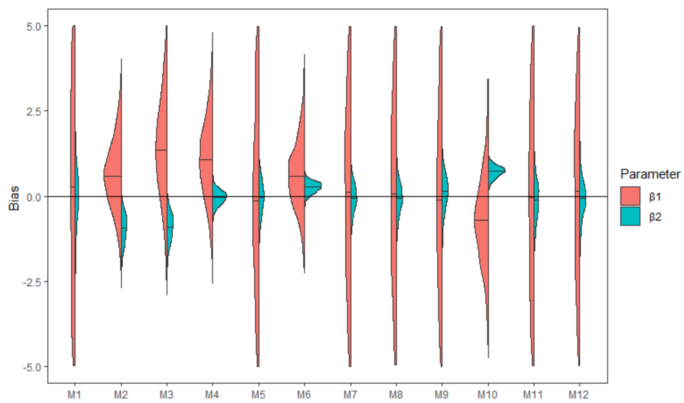
(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



(d) Bias under $n = 100$ and $m_1 = m_2 = 40\%$.

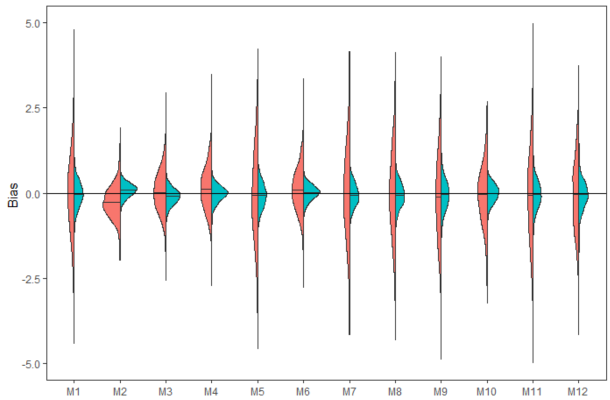


(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.

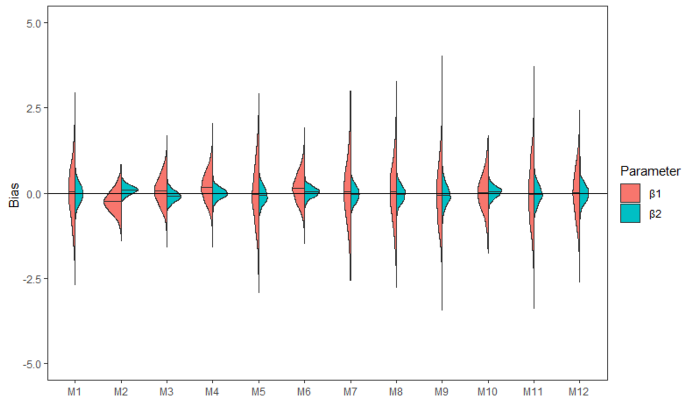


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

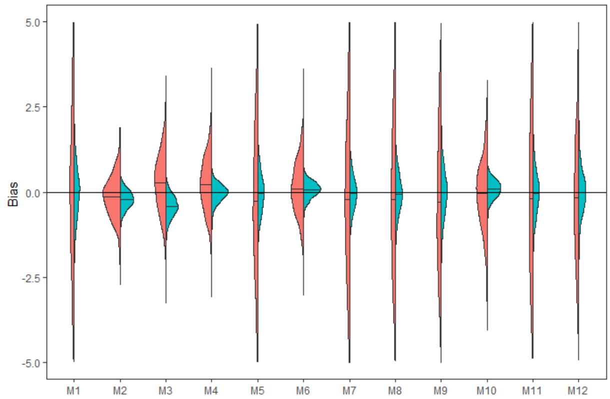
Figure S1. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to upper LOD.



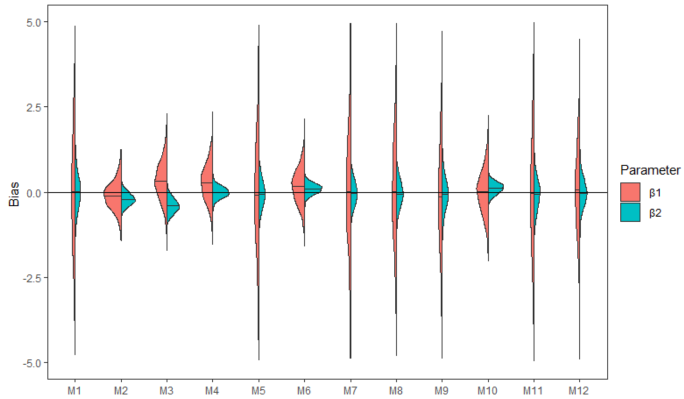
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



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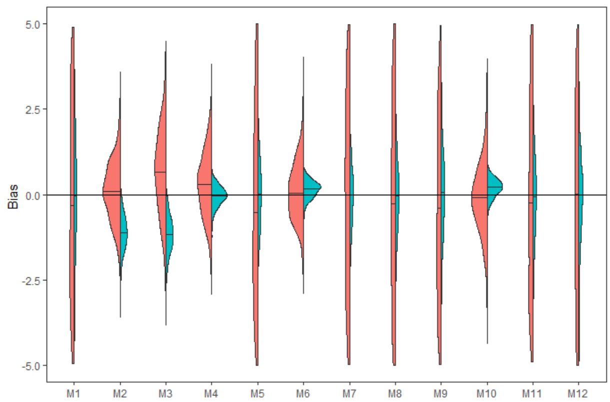

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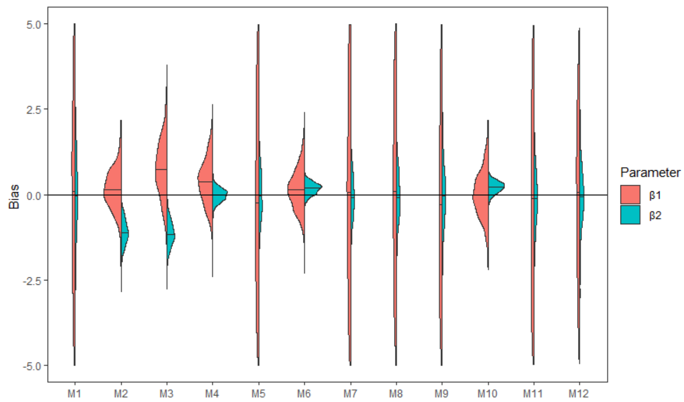
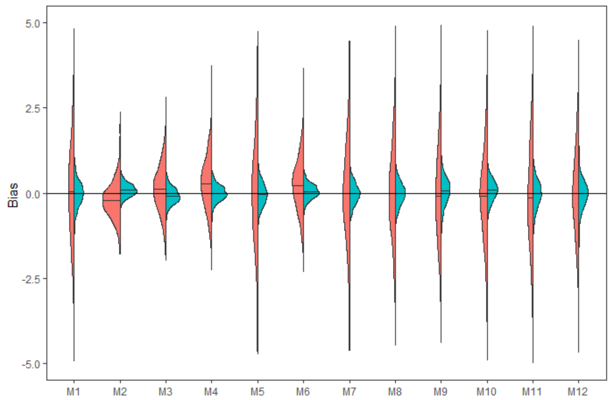
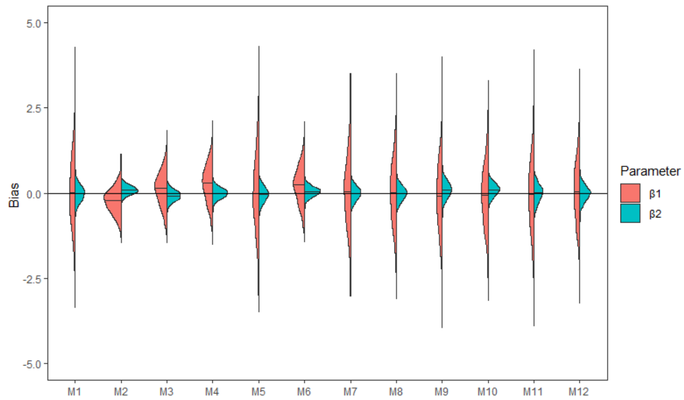


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

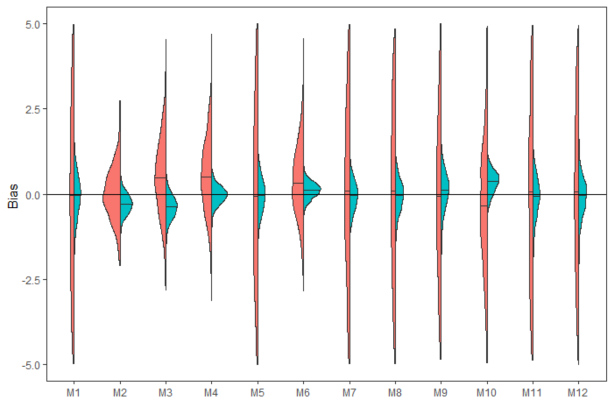
Figure S2. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to interval LOD.



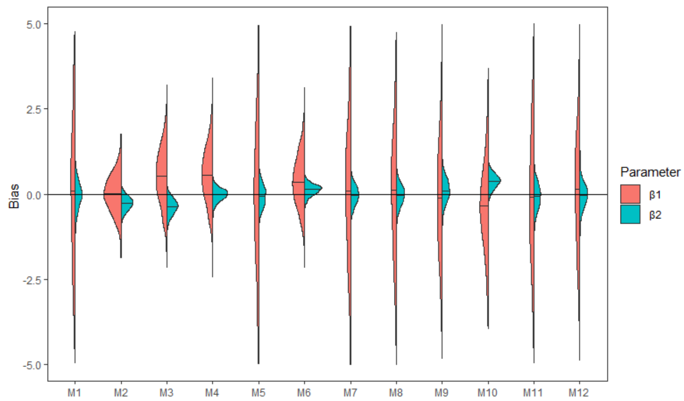
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



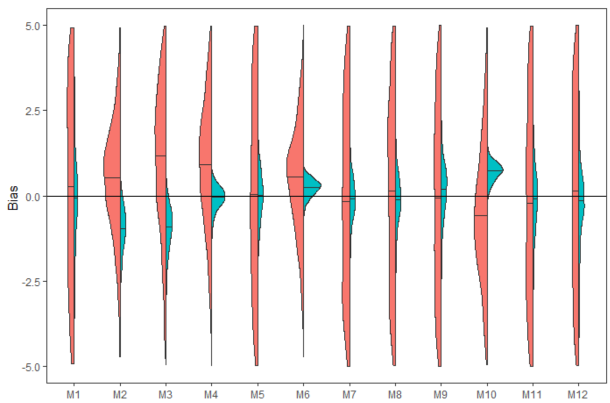
(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



(d) Bias under $n = 100$ and $m_1 = m_2 = 40\%$.



(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.

