



Article Density and Mass Function for Regular Rotating Electrically Charged Compact Objects Determined by Nonlinear Electrodynamics Minimally Coupled to Gravity

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Abstract: We address the question of the electromagnetic density and the mass function for regular rotating electrically charged compact objects as determined by dynamical equations of nonlinear electrodynamics minimally coupled to gravity. The rotating electrically charged compact objects are described by axially symmetric geometry, in which their electromagnetic fields are governed by four source-free equations for two independent field components of the electromagnetic tensor $F_{\mu\nu}$, with two constraints on the integration functions. An additional condition of compatibility of four dynamical equations for two independent field functions imposes the constraint on the Lagrange derivative $\mathcal{L}_F = d\mathcal{L}/dF$, directly related to the electromagnetic density. As a result, the compatibility condition determines uniquely the generic form of the electromagnetic density and the mass function for regular rotating electrically charged compact objects.

Keywords: mass function; regular electrically charged rotating black hole; electromagnetic spinning soliton



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1. Introduction

Axially symmetric metrics, which describe regular rotating compact objects, are most frequently constructed from spherical metrics by applying the Newman–Janis complex coordinate translation [1]

$$(r,t) \to (r,u); \ u = t - \int \frac{dr}{g(r)}; \ r \to r + ia\cos\theta; \ u \to u = ia\cos\theta.$$
 (1)

The Newman–Janis algorithm was proposed in 1965, and commented by authors as "There is no clear reason for this operation to yield a new solution..." [1]. However, the new solution was obtained in the same 1965 year, called the Kerr–Newman solution [2]

$$ds^{2} = \frac{(2mr - q^{2}) - \Sigma}{\Sigma} dt^{2} + \frac{\Sigma}{\Delta} dr^{2} + \Sigma d\theta^{2} - \frac{2a(2mr - q^{2})\sin^{2}\theta}{\Sigma} dt d\phi$$
$$+ \left(r^{2} + a^{2} + \frac{(2mr - q^{2})a^{2}\sin^{2}\theta}{\Sigma}\right)\sin^{2}\theta d\phi^{2};$$
(2)

$$\Sigma = r^2 + a^2 \cos^2 \theta; \ \Delta = r^2 - 2mr + a^2 + q^2.$$
(3)

It originates from the Reissner–Nordström solution and describes a rotating body with the specific angular momentum *a*, mass *m*, charge *q*, and the electromagnetic potential [2]

$$A_i = -(q/r)\Sigma[1;0,0,-a\sin^2\theta].$$
 (4)

Soon later Carter found that coupling of the parameter *a* with the mass *m* in the angular momentum J = ma, and with the charge *q* in an asymptotic magnetic momentum $\mu = qa$,

results in the same gyromagnetic ratio q/m, as measured by a distant observer, as required by the Dirac equation for a spinning particle [3].

In addition, Carter discovered the global causality violation in the spacetime of a charged spinning structure without horizons ($r_{\pm} = m \pm \sqrt{m^2 - (a^2 + q^2)}$) when $a^2 + q^2 > m^2$. In this case, $g_{\phi\phi} < 0$ for $2mr < q^2$, which leads to the existence of closed time-like curves in the interior region $r < q^2/2m$, which can be extended over the whole manifold [3].

The clear reason why the Newman–Janis operation yields a new solution was found by Gürses and Gürsey [4]. They developed the general formalism for the transition from a spherical to an axially symmetric solution and have shown that the Newman–Janis algorithm works for spherical metrics of the Kerr–Schild class, written below in the geometrical units c = G = 1 [5]

$$ds^{2} = g(r)dt^{2} - \frac{dr^{2}}{g(r)} - r^{2}d\Omega^{2}; \ g(r) = 1 - \frac{2\mathcal{M}(r)}{r}; \ \mathcal{M}(r) = 4\pi \int_{0}^{r} \rho(x)x^{2}dx$$
(5)

where $\rho(r)$ is the density and $\mathcal{M}(r)$ is the mass function.

In the spherically symmetric case, the electromagnetic stress-energy tensors, which generate the Kerr–Schild metrics (5), have the algebraic structure [6]

$$T_0^0 = T_1^1 (6)$$

where $T_0^0 = T_t^t = \rho$ and $T_1^1 = T_r^r = -p_r$, so that the radial pressure p_r and the density ρ are related by the equation of state $p_r = -\rho$, which is the basic generic feature of the spherical solutions of the Kerr–Schild class [6].

The Kerr–Schild metrics belong to the special class of algebraically degenerated solutions to the Einstein equations, which have the linear form. The stress-energy tensor satisfies $\partial_{\mu}T_{\nu}^{\mu} = 0$ [4], which yields the coordinate-dependent equation of state for the transversal pressure, $p_{\perp} = -\rho - r\rho'/2$, where $p_{\perp} = -T_{\theta}^{\theta} = -T_{\phi}^{\theta}$.

The electromagnetic field behaves thus as an intrinsically anisotropic medium due to the generic algebraic structure of its stress-energy tensor. In the axially symmetric spacetime, the relation (6) is not valid, but the equation of state for the radial pressure, $p_r = -\rho$ is obligatory, and the relation between p_{\perp} and ρ is described by the more complicated coordinate-dependent equation of state, as we shall see below.

Anisotropy exhibited by the structure of stress-energy tensors has been revealed for collisionless N-body systems of neutral particles when they behave as non-ideal fluids and can be applied for a wide class of astrophysical matter sources (for a recent review [7]). An intrinsically-relativistic kinetic mechanism for the generation of non-isotropic relativistic kinetic equilibria has been reported in [8]. In Schwarzschild geometry, a non-ideal fluid is characterized by a temperature anisotropy carried by the tangential component $T^{\phi\phi}$ of its stress-energy tensor, which determines the transversal pressure [7].