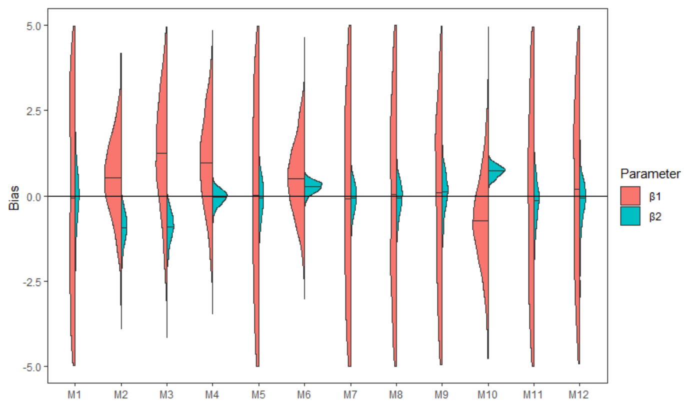


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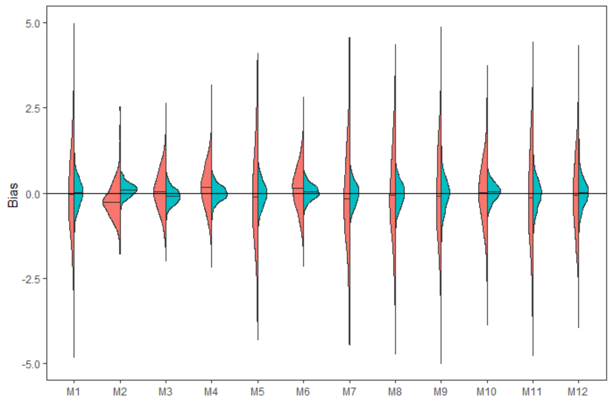
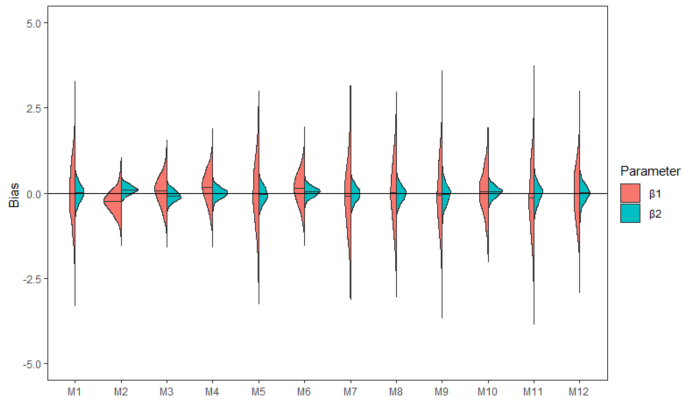
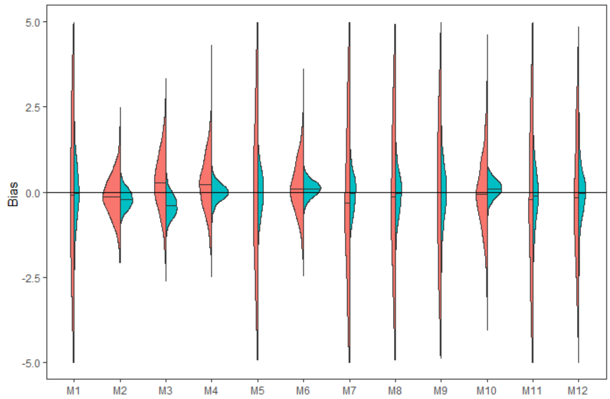
Figure S3. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are correlated and $X^*_{ij}, j = 1, 2$ is subjected to upper LOD.



(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



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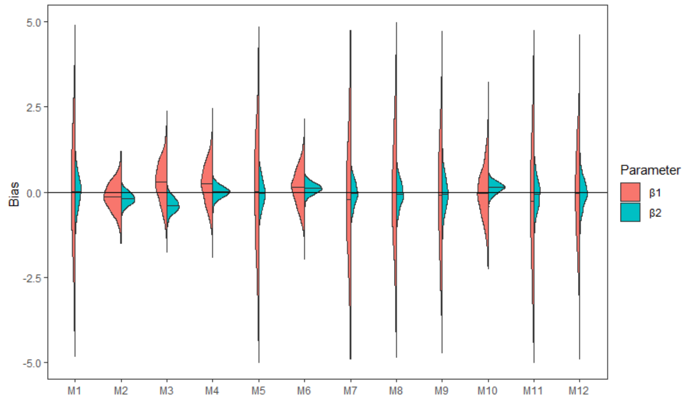

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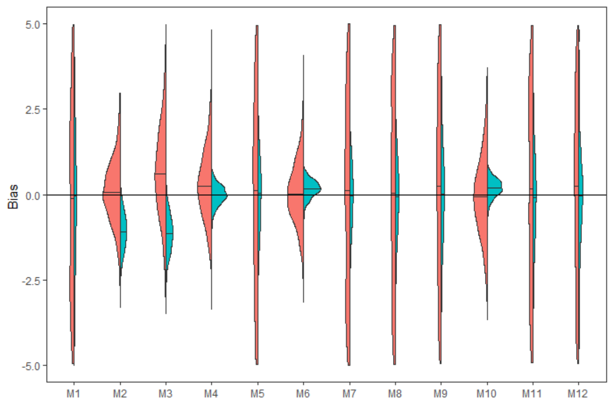

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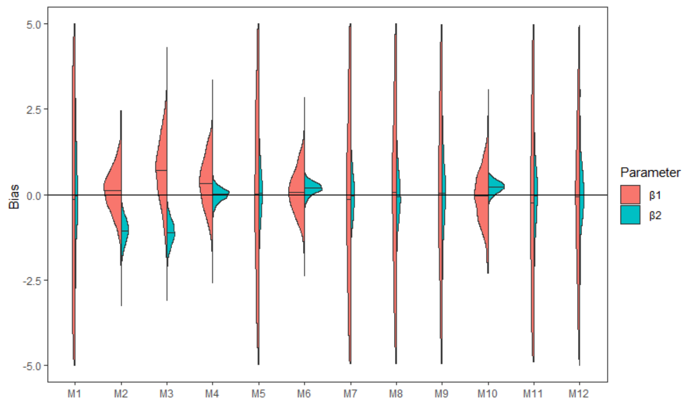
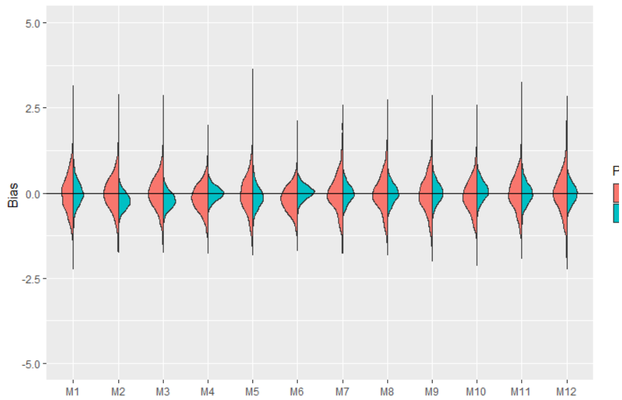
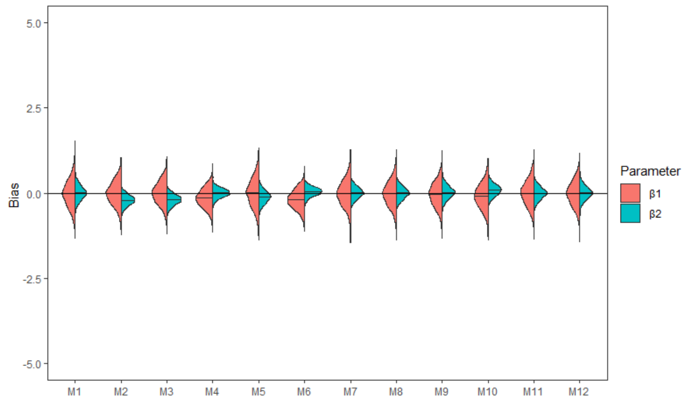


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

Figure S4. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are correlated and $X_{ij}^*, j = 1,2$ is subjected to interval LOD.



(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



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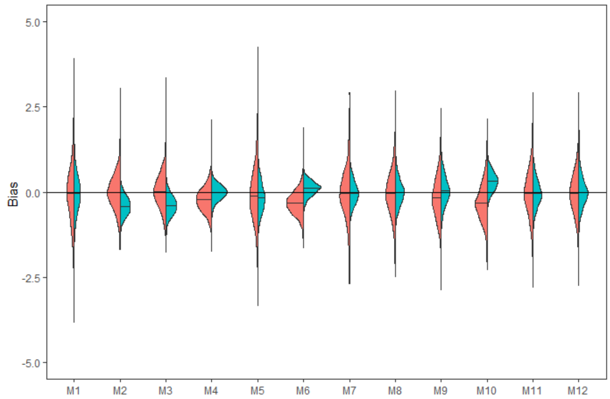
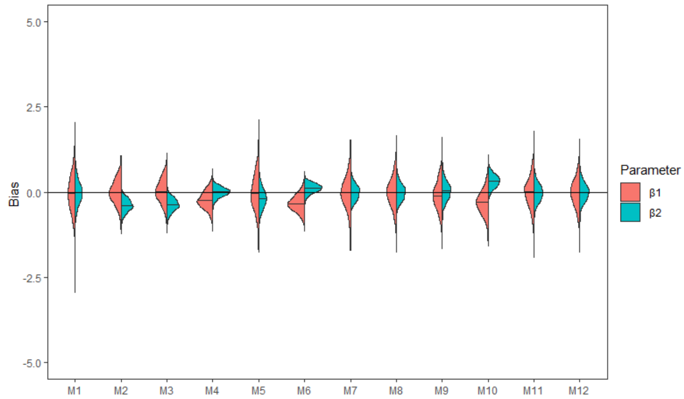


(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



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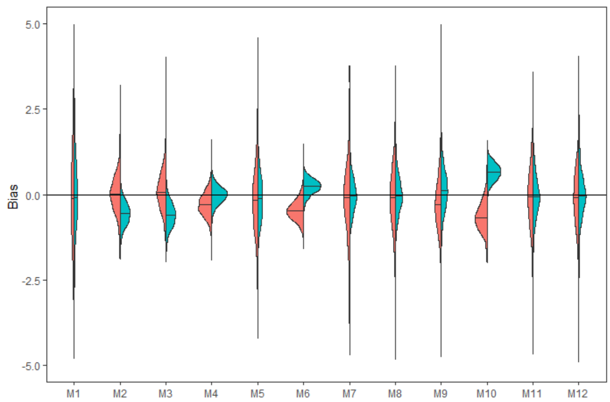