In Kerr geometry, a non-ideal fluid formed by relativistic collisionless neutral particles can generate non-isotropic equilibrium configurations presented by stress-energy tensors, which exhibit pressure and temperature anisotropies [9]. The structure of stress-energy tensors differs substantially from the spherical case due to the existence of the Killing tensor, characterized by the Carter invariant *K*. The non-isotropic character of $T\mu\nu$ appears as the direct unique consequence of the dependence of the kinetic distribution function on the invariant *K* for the relativistic equilibrium solution in the Kerr metric. In the paper [7], two kinds of deviations from the ideal fluid are revealed : (i) all the diagonal terms of the tensor $T^{\mu\nu}$ are different from each other, and (ii) $T^{\mu\nu}$ has also non-zero off diagonal components.

Geometry of rotating compact objects is described by the Gürses–Gürsey metric, which has, in geometrical units c = G = 1, the form [4]

$$ds^{2} = \frac{2f - \Sigma}{\Sigma} dt^{2} + \frac{\Sigma}{\Delta} dr^{2} + \Sigma d\theta^{2} - \frac{4af\sin^{2}\theta}{\Sigma} dt d\phi + \left(r^{2} + a^{2} + \frac{2fa^{2}\sin^{2}\theta}{\Sigma}\right)\sin^{2}\theta d\phi^{2}$$
(7)

$$\Sigma = r^2 + a^2 \cos^2 \theta; \ \Delta = r^2 + a^2 - 2f(r); \ f(r) = r\mathcal{M}(r).$$
(8)

The Boyer–Lindquist coordinates r, θ, ϕ , are connected with the Cartesian coordinates by $x^2 + y^2 = (r^2 + a^2) \sin^2 \theta$; $z = r \cos \theta$. The signature of the metric is [-+++]. The mass function $\mathcal{M}(r)$ comes from the original spherical solution of the Kerr–Schild class (5). For regular solutions $\mathcal{M}(r)$ increases monotonically from $\mathcal{M}(r) = (4\pi/3)\rho(0)r^3 \rightarrow 0$ as $r \rightarrow 0$ to the Reissner–Nordström mass function $\mathcal{M}(r) = m - q^2/2r \rightarrow m$ as $r \rightarrow \infty$, which ensures the causal safety on the whole spacetime manifold of a spinning structure without horizon since $g_{\phi\phi} > 0$ everywhere due to $f(r) \ge 0$ [6].

In the axially symmetric spacetime, the surfaces r = constant are the confocal ellipsoids

$$r^{4} - (x^{2} + y^{2} + z^{2} - a^{2})r^{2} - a^{2}z^{2} = 0$$
(9)

which at r = 0 reduces to the equatorial disk

$$x^2 + y^2 \le a^2, \ z = 0 \tag{10}$$

confined by the ring [10]

$$x^2 + y^2 = a^2, \ z = 0 \tag{11}$$

which in the Kerr–Newman geometry comprises its ring singularity [2].

Horizons in the spacetime with the metric (7) are defined by the equation $\Delta(r) = 0$ which gives

$$r_{+,-} = \mathcal{M}(r) \pm \sqrt{\mathcal{M}^2 - a^2}; \ r_{\pm} = \mathcal{M}(r_{\pm})$$
 (12)

where r_+ is the event horizon, $r_- < r_+$ is the internal horizon, and r_\pm is the double horizon of the extreme black hole with $a = a_{dh}$. Normalizing distances to *GM*, the mass function to the total mass *m*, and the spherical electromagnetic density $\tilde{\rho}(r)$ to the scale of the interior de Sitter vacuum $\tilde{\rho}_0$, we introduce the characteristic parameter $x_g = r_g/r_0$, where $r_g = 2Gm$ is the Schwarzschild radius and $r_0 = \sqrt{\frac{3}{8\pi G\tilde{\rho}_0}}$ is the de Sitter radius, so that the characteristic parameter x_g depends on both the mass of an object and on the density of its interior de Sitter vacuum. As follows from $\Delta = r^2 + a^2 - 2r\mathcal{M}(r) = 0$, with taking into account (12), $a = a_{dh} = \mathcal{M}(r_{\pm}) = r_{\pm}$. As a result, for each value of the parameter x_g , there exists the value of the angular momentum a_{dh} characterizing the double horizon, which presents the boundary between black holes and solitons. The scale of the interior de Sitter vacuum is fixed as a fundamental scale for a limiting density ([11,12] and references therein), and the relation of a_{dh} with x_g gives its relation with the mass of an object. In Figure 1, we show the dependence of horizons on the parameter x_g for several values of a(Left), and on the angular momentum *a* for several values of x_g (Right) [13].



Figure 1. Horizons dependently on x_g for given *a* (Left) and on *a* for given x_g (Right).

Regular axially symmetric solutions have been typically obtained by choosing an attractive spherical mass function (the favored choice [14–17]), applying the Newman–Janis algorithm, and calculating the electromagnetic field from a modified potential (4).

Regular spherical models for electrically and magnetically charged black holes and magnetic monopoles have been presented in [18–30].

In regular solutions for rotating charged black holes and soliton-like objects [31–42], geometrical singularity is successfully removed by applying regular spherical mass functions in the Newman–Janis algorithm, but there remains inconsistency concerning the Lagrange dynamics directly related to the behavior of the electromagnetic field [37], calculated from the electromagnetic potential "postulated as an ad hoc result" in the Newman–Janis algorithm [43]. As a result, one obtains an approximate solution for electromagnetic tensor $F_{\mu\nu}$, which does not satisfy the whole system of the field equations [37].

For the electrically charged spherical objects, the problem with the Lagrange dynamics was posed in [22] as the existential problem with the conclusion of the non-existence of regular solutions with the non-zero electric charge, due to non-Maxwellian behavior of fields in their centers. Detailed analysis of this case has shown that regular spherical solutions for electrically charged objects exist and have the obligatory de Sitter centers [23] required by the algebraic structure of their electromagnetic stress-energy tensors (6). Regularity leads to the vanishing of the field unvariant *F* at the de Sitter center and at infinity. The non-monotonic behavior of the invariant *F* inevitably leads to a branching of the Lagrange density $\mathcal{L}(F)$ [44,45].