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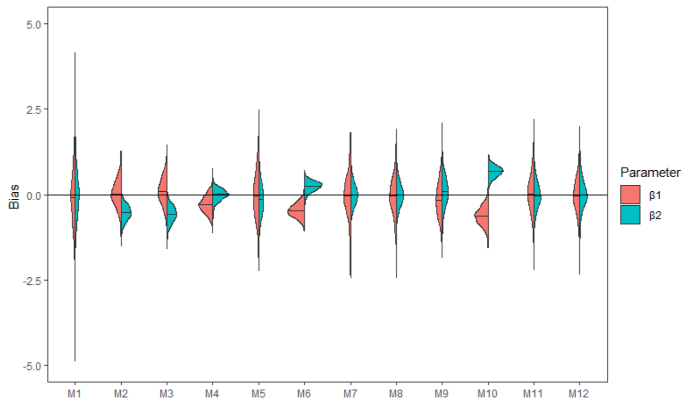
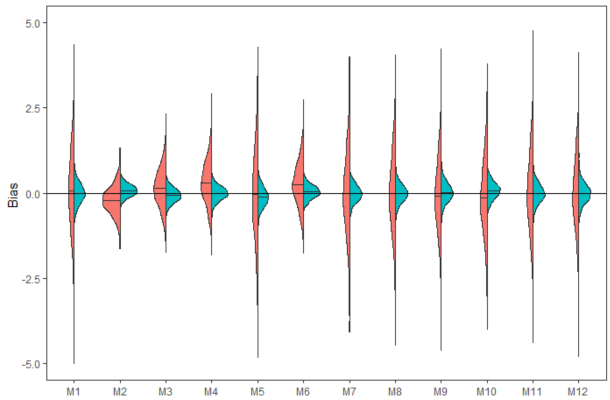
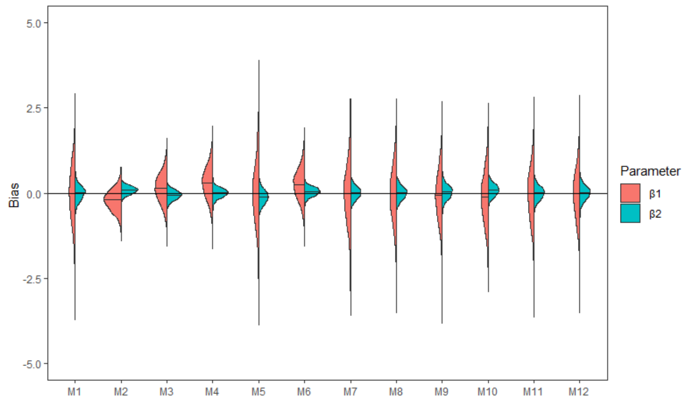


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

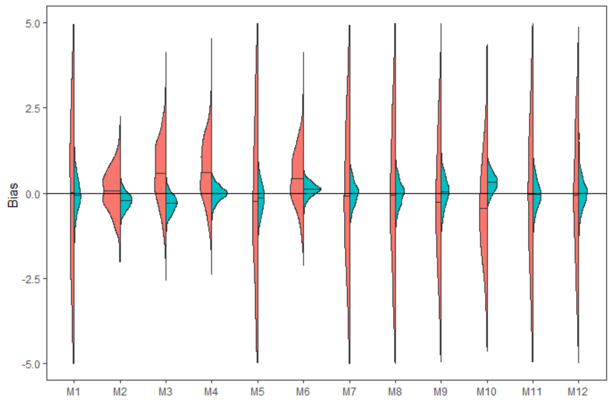
Figure S5. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X_{ij}^*, j = 1, 2$ is subjected to lower LOD in linear model.



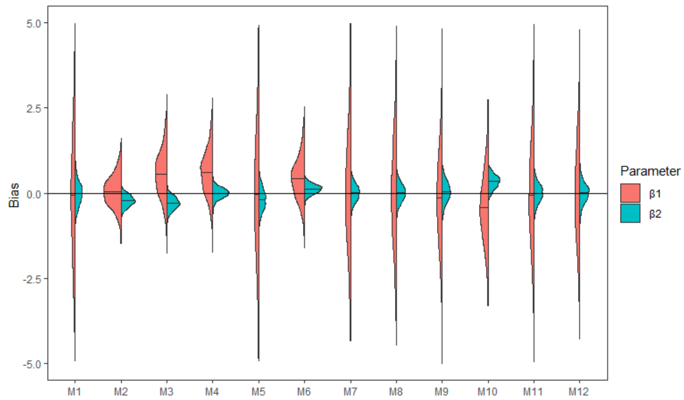
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



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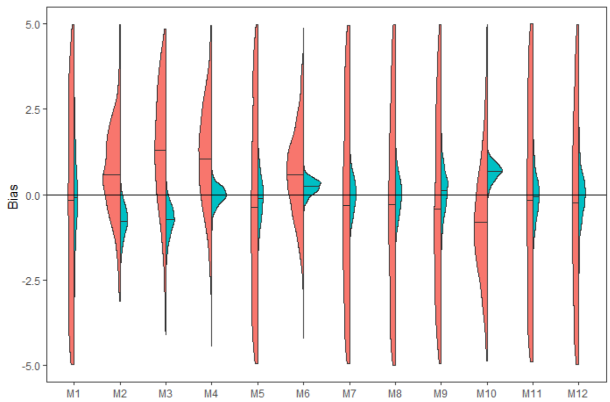
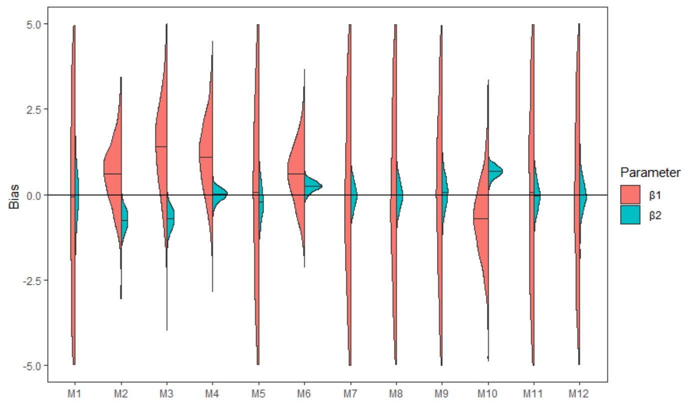
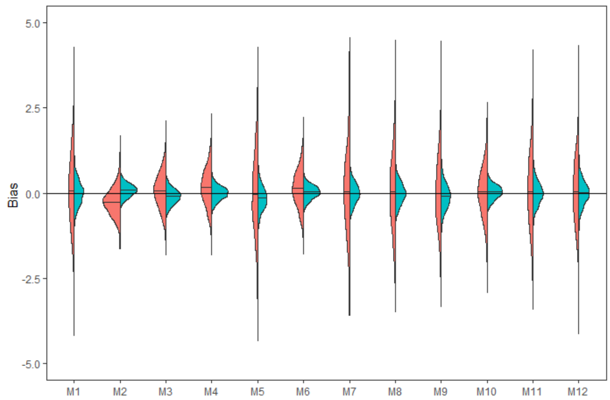


(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.

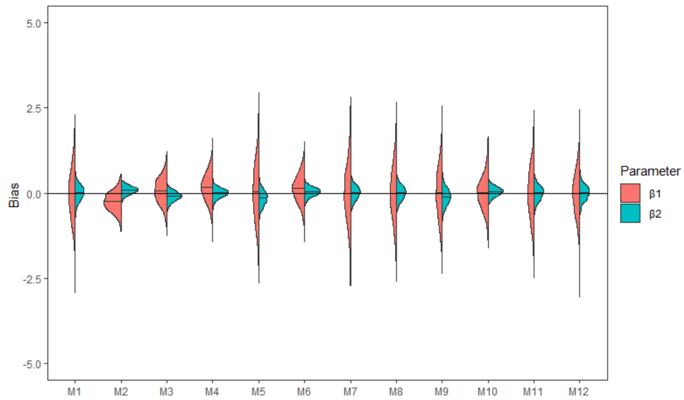


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

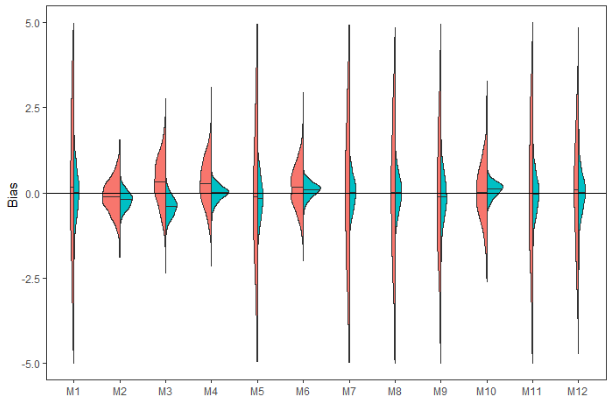
Figure S6. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to upper LOD in linear model.



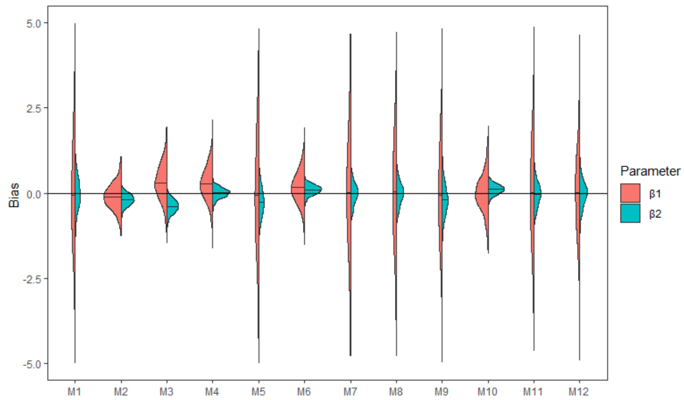
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



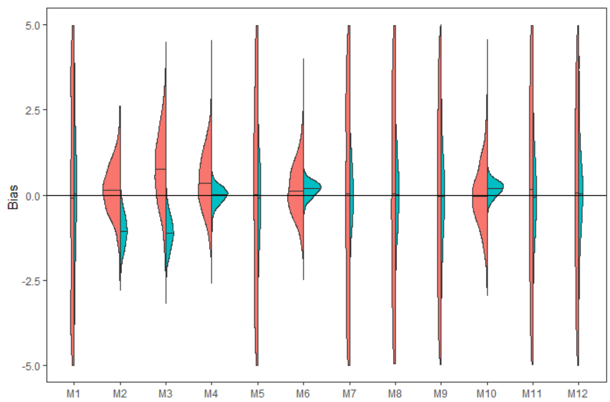
(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



(d) Bias under $n = 100$ and $m_1 = m_2 = 40\%$.



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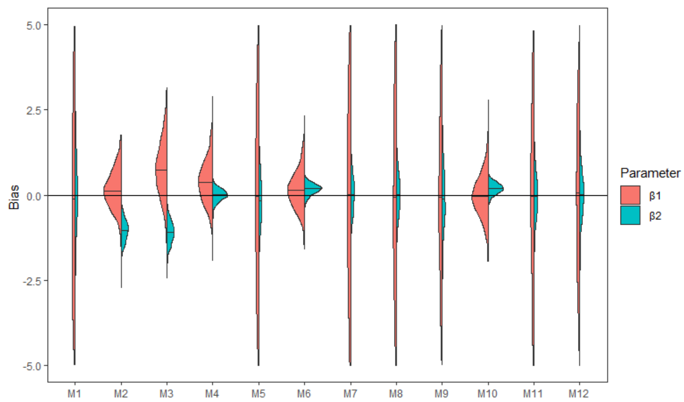
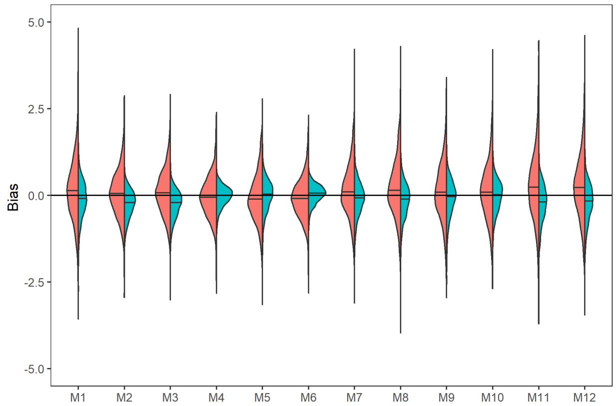


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

Figure S7. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to interval LOD in linear model.