Regular models of a soliton-like spinning object go back to the early models of the electron as an extended particle. The basic necessary condition for its existence required introducing an additional non-electromagnetic cohesive force to balance the Coulomb repulsion [46–48]. Analysis of physical reasons for the models of the additional force [49] led to the appearance of the Dirac nonlinear electrodynamics [50,51], which admits spherical solutions describing extended charged particles [52]. Generalization of the Dirac nonlinear electrodynamics [53] admits solutions describing spinning charged particles with the behavior typical for solitons: attainability of a particle interior for another particle [53]. Contemporary models based on the spin dynamics have been developed in [54,55] (for a review [56]).

Independently, nonlinear electrodynamics was proposed by Born and Infeld, as motivated by the fundamental aim to describe electromagnetic field and particles in the frame of one physical entity, which is the electromagnetic field. Another aim appealed to the principle of finiteness: "a satisfactory theory should avoid letting physical quantities become infinite" [57]. In quantum electrodynamics, based on the conception of a point charge, infinities originate from its infinite self-energy. Born and Infeld obtained finite total energy for the field around a point charge by imposing an upper limit on the electric field strength, but geometry remained singular [57] (Later, it was shown that nonlinear electrodynamics theories appear as the low-energy effective limits in certain class of string/M-theories [58–60]).

Both the aims, formulated by Born and Infeld, can be achieved in nonlinear electrodynamics minimally coupled to gravity (NED-GR), which describes, in a self-consistent way and without additional assumptions concerning coupling between electromagnetic field and gravity, regular electrically charged black holes and electromagnetic solitons replacing naked singularities. Their electromagnetic fields are governed by the source-free NED-GR equations for the electromagnetic tensor $F_{\mu\nu}$, while their stress-energy tensors for these fields generate the gravitational fields as the source terms in the Einstein equations [6,23,61].

Electromagnetic spinning solitons, which are lumps of a nonlinear electromagnetic field, related by electromagnetic and gravitational interaction, are qualified as physical solitons in the spirit of the Coleman lumps [62] as non-singular non-dissipative particle-like objects, keeping themselves together by their own self-interaction.

For rotating regular electrically charged objects, the axial symmetry of geometry (7) transforms the de Sitter center r = 0 into the de Sitter equatorial disk (10), which

is the fundamental generic ingredient of regular rotating electrically charged NED-GR objects, uniquely determined by the algebraic structure of electromagnetic stress-energy tensors [6,61]. As a result, their electromagnetic mass, $m = 4\pi \int_0^\infty \tilde{\rho}_{em}(r)r^2dr$, is intrinsically related to gravity and breaking of spacetime symmetry from the de Sitter group [23], characteristic for all regular objects with the de Sitter interiors [63,64] (for a review [11]). The generic relation of the mass of electromagnetic spinning solitons [65] with the breaking of spacetime symmetry suggests the existence of the relation of the Higgs mechanism with gravity and spacetime symmetry [66] due to intrinsic involvement of the de Sitter vacuum as the false vacuum state $p = -\rho$ of Higgs scalar fields, which supply fermions with masses via their spontaneous symmetry breaking from a false vacuum state [67–69] (for a review [70]).

In nonlinear electrodynamics, minimally coupled to gravity, the electromagnetic field of regular rotating electrically charged objects is described by four dynamical equations for two independent components of the electromagnetic tensor $F_{\mu\nu}$, restricted by the necessary and sufficient condition of their compatibility, imposed on the Lagrange derivative $\mathcal{L}_F = d\mathcal{L}dF$, and by two constraints imposed on the integration functions in general solutions [71]. As a result, three independent dynamical equations uniquely define three basic field functions: two independent components of $F_{\mu\nu}$ and the Lagrange derivative \mathcal{L}_F , which is directly related with the spherical electromagnetic density $\tilde{\rho}(r)$ and the mass function $\mathcal{M}(r)$.

General solutions for two independent field components have been found in our recent paper [71]. The aim of this paper is to show that NED-GR determines uniquely the electromagnetic density $\tilde{\rho}(r)$ and the mass function $\mathcal{M}(r)$, and to obtain the electromagnetic density and mass function from the compatibility condition.

In Section 2, we present the basic NED-GR equations and the general solutions for the electromagnetic tensor $F_{\mu\nu}$, needed for derivation and further analysis of the electromagnetic density. In Section 3, we apply the general solutions in the compatibility condition and obtain the electromagnetic density $\tilde{\rho}(r)$ and the mass function $\mathcal{M}(r)$. Section 4 contains conclusions.

2. Basic Equations

In the axially symmetric geometry, non-zero field components F_{01} , F_{02} , F_{13} , F_{23} are related by

$$F_{31} = a \sin^2 \theta F_{10}; \ aF_{23} = (r^2 + a^2)F_{02}.$$
 (13)

The stress-energy tensor of an electromagnetic field $F_{\mu\nu}$, calculated in the standard way [72,73], has the general form [6]

$$8\pi T^{\mu}_{\nu} = 2\mathcal{L}_F F_{\nu\alpha} F^{\mu\alpha} - \frac{1}{2} \delta^{\mu}_{\nu} \mathcal{L}$$
⁽¹⁴⁾

where \mathcal{L} is the Lagrangian, and \mathcal{L}_F is its derivative, $\mathcal{L}_F = d\mathcal{L}/dF$.

The condition (6) is not satisfied in the axially symmetric geometry where [71]

$$8\pi T_0^0 = -\frac{2\mathcal{L}_F}{\Sigma} \Big[(r^2 + a^2)F_{10}^2 + F_{20}^2 \Big] - \frac{\mathcal{L}}{2}; \quad 8\pi T_1^1 = -2\mathcal{L}_F F_{10}^2 - \frac{\mathcal{L}}{2}.$$
(15)

However, the equation of state $p_r = -\rho$ remains valid and characterizes the algebraic structure of stress-energy tensors of electromagnetic fields.

The stress-energy tensors, which generate geometry (7), can be presented as [4]

$$T_{\mu\nu} = (\rho + p_{\perp})(u_{\mu}u_{\nu} - l_{\mu}l_{\nu}) + p_{\perp}g_{\mu\nu}$$
(16)

in the orthonormal basis of the tangent space at each point, presented by the tetrad $[u^{\mu}, l^{\mu}, n^{\mu}, m^{\mu}]$, directly related to the spacetime metric [72]. For the metric (7) it is given by [4]

$$u^{\mu} = \frac{1}{\sqrt{\pm\Delta\Sigma}} [(r^{2} + a^{2})\delta_{0}^{\mu} + a\delta_{3}^{\mu}]; \quad l^{\mu} = \sqrt{\frac{\pm\Delta}{\Sigma}}\delta_{1}^{\mu};$$
$$n^{\mu} = \frac{1}{\sqrt{\Sigma}}\delta_{2}^{\mu}; \quad m^{\mu} = \frac{-1}{\sqrt{\Sigma}\sin\theta} [a\sin^{2}\theta\delta_{0}^{\mu} + \delta_{3}^{\mu}].$$
(17)

For black holes, the sign plus refers to the regions outside the event horizon and inside the Cauchy horizon, where the vector u^{μ} is time-like. The sign minus refers to the regions between the horizons where the vector l^{μ} is time-like. The vectors m^{μ} and n^{μ} are space-like everywhere. In the geometry without horizons, the vector u^{μ} is time-like.

The eigenvalues of the stress-energy tensor (16) in the co-moving references frame, rotating with the angular velocity $\omega(r) = u^{\phi}/u^t = a/(r^2 + a^2)$, define the density ρ , and the principal pressures p_r and p_{\perp} by

$$T_{\mu\nu}u^{\mu}u^{\nu} = \rho(r,\theta); \ T_{\mu\nu}l^{\mu}l^{\nu} = p_{r} = -\rho; \ T_{\mu\nu}n^{\mu}n^{\nu} = T_{\mu\nu}m^{\mu}m^{\nu} = p_{\perp}(r,\theta).$$
(18)

The electromagnetic density and pressures obtained as the eigenvalues of the electromagnetic stress-energy tensor (14) are given by [71]

$$8\pi\rho = \frac{1}{2}\mathcal{L} + 2\mathcal{L}_F F_{10}^2; \quad p_r = T_1^1 = -\rho; \quad 8\pi p_\perp = -\frac{1}{2}\mathcal{L} + 2\mathcal{L}_F \frac{F_{20}^2}{a^2 \sin^2 \theta}.$$
 (19)

Nonlinear electromagnetic field represents thus an anisotropic continuous medium.

Equation (19) yields the basic relation [6]

$$4\pi(p_{\perp}+\rho) = \mathcal{L}_F\left(F_{10}^2 + \frac{F_{20}^2}{a^2\sin^2\theta}\right).$$
 (20)

The weak energy condition (WEC) requires $\rho \ge 0$ and $p_k + \rho \ge 0$ for the principal pressures [74]. In the Equation (20) this depends on the sign of the Lagrange derivative \mathcal{L}_F .

The vectors of the electromagnetic field strengths $\mathbf{E} = \{F_{\beta 0}\}$, $\mathbf{H} = \{\mathcal{L}_F^* F_{0\beta}\}$ and the vectors of the electric and magnetic induction $\mathbf{D} = \{\mathcal{L}_F F^{0\beta}\}$; $\mathbf{B} = \{^*F^{\beta 0}\}$ are related by $D^{\alpha} = \epsilon^{\alpha}_{\beta} E^{\beta}$; $B^{\alpha} = \mu^{\alpha}_{\beta} H^{\beta}$, where $\epsilon^{\beta}_{\alpha}$ and μ^{β}_{α} are the tensors of the dielectric and magnetic permeability. Their independent eigenvalues are given by [6]

$$\epsilon_r^r = \frac{(r^2 + a^2)}{\Delta} \mathcal{L}_F, \ \epsilon_\theta^\theta = \mathcal{L}_F; \ \mu_r^r = \frac{(r^2 + a^2)}{\Delta \mathcal{L}_F}, \ \mu_\theta^\theta = (\mathcal{L}_F)^{-1}$$
(21)

Violation of WEC in (20) would involve the negative values of the dielectric and magnetic permeability, which is incompatible with the basic requirements of electrodynamics of continuous media [75]). Therefore the NED-GR electrically charged objects always satisfy WEC [71].

In geometry with the metric (7), the eigenvalues of the stress-energy tensor (18) are related with the electromagnetic density $\tilde{\rho}$ and with the master function $f(r) = r\mathcal{M}(r)$ as [76]

$$\rho(r,\theta) = \frac{r^{4}}{\Sigma^{2}}\tilde{\rho}(r) = \frac{2(f'r-f)}{\Sigma^{2}};$$

$$p_{\perp}(r,\theta) = \left(\frac{r^{4}}{\Sigma^{2}} - \frac{2r^{2}}{\Sigma}\right)\tilde{\rho}(r) - \frac{r^{3}}{2\Sigma}\tilde{\rho}'(r) = \frac{2(f'r-f) - f''\Sigma}{\Sigma^{2}}$$
(22)

As follows from (22), in the equatorial plane [6]

$$p_{\perp}(r,\theta) + \rho(r,\theta) = \tilde{p}_{\perp}(r) + \tilde{\rho}(r) = -r\tilde{\rho}'(r)/2.$$
(23)

Regularity requires $r\tilde{\rho}'(r) \rightarrow 0$ as $r \rightarrow 0$ [23]. This gives the equation of state on the disk

$$p_{\perp} = -\rho; \ p_r = -\rho; \ \rightarrow p = -\rho; \ \rho = \tilde{\rho}_0$$
 (24)

which represents the rotating de Sitter vacuum in the co-rotating frame [6].

Simultaneously, the master function f(r) in (7) achieves for $r \rightarrow 0$ the de Sitter asymptotic [23]

$$f(r) = \frac{r^4}{2r_0^2}; \quad r_0^2 = \frac{3}{8\pi G\tilde{\rho}_0}.$$
(25)

The equatorial de Sitter vacuum disk is the basic generic constituent of all regular rotating electrically charged NED-GR compact objects [6,61].