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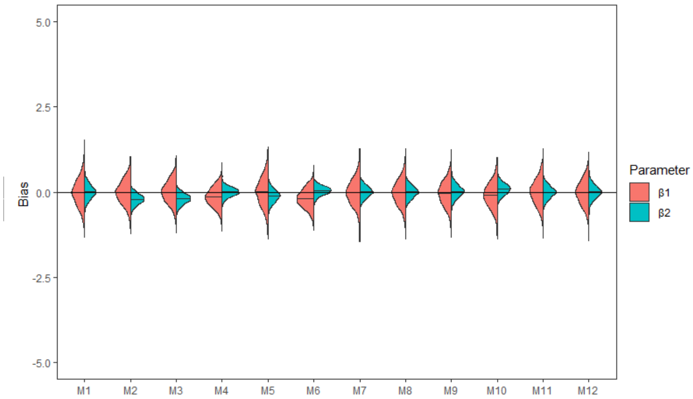
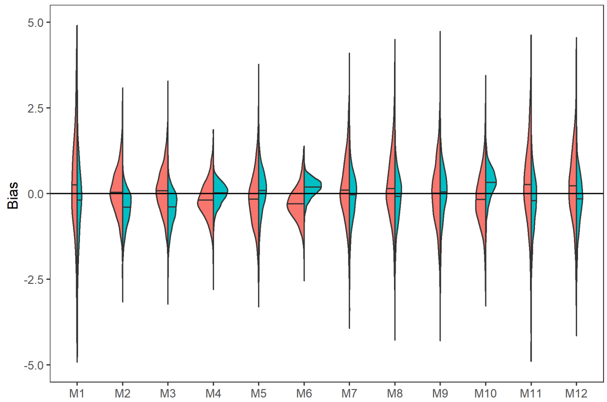
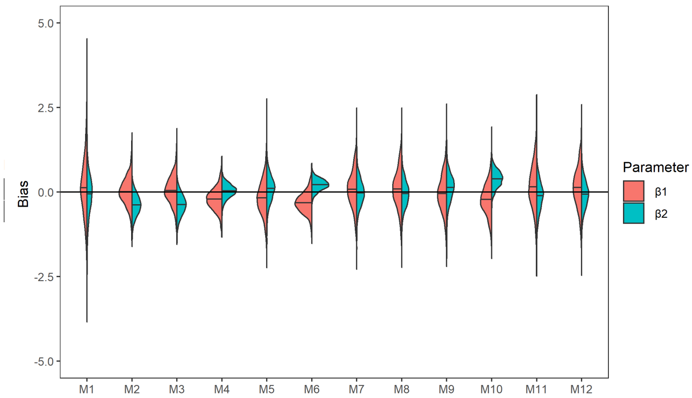


(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



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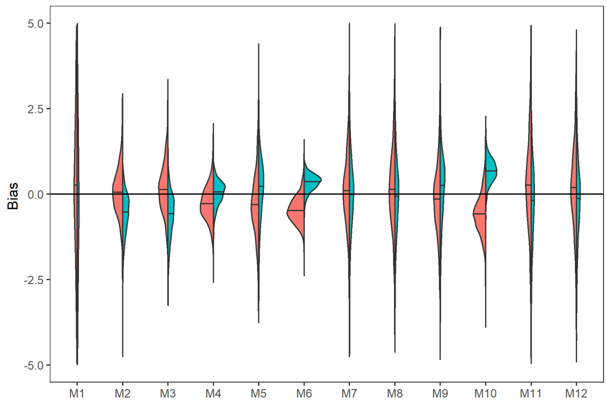
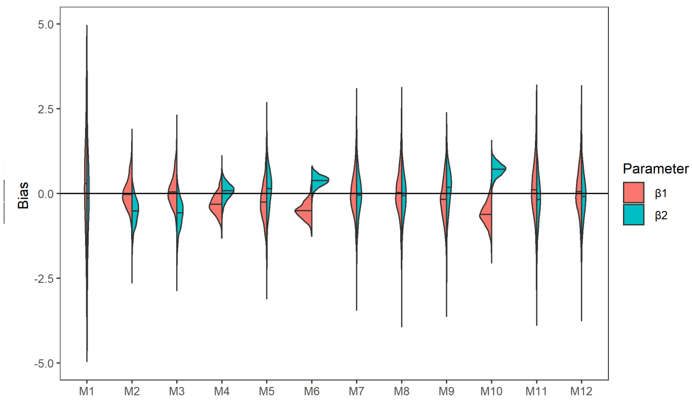
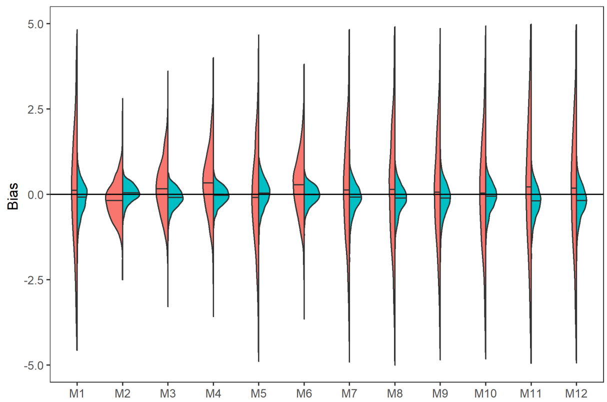


(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.

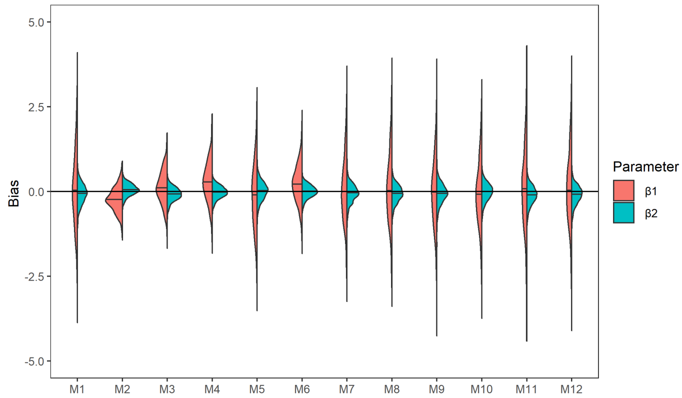


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

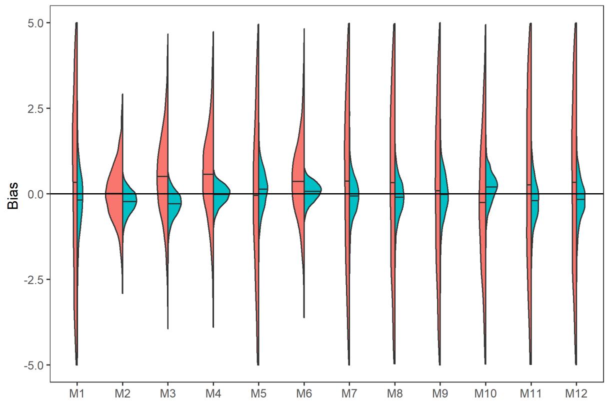
Figure S8. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to lower LOD in Cox PH model.



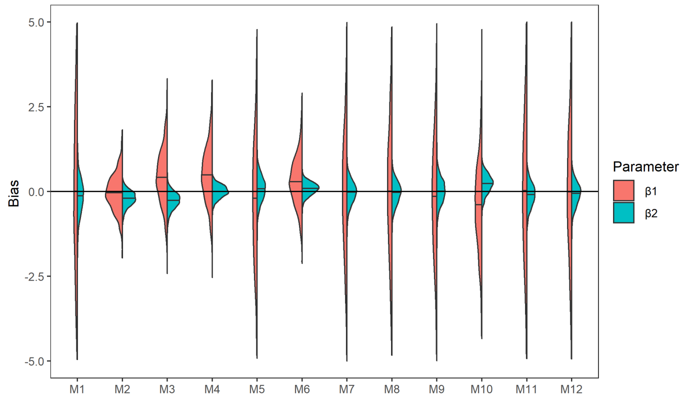
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



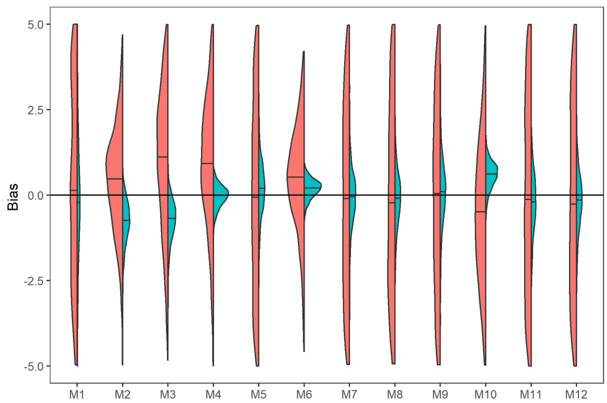
(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



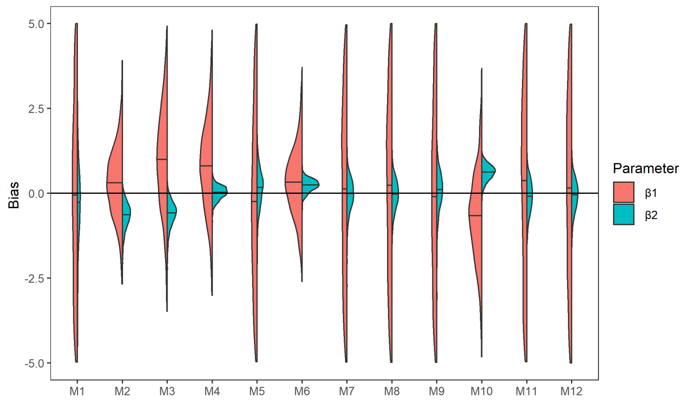
(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



(d) Bias under $n = 100$ and $m_1 = m_2 = 40\%$.

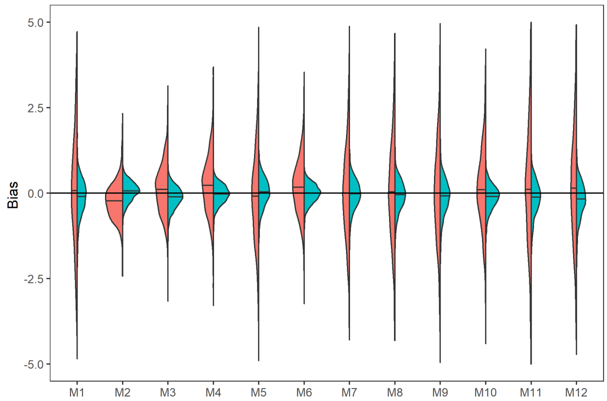


(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.

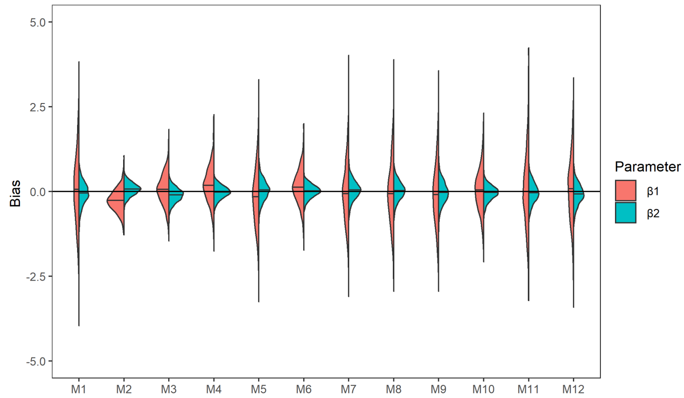


(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

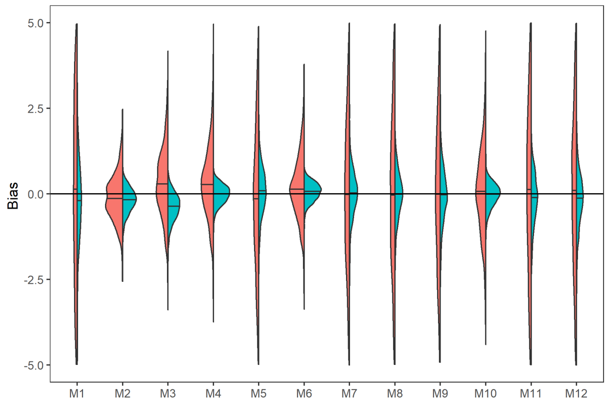
Figure S9. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to upper LOD in Cox PH model.



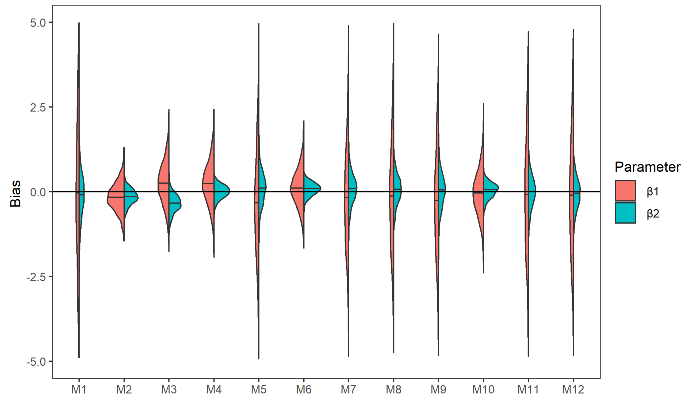
(a) Bias under $n = 50$ and $m_1 = m_2 = 20\%$.



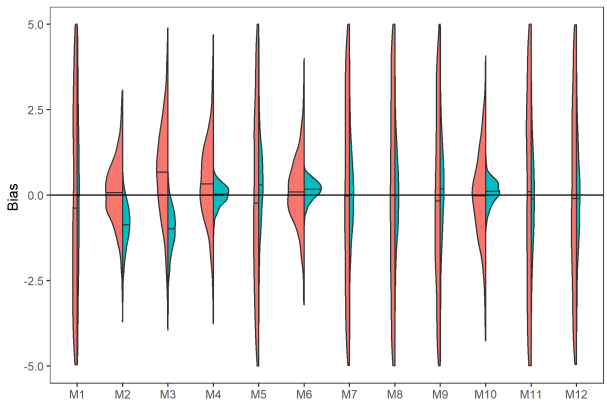
(b) Bias under $n = 100$ and $m_1 = m_2 = 20\%$.



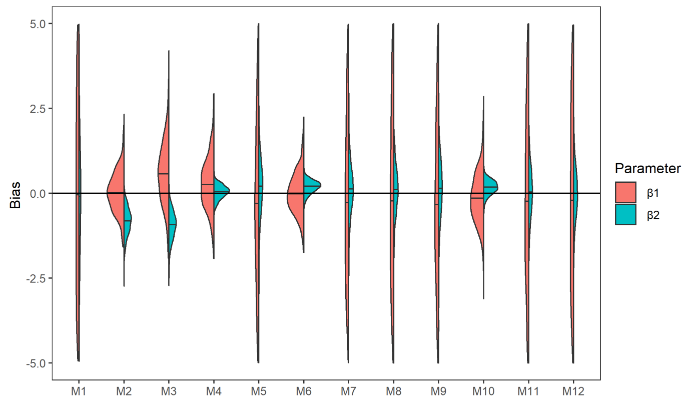
(c) Bias under $n = 50$ and $m_1 = m_2 = 40\%$.



(d) Bias under $n = 100$ and $m_1 = m_2 = 40\%$.



(e) Bias under $n = 50$ and $m_1 = m_2 = 60\%$.



(f) Bias under $n = 100$ and $m_1 = m_2 = 60\%$.

Figure S10. Violin plots showing the empirical distribution of the bias associated with MLE of β_1 (red) and β_2 (green) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to interval LOD in Cox PH model.

Table S1. Summary of the AAB (×1000) when covariates are independent and $X_{ij}^{*}, j = 1, 2$ is subjected to upper LOD in the AFT model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	18	225	108	253	81	218	34	42	117	135	74	42
	β_2	42	68	102	29	66	17	41	35	59	60	31	50
	γ_1	1	2	0	2	7	3	2	2	26	7	30	13
							$m_1 = m_2 = 40\%$						
	β_1	46	18	504	536	240	397	77	59	231	450	117	92
	β_2	47	319	421	51	73	89	49	46	108	360	73	65
	γ_1	18	1	2	2	2	4	2	24	78	6	88	1
							$m_1 = m_2 = 60\%$						
	β_1	7867	488	1258	958	142	521	143	133	212	672	45	85
	β_2	1230	1114	1065	77	79	226	126	139	109	721	204	157
	γ_1	31	4	5	1	20	1	3	75	151	3	177	11
							$m_1 = m_2 = 20\%$						
100	β_1	58	197	153	313	13	269	35	34	68	54	7	36
	β_2	6	84	80	8	52	41	12	11	78	92	15	15
	γ_1	8	9	11	10	22	10	10	10	17	16	19	17
							$m_1 = m_2 = 40\%$						
	β_1	152	56	571	617	32	455	62	69	156	338	2	55
	β_2	9	286	379	26	55	119	17	16	116	381	58	26
	γ_1	21	14	15	12	20	11	12	32	67	28	73	31
							$m_1 = m_2 = 60\%$						
	β_1	147	607	1427	1102	249	632	33	16	272	688	20	3
	β_2	84	978	930	41	67	256	59	61	112	720	130	74
	γ_1	54	17	17	14	11	11	14	67	144	30	164	35

Table S2. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to upper LOD. M1 is complete-case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSEs

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	2277	305	518	722	2104	678	1708	1722	2072	1552	2301	1814
	β_2	158	50	76	59	156	56	122	125	155	116	182	141
	γ_1	57	32	32	32	44	32	34	54	142	107	150	112
							$m_1 = m_2 = 40\%$						
	β_1	>9999	559	1515	1621	8078	1306	6702	7085	6799	3310	8728	7011
	β_2	524	221	328	77	346	69	281	296	315	256	416	325
	γ_1	171	35	35	35	49	35	35	105	197	127	203	133
							$m_1 = m_2 = 60\%$						
	β_1	>9999	2274	6269	4212	>9999	2543	>9999	>9999	>9999	5077	>9999	>9999
	β_2	>9999	2008	1809	112	834	115	805	892	724	599	1193	877
	γ_1	836	42	40	38	54	39	39	246	377	152	404	161
							$m_1 = m_2 = 20\%$						
100	β_1	998	158	255	400	1036	364	801	811	962	706	1046	822
	β_2	68	27	36	25	67	26	56	57	71	57	78	59
	γ_1	24	15	15	15	23	15	15	23	60	42	62	42
							$m_1 = m_2 = 40\%$						
	β_1	6033	252	888	978	4283	730	3295	3381	3714	1486	4427	3329
	β_2	187	133	207	32	144	40	108	110	134	195	161	113
	γ_1	51	17	17	16	26	17	17	42	86	55	90	55
							$m_1 = m_2 = 60\%$						
	β_1	>9999	1348	4278	2801	>9999	1479	>9999	>9999	>9999	2124	>9999	>9999
	β_2	1017	1218	1082	46	330	94	261	275	283	553	409	279
	γ_1	219	21	21	19	27	19	19	89	171	64	182	68

Table S3. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to interval LOD in the AFT model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	2	257	38	126	31	99	11	13	84	33	218	10
	β_2	25	82	107	20	66	11	55	52	47	9	6	44
	γ_1	7	8	5	3	0	4	5	25	35	1	67	5
						$m_1 = m_2 = 40\%$							
	β_1	35	133	277	227	219	125	76	85	200	55	113	79
	β_2	28	223	422	30	44	64	36	40	20	78	41	13
	γ_1	25	5	4	2	6	2	4	49	70	6	64	6
						$m_1 = m_2 = 60\%$							
	β_1	1595	88	659	306	499	99	207	208	426	70	367	214
	β_2	812	1136	1176	34	2	153	67	65	25	191	94	44
	γ_1	39	4	3	2	20	4	2	115	138	6	139	6
						$m_1 = m_2 = 20\%$							
100	β_1	34	232	82	182	0	149	34	30	35	23	4	17
	β_2	3	98	88	4	51	31	23	21	44	31	19	3
	γ_1	7	7	8	10	17	9	11	32	22	17	18	16
						$m_1 = m_2 = 40\%$							
	β_1	49	96	332	290	89	183	42	49	108	24	15	66
	β_2	14	202	397	5	54	95	45	49	57	116	70	19
	γ_1	5	9	10	10	14	10	13	56	63	25	61	25
						$m_1 = m_2 = 60\%$							
	β_1	326	148	751	388	219	163	55	115	277	19	121	66
	β_2	75	1101	1143	11	9	177	65	67	0	215	109	40
	γ_1	35	12	12	12	9	11	14	90	100	26	98	29

Table S4. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to interval LOD. M1 is complete-case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSEs

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	1678	291	413	507	1824	485	1717	1766	2156	722	>9999	1495
	β_2	163	55	76	56	167	53	146	152	202	87	1291	152
	γ_1	53	33	32	32	46	32	41	55	178	98	1952	102
							$m_1 = m_2 = 40\%$						
	β_1	>9999	386	863	789	6163	683	6426	6564	7332	978	8707	5223
	β_2	682	142	300	68	409	64	339	377	497	92	492	343
	γ_1	168	35	35	34	50	34	46	126	307	104	316	113
							$m_1 = m_2 = 60\%$						
	β_1	>9999	760	2186	1235	>9999	884	>9999	>9999	>9999	1120	>9999	>9999
	β_2	>9999	1660	1730	87	1474	91	1338	1509	1691	121	1871	1315
	γ_1	1161	38	37	36	54	36	50	273	586	113	636	116
							$m_1 = m_2 = 20\%$						
100	β_1	781	162	202	270	811	247	769	766	1001	295	1021	652
	β_2	79	31	37	25	76	25	65	66	90	33	87	65
	γ_1	24	14	14	15	22	15	18	26	75	38	75	40
							$m_1 = m_2 = 40\%$						
	β_1	4008	176	457	423	2996	331	3021	3054	3582	392	4104	2377
	β_2	283	84	213	28	212	35	178	184	247	45	247	167
	γ_1	45	16	16	16	24	16	21	46	132	42	133	44
							$m_1 = m_2 = 60\%$						
	β_1	>9999	359	1348	668	>9999	432	>9999	>9999	>9999	473	>9999	>9999
	β_2	2382	1391	1469	39	779	62	596	642	863	79	848	562
	γ_1	228	17	17	17	27	18	22	100	266	47	266	48