The eigenvalues of the electromagnetic stress-energy tensor (19) define the general form of the Lagrangian as

$$\mathcal{L} = 2\rho - 4\mathcal{L}_F F_{10}^2 \tag{26}$$

On the disk, regularity requires $p_{\perp} + \rho = 0$, by virtue of (24), and Equation (20) gives $F_{10}^2 = F_{20}^2/a^2 = 0$ since $\mathcal{L}_F \neq 0$, and the Equation (26) leads to

$$F_{10}^2 = \frac{2\rho - \mathcal{L}}{4\mathcal{L}_F} = 0.$$
 (27)

For a general form of the Lagrangian \mathcal{L} it is possible if and only if $\mathcal{L}_F \to \infty$, which corresponds to the strong field regime and provides the natural realization on the disk ([71] and references therein) of the underlying hypothesis of non-linearity replacing a singularity [57].

A physical consequence of the fact that $\mathcal{L}_F \to \infty$ on the disk, is that the dielectric and magnetic permeability, given by Equation (21), behave as $\epsilon_r^r = \epsilon_{\theta}^{\theta} = \mathcal{L}_F \to \infty$, $\mu_r^r = \mu_{\theta}^{\theta} = \mu = (\mathcal{L}_F)^{-1} \to 0$, respectively, and the de Sitter vacuum disk displays the behavior of the perfect conductor and the ideal diamagnetic [6].

The field invariant is determined by two independent field components as [6]

$$F = 2\left(\frac{F_{20}^2}{a^2 \sin^2 \theta} - F_{10}^2\right).$$
 (28)

The Lagrangian given by the general formula (26), behaves as $\mathcal{L} \to 2\tilde{\rho}(0)$ at approaching the disk, and at $r \to \infty$ it tends to $\mathcal{L} = 2\rho \to 0$, since $\rho \to 0$ for compact objects with finite mass. The field invariant F = 0 on the disk, by virtue of (20), which requires $F_{10}^2 = F_{20}^2 = 0$. This also concerns the limit $r \to \infty$, where $\rho \to 0$ and $(p_{\perp} + \rho) \to 0$ for compact objects.

The field invariant *F* evolves between two zero values, and its non-monotonic behavior results in branching of a Lagrangian $\mathcal{L}(F)$ on a surface where the invariant *F* achieves its minimum ([71] and references therein).

As a result, the Lagrange dynamics is presented by the non-uniform action [44,45,71]

$$\mathcal{I} = \mathcal{I}_{int} + \mathcal{I}_{ext} = \frac{1}{16\pi} \left[\int_{\Omega_{int}} \left(R - \mathcal{L}_{int}(F) \right) \sqrt{-g} d^4 x + \int_{\Omega_{ext}} \left(R - \mathcal{L}_{ext}(F) \right) \sqrt{-g} d^4 x \right]$$
(29)

where *R* is the scalar curvature and *g* is the determinant of the metric tensor. The electromagnetic Lagrangian $\mathcal{L}(F)$ should have the Maxwell limit in the weak field regime.

Each part of the manifold, Ω_{int} and Ω_{ext} , is confined by the space-like hypersurfaces $t = t_{in}$ and $t = t_{fin}$, and by the time-like 3-hypersurface at infinity, where electromagnetic fields vanish. The internal boundary between Ω_{int} and Ω_{ext} is defined as a time-like hypersurface Σ_c , at which the standard boundary conditions read [44]

$$\int_{\Sigma_c} \left(\mathcal{L}_{F(int)} F_{\mu\nu(int)} - \mathcal{L}_{F(ext)} F_{\mu\nu(ext)} \right) \sqrt{-g} \delta A^{\mu} d\sigma^{\nu} = 0;$$

$$\mathcal{L}_{int} - 2\mathcal{L}_{F(int)}F_{int} = \mathcal{L}_{ext} - 2\mathcal{L}_{F(ext)}F_{ext}.$$
(30)

Variation in the action (29) gives in both regions, Ω_{int} and Ω_{ext} , the source-free equations for the electromagnetic field [6,22]

$$\nabla_{\mu}(\mathcal{L}_F F^{\mu\nu}) = 0; \tag{31}$$

$$\nabla_{\mu}{}^{*}F^{\mu\nu} = 0; \ {}^{*}F^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}; \ \eta^{0123} = -\frac{1}{\sqrt{-g}}$$
(32)

and the Einstein equations $G_{\nu}^{\mu} = -8\pi T_{\nu}^{\mu}$ with the electromagnetic tensor (14) as the source. The field Equations (31) and (32), with taking into account Equation (13), present the

system of four equations for two independent field functions F_{10} , F_{20} [61]

$$\frac{\partial}{\partial r}[(r^2 + a^2)\sin\theta\mathcal{L}_F F_{10}] + \frac{\partial}{\partial\theta}[\sin\theta\mathcal{L}_F F_{20}] = 0;$$
(33)

$$\frac{\partial}{\partial r}[a\sin\theta\mathcal{L}_F F_{10}] + \frac{\partial}{\partial\theta}[\frac{1}{a\sin\theta}\mathcal{L}_F F_{20}] = 0, \tag{34}$$

$$\frac{\partial F_{10}}{\partial \theta} - \frac{\partial F_{20}}{\partial r} = 0, \tag{35}$$

$$\frac{\partial}{\partial \theta} [a^2 \sin^2 \theta F_{10}] - \frac{\partial}{\partial r} [(r^2 + a^2) F_{20}] = 0.$$
(36)

Compatibility of this system has been studied in [61] by resolving it with respect to the derivatives $\partial F_{10}/\partial r$, $\partial F_{20}/\partial r$, $\partial F_{10}/\partial \theta$, $\partial F_{20}/\partial \theta$, with the coefficients depending on F_{10} and F_{20} . Equality of the mixed second derivatives gives the uniform system of two algebraic equations for F_{10} and F_{20} , which has a non-trivial solution if and only if its determinant is equal to zero. This yields the necessary and sufficient compatibility condition [61]

$$\frac{\partial}{\partial r} \left(\frac{1}{\mathcal{L}_F} \frac{\partial \mathcal{L}_F}{\partial \theta} \right) \frac{\partial}{\partial \theta} \left(\frac{1}{\mathcal{L}_F} \frac{\partial \mathcal{L}_F}{\partial r} \right) + \frac{4a^2 \sin^2 \theta}{\Sigma^2} \frac{1}{\mathcal{L}_F^2} \left[r \frac{\partial \mathcal{L}_F}{\partial r} + \cot \theta \frac{\partial \mathcal{L}_F}{\partial \theta} \right]^2 = 0.$$
(37)

The general solution to the system (31) and (32) is given by [71]

$$F_{20}(r,\theta) = \frac{r}{\mathcal{L}_F \Sigma^2} \bigg[\Psi(\theta) + \sin \theta \int \Phi(r) \Sigma(r,\theta) dr \bigg],$$
(38)

$$F_{10}(r,\theta) = \frac{1}{2a^2 \sin^3 \theta \, \mathcal{L}_F \Sigma^2} \bigg[\sin \theta \, \Sigma \Psi'(\theta) - \cos \theta \, [\Sigma - 2a^2 \sin^2 \theta] \Psi(\theta) \bigg] \\ + \frac{\cos \theta}{\mathcal{L}_F \Sigma^2} \bigg[\int \Phi(r) r^2 dr - r^2 \int \Phi(r) dr \bigg].$$
(39)

Two integration functions $\Phi(r)$ and $\Psi(\theta)$ are restricted by two basic constraints, which ensure the satisfaction of the full dynamical system (33)–(36) by solutions (38) and (39).

The 1-st constraint is given by the integro-differential equation [71]

$$\Psi'(\theta) + (\tan\theta - \cot\theta\Psi(\theta) - r\sin\theta\tan\theta\Sigma\Phi(r) + \sin\theta\tan\theta\int\Phi(r)r^2dr$$

$$-a^{2}\sin^{2}\theta\cos\theta\int\Phi(r)dr = -\frac{r\tan\theta}{\mathcal{L}_{F}}\frac{\partial\mathcal{L}_{F}}{\partial r}\bigg[\Psi(\theta) + \sin\theta\int\Phi(r)\Sigma(r,\theta)dr\bigg].$$
 (40)

The 2-nd constraint reads [71]

$$\frac{d}{dr}\left(r^{2}\sin\theta\Phi(r)\right) = D_{1}\Phi(r) + D_{2}\left[\Psi'(\theta) - a^{2}\sin\theta\sin2\theta\int\Phi(r)dr\right] + D_{3}\Psi(\theta) + D_{4}\int\Phi(r)\Sigma(r,\theta)dr$$
(41)

$$D_{1} = \frac{r \sin \theta (a^{2} \cos^{2} \theta - r^{2})}{\mathcal{L}_{F} \Sigma^{2}} \left[r \frac{\partial \mathcal{L}_{F}}{\partial r} + \cot \theta \frac{\partial \mathcal{L}_{F}}{\partial \theta} \right]; \quad D_{2} = \frac{r \cot \theta}{\mathcal{L}_{F} \Sigma} \frac{\partial \mathcal{L}_{F}}{\partial r}; \tag{42}$$

$$D_{3} = \frac{1}{\mathcal{L}_{F}} \frac{\partial \mathcal{L}_{F}}{\partial r} \frac{r}{\Sigma^{2} \sin^{2} \theta} \left(2a^{2} \cos^{2} \theta \sin^{2} \theta - \Sigma \cos 2\theta \right) + \frac{\cot \theta}{\mathcal{L}_{F}} \frac{\partial \mathcal{L}_{F}}{\partial \theta} \frac{\partial \mathcal{L}_{F}}{\Sigma^{2}} + \frac{r}{\mathcal{L}_{F} \Sigma} \left[\cot \theta \left(\frac{\partial}{\partial r} \frac{\partial \mathcal{L}_{F}}{\partial \theta} - \frac{2}{\mathcal{L}_{F}} \frac{\partial \mathcal{L}_{F}}{\partial r} \frac{\partial \mathcal{L}_{F}}{\partial \theta} \right) + r \left(\frac{\partial}{\partial r} \frac{\partial \mathcal{L}_{F}}{\partial r} - \frac{1}{\mathcal{L}_{F}} \left(\frac{\partial \mathcal{L}_{F}}{\partial r} \right)^{2} \right) \right]; \quad (43)$$

$$D_{4} = \frac{r}{\mathcal{L}_{F}\Sigma^{2}} \frac{\partial \mathcal{L}_{F}}{\partial r} (\Sigma + 2a^{2}\cos^{2}\theta) \sin\theta + \frac{\cot\theta}{\mathcal{L}_{F}} \frac{\partial \mathcal{L}_{F}}{\partial \theta} \frac{(a^{2}\cos^{2}\theta - r^{2})\sin\theta}{\Sigma^{2}} + \frac{r\sin\theta}{\mathcal{L}_{F}\Sigma} \left[\cot\theta \left(\frac{\partial}{\partial r}\frac{\partial \mathcal{L}_{F}}{\partial \theta} - \frac{2}{\mathcal{L}_{F}}\frac{\partial \mathcal{L}_{F}}{\partial r}\frac{\partial \mathcal{L}_{F}}{\partial \theta}\right) + r \left(\frac{\partial}{\partial r}\frac{\partial \mathcal{L}_{F}}{\partial r} - \frac{r}{\mathcal{L}_{F}}\left(\frac{\partial \mathcal{L}_{F}}{\partial r}\right)^{2}\right)\right].$$
(44)

The integration functions $\Phi(r)$ and $\Psi(\theta)$ do not depend on the Lagrange density \mathcal{L}_F and can be obtained in the Maxwell limit $\mathcal{L}_F = 1$, which gives $\Phi(r) = C_1/r^2$; $\Psi(\theta) = C_2 \sin 2\theta$, where C_1 and C_2 are the arbitrary constants. As a result, the general solution (38)-(39) reads [71]

$$F_{10} = C_1 \frac{2r\cos\theta}{\mathcal{L}_F \Sigma^2} + C_2 \frac{(a^2\cos^2\theta - r^2)}{a^2 \mathcal{L}_F \Sigma^2}; \quad F_{20} = C_1 \frac{\sin\theta(r^2 - a^2\cos^2\theta)}{\mathcal{L}_F \Sigma^2} + C_2 \frac{r\sin2\theta}{\mathcal{L}_F \Sigma^2}.$$
 (45)

Choice of the constants $C_1 = 0$, $C_2 = -qa^2$, motivated by the known asymptotic solutions [3,77], results in $\Phi(r) = 0$; $\Psi(\theta) = -qa^2 \sin 2\theta$, and the general solution takes the form

$$F_{01} = -\frac{q(r^2 - a^2\cos^2\theta)}{\Sigma^2 \mathcal{L}_F}; \quad F_{02} = \frac{qa^2r\sin 2\theta}{\Sigma^2 \mathcal{L}_F}; \quad F_{31} = a\sin^2\theta F_{10}; \quad aF_{23} = (r^2 + a^2)F_{02} \quad (46)$$

which satisfies the full dynamical system (33)–(36), and coincides with the known solution to the Maxwell–Einstein equations [3,77] in the weak field limit $\mathcal{L}_F = 1$.