Table S5. Summary of the AAB ($\times$1000) when covariates are correlated and $X^*_{ij}, j = 1, 2$ are subjected to upper LOD in the AFT model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	46	210	124	275	26	233	24	11	54	72	63	13
	β_2	19	90	75	1	38	45	24	19	54	77	39	34
	γ_1	6	8	2	7	62	4	1	3	8	8	3	6
							$m_1 = m_2 = 40\%$						
	β_1	88	2	494	544	81	388	75	93	42	364	88	69
	β_2	67	292	390	25	48	112	51	60	86	348	112	78
	γ_1	14	13	6	2	69	10	8	11	14	28	37	3
							$m_1 = m_2 = 60\%$						
	β_1	1417	491	1265	957	279	519	107	320	83	514	345	309
	β_2	56	1078	1028	56	27	238	131	155	139	709	189	177
	γ_1	40	32	30	29	57	47	36	72	63	25	90	3
							$m_1 = m_2 = 20\%$						
100	β_1	40	220	124	285	14	242	44	33	61	48	27	49
	β_2	13	88	75	3	40	45	10	10	89	97	8	15
	γ_1	5	15	4	2	65	2	5	7	0	12	11	1
							$m_1 = m_2 = 40\%$						
	β_1	68	12	519	564	6	384	78	77	93	331	26	114
	β_2	33	279	374	15	59	128	35	34	105	377	76	45
	γ_1	7	16	9	6	88	17	14	16	34	27	56	2
							$m_1 = m_2 = 60\%$						
	β_1	67	513	1288	976	24	511	23	77	195	718	309	157
	β_2	86	989	939	40	73	257	86	96	86	720	194	111
	γ_1	16	26	24	24	92	42	30	33	79	28	110	2

Table S6. Summary of the MSE (×1000) when covariates are correlated and $X^*_{ij}, j = 1, 2$ is subjected to upper LOD. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	2768	313	526	740	2546	698	1922	2022	2534	1711	2853	2137
	β_2	154	52	68	54	143	52	127	131	163	124	188	150
	γ_1	57	34	35	35	54	35	38	53	146	92	157	98
							$m_1 = m_2 = 40\%$						
	β_1	>9999	594	1625	1764	>9999	1418	8050	8764	9078	3593	>9999	9029
	β_2	487	198	293	70	320	72	256	269	326	247	449	301
	γ_1	146	39	40	39	63	38	43	93	216	104	224	120
							$m_1 = m_2 = 60\%$						
	β_1	>9999	2720	7424	5145	>9999	3018	>9999	>9999	>9999	4923	>9999	>9999
	β_2	>9999	1940	1757	112	829	121	857	994	773	593	1334	960
	γ_1	2566	54	53	50	78	49	55	269	409	135	451	176
							$m_1 = m_2 = 20\%$						
100	β_1	1159	182	285	439	1283	398	941	969	1197	819	1284	1005
	β_2	60	27	33	24	63	25	52	53	73	54	75	57
	γ_1	27	17	17	17	29	17	18	26	60	41	62	42
							$m_1 = m_2 = 40\%$						
	β_1	6347	308	994	1074	4711	775	3811	3932	3978	1695	5304	4128
	β_2	175	128	202	32	142	44	108	112	134	194	162	121
	γ_1	55	19	20	19	35	19	20	42	86	47	92	53
							$m_1 = m_2 = 60\%$						
	β_1	>9999	1512	4620	3009	>9999	1534	>9999	>9999	>9999	2469	>9999	>9999
	β_2	1009	1273	1121	47	388	95	315	337	348	555	518	346
	γ_1	217	25	24	22	41	22	24	82	178	54	187	67

Table S7. Summary of the AAB (×1000) when covariates are correlated and $X^*_{ij}, j = 1, 2$ is subjected to interval LOD. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. AAB less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	34	231	65	166	53	133	103	22	196	18	85	12
	β_2	10	102	82	4	17	33	15	17	37	27	29	1
	γ_1	6	6	4	15	56	12	67	14	226	6	37	9
						$m_1 = m_2 = 40\%$							
	β_1	44	128	287	250	29	144	266	58	27	11	267	132
	β_2	25	199	397	0	34	94	75	78	20	96	125	61
	γ_1	12	5	2	9	72	0	85	31	56	21	55	23
						$m_1 = m_2 = 60\%$							
	β_1	934	74	651	305	130	75	68	118	257	35	23	73
	β_2	70	1100	1144	13	11	166	69	87	8	192	128	57
	γ_1	37	9	6	2	69	19	89	87	4	35	10	43
						$m_1 = m_2 = 20\%$							
100	β_1	23	249	61	170	5	132	80	33	43	41	113	6
	β_2	8	105	80	3	40	38	20	20	36	37	13	2
	γ_1	5	15	2	10	65	6	80	30	38	8	39	4
						$m_1 = m_2 = 40\%$							
	β_1	1	127	295	254	27	137	148	33	28	19	207	44
	β_2	13	192	386	6	43	104	42	51	43	123	61	16
	γ_1	11	10	3	5	88	6	98	56	37	27	41	27
						$m_1 = m_2 = 60\%$							
	β_1	85	102	690	336	167	93	142	68	247	58	82	7
	β_2	10	1075	1122	1	33	185	41	51	33	217	65	6
	γ_1	25	15	11	7	99	26	104	82	6	43	5	46

Table S8. Summary of the MSE (×1000) when covariates are correlated and $X^*_{ij}, j = 1, 2$ is subjected to interval LOD. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	2113	312	431	541	2049	511	1939	2048	>9999	832	2573	1698
	β_2	172	57	69	52	161	53	145	148	356	84	208	152
	γ_1	64	34	34	35	51	35	50	67	>9999	92	179	97
							$m_1 = m_2 = 40\%$						
	β_1	>9999	400	902	862	6634	729	6742	7071	>9999	1154	9128	5438
	β_2	801	133	279	62	501	66	461	488	6024	91	669	440
	γ_1	165	35	36	37	59	37	55	116	>9999	98	291	105
							$m_1 = m_2 = 60\%$						
	β_1	>9999	818	2350	1374	>9999	987	>9999	>9999	>9999	1271	>9999	>9999
	β_2	7727	1606	1671	85	1576	97	1498	1682	1912	119	2112	1440
	γ_1	1612	41	41	41	62	39	57	263	598	94	625	98
							$m_1 = m_2 = 20\%$						
100	β_1	909	177	221	305	938	281	928	922	1213	404	1271	770
	β_2	66	31	33	24	67	24	59	58	88	33	80	58
	γ_1	28	16	16	17	27	17	27	30	86	40	83	44
							$m_1 = m_2 = 40\%$						
	β_1	4113	199	496	479	3279	371	3236	3333	3863	528	4489	2501
	β_2	245	76	200	28	194	37	162	168	234	48	224	154
	γ_1	59	18	18	18	33	18	30	58	132	42	132	44
							$m_1 = m_2 = 60\%$						
	β_1	>9999	419	1462	757	>9999	479	>9999	>9999	>9999	621	>9999	>9999
	β_2	1730	1321	1412	38	751	65	597	627	884	82	873	563
	γ_1	188	20	20	20	36	20	32	107	289	42	294	43

Table S9. The proportion (%) of convergence cases for the AFT model's complete case analysis.

n	mr2/mr1	Lower LOD			Upper LOD			Interval LOD		
		0.2	0.4	0.6	0.2	0.4	0.6	0.2	0.4	0.6
		Independent covariates								
50	0.2	100	100	100	100	100	99	100	100	100
	0.4	100	100	98	100	100	94	100	100	98
	0.6	100	98	85	100	94	71	100	97	75
100	0.2	100	100	100	100	100	100	100	100	100
	0.4	100	100	100	100	100	100	100	100	100
	0.6	100	100	100	100	100	99	100	100	99
		Correlated covariates								
50	0.2	100	100	100	100	100	99	100	100	100
	0.4	100	100	98	100	100	94	100	100	97
	0.6	99	96	80	100	98	76	100	98	81
100	0.2	100	100	100	100	100	100	100	100	100
	0.4	100	100	100	100	100	100	100	100	100
	0.6	100	100	99	100	100	99	100	100	100

Table S10. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to lower, upper, and interval LOD in the AFT model for a large sample $n = 500$. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
LOD		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
Lower	β_1	13	7	2	147	62	196	12	12	106	104	31	10
	β_2	1	190	172	9	63	38	1	1	65	60	30	1
	γ_1	0	5	5	2	5	1	1	1	47	4	52	0
						$m_1 = m_2 = 40\%$							
	β_1	14	1	34	244	85	353	13	14	164	342	14	11
	β_2	4	354	336	15	57	113	1	0	117	274	27	1
	γ_1	0	7	7	4	7	2	4	4	57	5	74	3
						$m_1 = m_2 = 60\%$							
	β_1	13	20	104	312	108	503	6	6	206	665	26	8
	β_2	13	488	527	19	32	234	6	7	153	621	6	7
	γ_1	3	7	7	5	10	2	5	10	68	2	96	5
						$m_1 = m_2 = 20\%$							
Upper	β_1	6	223	121	281	16	237	7	7	76	93	29	4
	β_2	1	86	77	6	59	45	6	5	86	105	5	7
	γ_1	3	1	3	2	21	1	2	6	49	3	51	6
						$m_1 = m_2 = 40\%$							
	β_1	54	29	536	585	37	412	99	101	85	352	62	92
	β_2	7	274	365	16	75	131	20	21	97	388	59	24
	γ_1	1	7	8	5	19	3	5	15	90	5	98	12
						$m_1 = m_2 = 60\%$							
	β_1	66	569	1384	1072	24	580	189	193	106	708	138	178
	β_2	11	908	871	33	62	266	33	38	87	728	125	42
	γ_1	5	8	9	6	14	3	6	34	174	7	189	16
						$m_1 = m_2 = 20\%$							
Interval	β_1	1	261	49	153	16	117	1	2	70	0	41	3
	β_2	4	103	85	1	72	36	21	24	60	45	12	1
	γ_1	4	3	0	1	16	0	3	31	62	3	59	4
						$m_1 = m_2 = 40\%$							
	β_1	5	121	301	264	55	151	8	6	94	9	61	6
	β_2	5	198	393	4	89	97	36	40	94	125	43	8
	γ_1	3	0	2	3	13	1	2	45	105	5	103	6
						$m_1 = m_2 = 60\%$							
	β_1	57	120	718	361	55	123	119	122	63	17	73	112
	β_2	7	1081	1131	10	56	183	58	64	85	225	84	29
	γ_1	3	2	3	4	5	2	1	56	136	4	135	5