3. Density and Mass Function of Electromagnetic Field

Introducing the general solution (46) into the Equation (20) we obtain the intrinsic relation of the Lagrange derivative \mathcal{L}_F with the density and pressure [6]

$$\mathcal{L}_F = \frac{q^2}{4\pi(p_\perp + \rho)\Sigma^2}.$$
(47)

On the disk, where $\Sigma = 0$ due to r = 0, $\cos \theta = 0$, and $(p_{\perp} + \rho) = 0$ by virtue of (24), the Equation (47) requires $\mathcal{L}_F \to \infty$. This guarantees regularity of geometry in the strongly nonlinear regime, as well as has the clear physical sense: it identifies the disk as the perfect conductor (dielectric permeability $\epsilon = \mathcal{L}_F \to \infty$) and the ideal diamagnetic (magnetic permeability $\mu = (\mathcal{L}_F)^{-1} = 0$). The behavior $\mathcal{L}_F \to \infty$ provides also fulfillment of the compatibility condition (37) on the disk [61].

Outside the disk, the function \mathcal{L}_F is continuous as a function of the space variables r and θ [71]. Then

$$\frac{\partial}{\partial r} \left(\frac{1}{\mathcal{L}_F} \frac{\partial \mathcal{L}_F}{\partial \theta} \right) = \frac{\partial}{\partial \theta} \left(\frac{1}{\mathcal{L}_F} \frac{\partial \mathcal{L}_F}{\partial r} \right)$$
(48)

and the compatibility condition (37) takes the form

$$\left[\frac{\partial}{\partial r}\left(\frac{1}{\mathcal{L}_F}\frac{\partial\mathcal{L}_F}{\partial\theta}\right)\right]^2 + \frac{4a^2\sin^2\theta}{\Sigma^2}\frac{1}{\mathcal{L}_F^2}\left[r\frac{\partial\mathcal{L}_F}{\partial r} + \cot\theta\frac{\partial\mathcal{L}_F}{\partial\theta}\right]^2 = 0.$$
 (49)

The derivatives of the function \mathcal{L}_F , calculated from (47), are given by ($q^2/4\pi$ cancels)

$$\frac{\partial \mathcal{L}_F}{\partial r} = -\mathcal{L}_F \left[\frac{4r}{\Sigma} + \frac{1}{(p_\perp + \rho)} \frac{\partial}{\partial r} (p_\perp + \rho) \right];$$

$$\frac{\partial \mathcal{L}_F}{\partial \theta} = \mathcal{L}_F \left[\frac{2a^2 \sin 2\theta}{\Sigma} - \frac{1}{(p_\perp + \rho)} \frac{\partial}{\partial \theta} (p_\perp + \rho) \right];$$

$$\frac{\partial}{\partial \theta} \left(\frac{1}{\mathcal{L}_F} \frac{\partial \mathcal{L}_F}{\partial r} \right) = -\frac{4ra^2 \sin 2\theta}{\Sigma^2} - \frac{1}{(p_\perp + \rho)} \frac{\partial^2 (p_\perp + \rho)}{\partial r \partial \theta}$$

$$+ \frac{1}{(p_\perp + \rho)^2} \frac{\partial (p_\perp + \rho)}{\partial r} \frac{\partial (p_\perp + \rho)}{\partial \theta}.$$
(51)

Then, after some simple algebra, we express the compatibility condition (37) in terms of the electromagnetic density and pressure $(p_{\perp} + \rho)$

$$\left[\frac{4ra^{2}\sin 2\theta}{\Sigma^{2}} - \frac{1}{(p_{\perp} + \rho)^{2}}\frac{\partial(p_{\perp} + \rho)}{\partial r}\frac{\partial(p_{\perp} + \rho)}{\partial \theta} + \frac{1}{(p_{\perp} + \rho)}\frac{\partial^{2}(p_{\perp} + \rho)}{\partial r\partial \theta}\right]^{2}$$

$$+\frac{4a^2\sin^2\theta}{\Sigma^2} \left[\frac{4(r^2-a^2\cos^2\theta)}{\Sigma} + \frac{r}{(p_{\perp}+\rho)}\frac{\partial(p_{\perp}+\rho)}{\partial r} + \frac{\cot\theta}{(p_{\perp}+\rho)}\frac{\partial(p_{\perp}+\rho)}{\partial\theta}\right]^2 = 0 \quad (52)$$

A vanishing of the sum of two squared quantities in (52) yields two equations

$$\frac{r}{(p_{\perp}+\rho)}\frac{\partial(p_{\perp}+\rho)}{\partial r} + \frac{\cot\theta}{(p_{\perp}+\rho)}\frac{\partial(p_{\perp}+\rho)}{\partial\theta} = -\frac{4(r^2 - a^2\cos^2\theta)}{\Sigma};$$
(53)

$$\frac{1}{(p_{\perp}+\rho)^2}\frac{\partial(p_{\perp}+\rho)}{\partial r}\frac{\partial(p_{\perp}+\rho)}{\partial\theta} - \frac{1}{(p_{\perp}+\rho)}\frac{\partial^2(p_{\perp}+\rho)}{\partial r\partial\theta} = \frac{4ra^2\sin\,2\theta}{\Sigma^2}.$$
 (54)