Table S11. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to lower, upper, and interval LOD in the AFT model for a large sample $n = 500$. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
Lower	β_1	37	27	27	40	42	55	33	33	50	38	45	33
	β_2	10	43	36	5	13	6	8	8	15	11	12	8
	γ_1	4	3	3	3	3	3	3	4	13	4	14	4
						$m_1 = m_2 = 40\%$							
	β_1	70	29	32	75	66	137	48	48	76	141	64	48
	β_2	23	135	122	5	24	18	14	13	34	85	20	14
	γ_1	8	3	3	3	4	3	3	7	22	4	24	4
						$m_1 = m_2 = 60\%$							
	β_1	178	33	51	112	114	262	83	84	120	455	116	84
	β_2	69	251	292	6	53	60	25	25	69	393	39	25
	γ_1	17	3	3	3	5	3	3	12	43	3	47	3
						$m_1 = m_2 = 20\%$							
Upper	β_1	182	70	55	132	192	106	144	143	176	130	184	144
	β_2	12	11	12	5	17	7	10	10	19	20	14	11
	γ_1	5	3	3	3	5	3	3	4	13	8	13	8
						$m_1 = m_2 = 40\%$							
	β_1	972	46	391	451	812	262	570	572	620	362	740	564
	β_2	31	86	146	7	36	23	20	21	33	160	33	21
	γ_1	9	3	3	3	5	3	3	7	23	9	24	9
						$m_1 = m_2 = 60\%$							
	β_1	7781	513	2349	1440	3821	534	2863	2886	3053	782	3841	2875
	β_2	104	872	798	10	82	77	47	48	70	536	90	50
	γ_1	22	4	4	4	5	4	4	15	59	11	64	11
						$m_1 = m_2 = 20\%$							
Interval	β_1	139	86	36	65	154	53	140	141	178	49	172	109
	β_2	13	15	13	5	19	6	13	13	22	8	16	11
	γ_1	4	3	3	3	4	3	4	6	18	7	18	7
						$m_1 = m_2 = 40\%$							
	β_1	675	45	155	131	594	76	520	521	709	64	696	412
	β_2	48	48	166	6	53	15	34	34	59	22	43	30
	γ_1	8	3	3	3	5	3	4	10	33	7	33	7
						$m_1 = m_2 = 60\%$							
	β_1	4424	79	664	226	2637	89	2442	2444	3362	78	3378	1879
	β_2	249	1202	1312	8	164	40	112	115	183	57	148	100
	γ_1	20	3	3	3	5	3	4	19	78	7	78	8

Table S12. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to lower LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	12	14	4	133	37	170	16	10	51	79	1	4
	β_2	4	225	208	2	107	36	1	2	25	82	6	2
	γ_1	2	0	0	0	1	0	0	2	5	2	5	0
						$m_1 = m_2 = 40\%$							
	β_1	32	9	26	219	62	312	41	38	135	301	15	37
	β_2	32	408	388	9	138	106	1	1	53	320	20	2
	γ_1	5	1	1	0	2	1	1	3	18	4	19	4
						$m_1 = m_2 = 60\%$							
	β_1	91	14	95	277	155	456	90	87	272	639	76	82
	β_2	126	559	604	15	104	229	4	1	122	661	32	2
	γ_1	7	0	0	1	5	1	0	9	23	4	18	1
						$m_1 = m_2 = 20\%$							
100	β_1	7	3	5	136	1	177	11	13	19	89	4	14
	β_2	6	214	198	3	110	43	5	5	27	88	3	6
	γ_1	1	1	0	0	2	0	1	2	6	3	7	2
						$m_1 = m_2 = 40\%$							
	β_1	27	4	37	223	10	321	13	13	47	300	9	16
	β_2	12	393	374	0	175	117	2	1	27	324	13	4
	γ_1	2	1	1	0	3	0	1	1	8	2	8	1
						$m_1 = m_2 = 60\%$							
	β_1	101	25	105	285	52	465	17	21	68	628	18	20
	β_2	10	534	577	2	149	243	11	12	63	671	10	12
	γ_1	15	1	1	1	3	2	2	5	10	2	8	1

Table S13. Summary of the MSE (×1000) when covariates are independent and $X_{ij}^*, j = 1,2$ is subjected to lower LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	398	249	247	208	376	209	295	310	329	302	348	333
	β_2	104	119	105	40	110	40	79	81	87	82	87	85
	γ_1	36	23	22	22	28	22	22	35	69	44	75	45
						$m_1 = m_2 = 40\%$							
	β_1	1060	262	285	211	696	232	479	506	551	417	602	540
	β_2	345	266	245	48	254	56	155	165	177	203	178	167
	γ_1	86	24	24	23	34	23	24	62	100	37	109	37
						$m_1 = m_2 = 60\%$							
	β_1	7662	303	389	236	1241	307	973	1022	952	610	1177	1045
	β_2	3059	455	522	64	530	103	356	378	392	518	437	381
	γ_1	1018	26	26	25	41	25	26	131	181	34	196	33
						$m_1 = m_2 = 20\%$							
100	β_1	177	118	118	108	176	115	145	147	155	145	155	157
	β_2	44	75	66	18	53	20	34	34	37	40	37	36
	γ_1	17	12	12	11	14	12	12	17	30	20	31	20
						$m_1 = m_2 = 40\%$							
	β_1	358	124	134	125	307	166	213	218	237	220	239	224
	β_2	129	199	182	21	129	33	66	67	77	147	73	71
	γ_1	34	13	13	12	17	12	12	28	39	17	41	16
						$m_1 = m_2 = 60\%$							
	β_1	1415	143	181	153	497	262	357	366	398	472	418	374
	β_2	628	349	401	30	285	82	140	143	188	484	165	149
	γ_1	109	13	13	13	20	14	13	55	66	16	69	15

Table S14. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to upper LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	73	208	144	306	3	264	28	35	33	80	41	31
	β_2	12	80	65	4	112	34	3	6	26	82	7	3
	γ_1	1	1	1	1	1	1	2	1	2	2	2	5
						$m_1 = m_2 = 40\%$							
	β_1	97	70	595	641	232	468	20	1	250	388	75	8
	β_2	40	234	313	13	150	101	7	4	41	315	30	6
	γ_1	1	1	1	0	6	1	1	7	4	2	4	6
						$m_1 = m_2 = 60\%$							
	β_1	1063	604	1431	1108	447	608	38	84	410	758	44	25
	β_2	96	800	756	22	113	227	14	16	93	667	56	13
	γ_1	41	2	2	1	4	0	1	4	2	6	3	2
						$m_1 = m_2 = 20\%$							
100	β_1	25	206	143	307	5	263	26	20	59	121	17	21
	β_2	5	85	58	3	112	44	4	4	31	88	1	5
	γ_1	2	1	0	0	1	0	1	3	2	2	3	2
						$m_1 = m_2 = 40\%$							
	β_1	1	47	560	613	3	435	27	29	103	391	6	30
	β_2	13	224	299	2	168	119	5	5	48	332	3	6
	γ_1	0	0	0	0	2	0	1	2	3	2	3	1
						$m_1 = m_2 = 60\%$							
	β_1	95	599	1416	1096	42	608	4	2	31	707	25	7
	β_2	15	754	710	2	189	246	7	4	56	668	40	3
	γ_1	3	0	0	1	0	0	1	1	18	4	18	0

Table S15. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to upper LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSEs.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	1725	218	368	548	1822	508	1348	1379	1391	1137	1499	1397
	β_2	100	40	49	41	118	40	75	79	86	76	86	82
	γ_1	34	21	21	22	29	22	23	33	75	66	79	70
							$m_1 = m_2 = 40\%$						
	β_1	>9999	401	1272	1352	7825	1007	5835	5936	5433	2490	6492	6034
	β_2	323	127	183	51	260	55	154	160	180	191	181	169
	γ_1	79	24	24	23	36	24	24	59	96	88	101	91
							$m_1 = m_2 = 60\%$						
	β_1	>9999	2073	6004	3916	>9999	2169	>9999	>9999	>9999	3959	>9999	>9999
	β_2	6264	932	811	73	557	104	359	388	404	524	439	389
	γ_1	946	30	29	26	41	27	28	119	158	108	171	115
							$m_1 = m_2 = 20\%$						
100	β_1	838	135	209	345	868	307	674	689	669	584	708	689
	β_2	48	22	24	19	62	20	39	40	43	44	41	41
	γ_1	17	11	11	11	15	11	11	16	34	32	35	33
							$m_1 = m_2 = 40\%$						
	β_1	4480	215	800	887	3753	620	2740	2752	2502	1231	2893	2759
	β_2	122	84	129	24	151	35	76	77	92	156	80	78
	γ_1	31	12	13	12	17	13	13	27	45	42	46	42
							$m_1 = m_2 = 60\%$						
	β_1	>9999	1221	3998	2581	>9999	1252	>9999	>9999	>9999	1834	>9999	>9999
	β_2	600	703	614	33	342	86	171	176	218	481	199	179
	γ_1	112	15	14	13	20	14	14	53	62	49	65	50

Table S16. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to interval LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	79	253	64	174	23	139	65	67	30	30	83	60
	β_2	4	80	97	1	134	29	1	1	90	35	2	6
	γ_1	1	0	1	0	1	0	1	1	3	2	6	5
							$m_1 = m_2 = 40\%$						
	β_1	224	106	332	296	78	190	16	24	44	27	78	76
	β_2	9	204	394	3	152	84	5	8	100	102	30	5
	γ_1	6	1	0	0	0	0	3	6	0	7	3	6
							$m_1 = m_2 = 60\%$						
	β_1	384	152	769	405	250	173	137	116	228	3	98	2
	β_2	43	1055	1109	2	97	168	13	1	54	186	67	5
	γ_1	4	0	0	0	2	0	3	1	5	8	0	9
							$m_1 = m_2 = 20\%$						
100	β_1	1	249	67	176	42	139	10	9	19	16	13	5
	β_2	9	86	91	4	142	36	4	4	104	43	2	7
	γ_1	4	1	1	1	1	1	2	5	1	2	0	1
							$m_1 = m_2 = 40\%$						
	β_1	24	103	326	288	1	180	33	34	13	7	21	30
	β_2	3	199	388	6	240	96	9	9	186	108	32	6
	γ_1	3	0	0	1	2	0	1	8	1	2	3	1
							$m_1 = m_2 = 60\%$						
	β_1	225	154	758	394	105	161	5	4	110	3	0	0
	β_2	6	1046	1103	3	132	176	17	17	83	194	26	16
	γ_1	6	1	0	0	2	0	1	12	1	3	1	2

Table S17. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1, 2$ is subjected to interval LOD in the linear model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSEs.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	1386	230	303	389	1385	366	1344	1353	1373	518	1411	1169
	β_2	142	46	61	41	163	41	111	113	147	52	121	111
	γ_1	34	23	22	21	32	22	32	34	96	59	97	64
							$m_1 = m_2 = 40\%$						
	β_1	8135	267	660	631	5662	516	5404	5417	5391	694	5857	4008
	β_2	545	119	253	52	456	56	274	282	419	70	335	282
	γ_1	83	24	24	23	38	24	36	72	141	69	148	75
							$m_1 = m_2 = 60\%$						
	β_1	>9999	578	1895	1046	>9999	711	>9999	>9999	>9999	890	>9999	>9999
	β_2	6883	1411	1501	66	1512	83	1036	1094	1421	97	1313	1039
	γ_1	573	27	26	25	43	26	38	153	231	74	252	79
							$m_1 = m_2 = 20\%$						
100	β_1	646	143	159	224	726	201	669	670	678	236	650	529
	β_2	57	24	30	19	83	20	49	49	73	24	51	48
	γ_1	16	12	11	11	16	11	16	15	42	28	42	30
							$m_1 = m_2 = 40\%$						
	β_1	3087	151	407	375	2958	291	2695	2710	2710	304	2781	1969
	β_2	225	73	194	22	275	30	129	135	236	36	141	125
	γ_1	31	12	12	12	19	12	18	28	62	32	62	33
							$m_1 = m_2 = 60\%$						
	β_1	>9999	328	1277	605	>9999	369	>9999	>9999	>9999	377	>9999	9746
	β_2	1461	1225	1339	32	872	58	449	468	806	67	553	438
	γ_1	105	14	13	13	23	13	19	61	98	35	100	35

Table S18. Summary of the AAB (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to lower LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	186	69	94	35	52	76	161	204	134	104	278	263
	β_2	116	241	246	31	3	37	91	135	80	17	235	202
	γ_1	121	23	36	57	35	42	96	144	163	138	211	216
							$m_1 = m_2 = 40\%$						
	β_1	1359	63	109	167	131	273	167	218	27	143	302	274
	β_2	1189	451	436	5	41	166	89	140	14	295	283	208
	γ_1	2218	6	11	30	77	1	75	160	114	107	157	196
							$m_1 = m_2 = 60\%$						
	β_1	70	81	177	248	259	447	136	198	143	549	283	228
	β_2	>9999	602	657	23	143	331	85	155	127	678	338	220
	γ_1	>9999	16	20	7	122	55	31	199	52	67	56	145
							$m_1 = m_2 = 20\%$						
100	β_1	109	32	52	77	76	120	79	97	44	25	146	134
	β_2	49	195	207	8	45	73	34	54	33	77	104	76
	γ_1	39	26	13	6	83	11	28	46	52	41	77	79
							$m_1 = m_2 = 40\%$						
	β_1	173	22	62	199	148	305	82	101	23	201	174	136
	β_2	93	386	375	32	96	214	17	42	116	386	124	68
	γ_1	94	43	38	20	134	56	2	37	29	1	45	53
							$m_1 = m_2 = 60\%$						
	β_1	4625	14	92	297	206	492	50	77	136	603	154	100
	β_2	6030	548	600	62	99	373	60	81	128	710	216	108
	γ_1	4614	65	68	56	164	107	38	31	7	38	5	7

Table S19. Summary of the MSE (×1000) when covariates are independent and $X_{ij}^{*}, j = 1,2$ is subjected to lower LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	1000	475	487	386	567	364	689	770	837	737	1106	1017
	β_2	330	224	215	89	197	77	232	270	279	216	394	354
	γ_1	155	66	67	69	67	65	87	139	269	188	335	250
							$m_1 = m_2 = 40\%$						
	β_1	>9999	491	538	341	804	332	1020	1208	1062	743	1586	1440
	β_2	>9999	484	454	101	418	99	474	561	536	322	839	676
	γ_1	>9999	64	64	66	73	60	84	235	355	142	440	195
							$m_1 = m_2 = 60\%$						
	β_1	>9999	518	659	343	1425	371	2105	2270	1732	719	3099	2793
	β_2	>9999	820	949	143	834	179	1172	1534	1100	594	2306	1635
	γ_1	>9999	63	63	63	86	59	74	501	685	107	836	138
							$m_1 = m_2 = 20\%$						
100	β_1	365	174	180	147	235	147	250	264	271	244	332	309
	β_2	112	103	105	41	74	39	77	83	85	74	110	98
	γ_1	52	25	25	26	32	25	29	44	71	53	80	62
							$m_1 = m_2 = 40\%$						
	β_1	932	178	195	157	326	192	349	372	353	267	480	413
	β_2	420	256	241	45	161	75	151	164	184	217	237	184
	γ_1	144	27	27	25	46	26	27	64	106	39	119	46
							$m_1 = m_2 = 60\%$						
	β_1	>9999	204	252	193	512	307	571	631	528	492	748	663
	β_2	>9999	487	567	63	373	166	392	431	414	546	638	455
	γ_1	>9999	29	29	28	56	34	28	128	207	33	219	33

Table S20. Summary of the AAB (×1000) when covariates are independent and $X_{ij}^*, j = 1,2$ is subjected to upper LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	169	163	200	363	52	304	153	188	77	35	278	204
	β_2	119	33	101	44	27	22	99	136	132	81	222	198
	γ_1	125	50	50	50	73	40	96	139	183	148	238	216
							$m_1 = m_2 = 40\%$						
	β_1	1689	31	535	590	90	386	180	207	1	333	385	218
	β_2	788	247	317	34	123	56	78	116	21	182	204	170
	γ_1	762	13	6	19	112	6	60	124	165	110	219	190
							$m_1 = m_2 = 60\%$						
	β_1	6320	429	1213	1003	131	496	119	38	151	650	57	70
	β_2	>9999	791	732	31	187	193	70	122	81	592	222	173
	γ_1	>9999	87	67	27	157	74	7	107	152	61	200	138
							$m_1 = m_2 = 20\%$						
100	β_1	99	219	131	298	103	234	66	86	6	31	124	92
	β_2	68	54	75	19	22	2	49	67	60	12	111	94
	γ_1	36	2	2	1	125	12	18	38	78	43	101	72
							$m_1 = m_2 = 40\%$						
	β_1	146	25	443	518	263	306	9	3	170	403	77	6
	β_2	145	209	273	6	71	82	30	44	3	223	109	74
	γ_1	87	41	49	34	168	62	16	16	37	3	61	40
							$m_1 = m_2 = 60\%$						
	β_1	1542	310	1028	827	404	334	141	139	233	680	241	144
	β_2	2456	671	617	15	137	235	26	42	84	609	112	67
	γ_1	2928	134	116	77	194	125	62	2	27	49	45	11

Table S21. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to upper LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. MSEs less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSE.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	3988	380	766	1100	2639	983	3025	3432	3521	2806	4634	3947
	β_2	254	73	98	82	165	80	183	206	242	193	311	273
	γ_1	153	70	69	69	77	66	90	134	288	248	363	318
							$m_1 = m_2 = 40\%$						
	β_1	>9999	726	1971	2176	9205	1581	>9999	>9999	>9999	5519	>9999	>9999
	β_2	>9999	185	242	88	351	81	325	380	407	256	544	461
	γ_1	>9999	66	66	65	91	61	81	199	390	264	479	353
							$m_1 = m_2 = 60\%$						
	β_1	>9999	2665	7569	5371	>9999	2703	>9999	>9999	>9999	6648	>9999	>9999
	β_2	>9999	1116	929	110	723	118	676	906	880	514	1178	949
	γ_1	>9999	81	78	74	106	70	84	387	563	277	661	360
							$m_1 = m_2 = 20\%$						
100	β_1	1416	179	291	460	1087	387	1141	1196	1194	1007	1438	1307
	β_2	102	34	45	36	77	36	74	80	89	74	102	93
	γ_1	45	26	26	26	45	25	29	40	89	78	101	86
							$m_1 = m_2 = 40\%$						
	β_1	8918	300	881	990	4243	631	4222	4547	3983	1951	5364	4721
	β_2	312	100	139	39	153	44	122	130	150	134	169	139
	γ_1	123	28	29	27	57	29	28	59	111	85	121	97
							$m_1 = m_2 = 60\%$						
	β_1	>9999	1149	3572	2512	>9999	1102	>9999	>9999	>9999	2321	>9999	>9999
	β_2	>9999	626	526	44	336	90	221	238	323	427	314	250
	γ_1	>9999	45	41	32	69	39	32	105	193	96	206	107

Table S22. Summary of the AAB (×1000) when covariates are independent and $X_{ij}^*, j = 1, 2$ is subjected to interval LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller AAB.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
						$m_1 = m_2 = 20\%$							
50	β_1	176	212	125	243	83	194	54	94	35	132	181	219
	β_2	137	48	123	37	23	12	24	67	106	124	154	202
	γ_1	132	31	46	52	83	42	2	57	80	154	136	211
						$m_1 = m_2 = 40\%$							
	β_1	2935	106	335	322	238	194	46	31	165	75	42	61
	β_2	2391	189	389	19	86	54	14	29	35	32	145	171
	γ_1	1563	13	22	31	127	8	61	18	54	116	109	173
						$m_1 = m_2 = 60\%$							
	β_1	>9999	83	682	364	409	120	139	152	356	38	145	63
	β_2	5949	920	1029	1	257	151	34	7	134	94	126	120
	γ_1	>9999	30	16	5	155	33	102	23	24	70	75	134
						$m_1 = m_2 = 20\%$							
100	β_1	80	258	72	193	137	142	36	22	66	54	37	86
	β_2	52	68	100	19	44	8	46	27	24	41	29	76
	γ_1	38	15	2	4	133	6	73	35	17	48	8	68
						$m_1 = m_2 = 40\%$							
	β_1	151	147	271	260	300	126	117	91	245	25	49	21
	β_2	127	148	345	3	100	82	81	61	37	43	1	53
	γ_1	95	42	33	21	182	45	130	75	56	10	35	30
						$m_1 = m_2 = 60\%$							
	β_1	2441	20	582	282	361	16	180	153	324	107	74	77
	β_2	848	827	940	39	178	192	125	107	120	176	19	12
	γ_1	4481	84	71	49	201	89	164	79	72	45	49	32

Table S23. Summary of the MSE (×1000) when covariates are independent and $X^*_{ij}, j = 1,2$ is subjected to interval LOD in the Cox PH model. M1 is complete case analysis; M2–M4 are the different variants of the substitution methods; M5–M6 are the different variants of the MI methods; M7–M8 are the different variants of the MDI methods; M9–M12 are the different variants of MDI embedded MI (MI + MDI) methods. Absolute biases less than 0.1 are highlighted in gray, with darker tones corresponding to smaller MSEs.

			Substitution			MI		MDI		MI + MDI			
n		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
							$m_1 = m_2 = 20\%$						
50	β_1	3486	345	599	809	2098	733	2272	2589	2738	1405	3355	3279
	β_2	320	85	115	82	184	79	182	212	277	153	318	334
	γ_1	161	67	69	68	76	65	78	102	288	244	353	309
							$m_1 = m_2 = 40\%$						
	β_1	>9999	495	1186	1220	6587	952	8054	9155	8543	1646	>9999	>9999
	β_2	>9999	173	322	84	491	81	466	532	630	135	741	751
	γ_1	>9999	66	67	67	85	63	77	175	373	230	449	284
							$m_1 = m_2 = 60\%$						
	β_1	>9999	932	2715	1743	>9999	1097	>9999	>9999	>9999	1798	>9999	>9999
	β_2	>9999	1297	1504	103	1598	103	1539	1789	2110	138	2536	2301
	γ_1	>9999	67	67	68	94	63	81	354	643	219	751	282
							$m_1 = m_2 = 20\%$						
100	β_1	1109	178	231	328	822	280	843	880	929	423	1046	1022
	β_2	117	41	55	38	92	37	77	83	103	51	100	104
	γ_1	45	26	26	26	45	25	32	34	82	74	88	80
							$m_1 = m_2 = 40\%$						
	β_1	6299	208	492	483	2940	340	2880	3130	3146	469	3760	3323
	β_2	489	78	193	41	235	44	181	194	255	52	235	227
	γ_1	133	27	27	26	60	26	42	56	123	71	128	77
							$m_1 = m_2 = 60\%$						
	β_1	>9999	377	1236	674	>9999	392	>9999	>9999	>9999	600	>9999	>9999
	β_2	>9999	877	1077	48	752	74	553	592	834	77	776	680
	γ_1	>9999	32	30	27	68	30	53	105	230	64	243	67