The relation of the Lagrange derivative \mathcal{L}_F with the spherical electromagnetic density $\tilde{\rho}(r)$ follows from Equation (19), which relates $(p_{\perp} + \rho)$ with the spherical density and its derivative [6]

$$(p_{\perp} + \rho) = \frac{2r^2}{\Sigma^2} \left(\frac{r\Sigma}{4} |\tilde{\rho}'| - \tilde{\rho}a^2 \cos^2 \theta \right).$$
(55)

Calculating the partial derivatives of $(p_{\perp} + \rho)$ from (55), we introduce their relation with the spherical density $\tilde{\rho}(r)$

$$\frac{\partial(p_{\perp}+\rho)}{\partial r} = \frac{4r}{\Sigma^3} (r^2 - a^2 \cos^2 \theta) \tilde{\rho} a^2 \cos^2 \theta + \left[\frac{r^4}{\Sigma^2} - \frac{3r^2}{2\Sigma} - \frac{2r^2 a^2 \cos^2 \theta}{\Sigma^2}\right] \tilde{\rho}' - \frac{r^3}{2\Sigma} \tilde{\rho}''; \quad (56)$$

$$\frac{\partial(p_{\perp}+\rho)}{\partial\theta} = \frac{r^2 a^2 \sin 2\theta}{2\Sigma^3} \left[-r\Sigma\tilde{\rho}' + 4\tilde{\rho}(r^2 - a^2\cos^2\theta) \right]; \tag{57}$$

$$\frac{\partial^2 (p_{\perp} + \rho)}{\partial r \partial \theta} = \frac{a^2 \sin 2\theta}{2\Sigma^4} \bigg[-r^3 \Sigma^2 \tilde{\rho}'' + r^2 \Sigma (5r^2 - 7a^2 \cos^2 \theta) \tilde{\rho}' - 8r \bigg((r^2 - a^2 \cos^2 \theta)^2 - 2r^2 a^2 \cos^2 \theta \bigg) \tilde{\rho} \bigg].$$
(58)

Applying the relation (55) in Equation (53), we transform it to the equation for the density

$$\Sigma^2 r(r\tilde{\rho}'' + 5\tilde{\rho}') = 0 \tag{59}$$

which is satisfied on the disk due to $\Sigma = 0$ and r = 0, and gives the first equation for the electromagnetic density $\tilde{\rho}(r)$ outside the disk

$$r\tilde{\rho}'' + 5\tilde{\rho}' = 0. \tag{60}$$

Second equation for the density $\tilde{\rho}$ follows from (54), with taking into account (55) and (56)–(58) $[4a^2\cos^2\theta\tilde{\rho} + r\Sigma\tilde{\rho}'][4(r^2 - a^2\cos^2\theta)\tilde{\rho} - r\Sigma\tilde{\rho}'] =$

$$2\tilde{\rho}[8a^2\cos^2\theta(r^2 - a^2\cos^2\theta)\tilde{\rho} - r\Sigma(\Sigma + 6a^2\cos^2\theta)\tilde{\rho}' - r^2\Sigma^2\tilde{\rho}''].$$
(61)

In the equatorial plane it reduces to

$$2r\tilde{\rho}\tilde{\rho}'' + 6\tilde{\rho}\tilde{\rho}' - r(\tilde{\rho}')^2 = 0.$$
(62)

Introducing for convenience $\tilde{\rho} = y$, r = x in Equations (60) and (62), we write them in the standard form

$$xy'' + 5y' = 0; (63)$$

$$2xyy'' + 6yy' - 2(y')^2 = 0.$$
(64)

To solve these equations, we apply the standard approach introducing

$$y' = p; \ y'' = p \frac{dp}{dy} \tag{65}$$

and transform Equations (63) and (64) to the form

$$p\frac{dp}{dy} + \frac{5}{x}p = 0; \ p\frac{dp}{dy} - \frac{p^2}{2y} + \frac{3p}{x} = 0.$$

Here, we can cancel p, since it corresponds to $y' = \tilde{\rho}'$, which can vanish only on the disk, according to Equation (23). As a result, Equations (63) and (64) take the form

$$\frac{dp}{dy} + \frac{5}{x} = 0 \tag{66}$$

$$\frac{dp}{dy} - \frac{p}{2y} + \frac{3}{x} = 0. \tag{67}$$

Applying Equation (66) in Equation (67), we transform it to

$$\frac{dp}{dy} - \frac{5}{4}\frac{p}{y} = 0.$$

The solution to this equation is given by

$$p = Cy^{5/4} \rightarrow y' = Cy^{5/4}$$
 (68)

where *C* is an arbitrary constant.

It is easily to see that the function p(y) in (68) satisfies both Equations (66) and (67). Second integration yields

$$y = \frac{4^4}{(Cx + C_1)^4} \tag{69}$$

where C_1 is the second arbitrary integration constant.

Two arbitrary constants can be introduced in such a way to rewrite the density profile $\tilde{\rho}(r)$ (denoted above as y(x) for convenience in dealing with the differential equations) in the form, involving the regularization parameter, and, at the same time, leading to the Maxwell density in the weak field limit. Transforming y(x) as

$$y = \frac{4^4}{(Cx+C_1)^4} = \left(\frac{4}{C}\right)^4 \frac{1}{(x+C_1/C)^4} = \frac{A}{(x+r_q)^4}; \ A = \left(\frac{4}{C}\right)^4; \ r_q = \frac{C_1}{C}$$
(70)

we obtain the density profile

$$\tilde{p}(r) = \frac{A}{(r+r_q)^4}.$$
(71)

The integration constant *A* is identified from Equation (47) in the weak field limit $\mathcal{L}_F \to 1$. According to Equation (23), in the equatorial plane $p_{\perp}(r,\theta) + \rho(r,\theta) = \tilde{p}_{\perp}(r) + \tilde{\rho}(r) = -r\tilde{\rho}'(r)/2$,; putting this in the denominator in Equation (47), we obtain

ŀ

$$\mathcal{L}_F = \frac{q^2}{4\pi(\tilde{p}_\perp + \tilde{\rho})r^4} = \frac{q^2}{8\pi A} \frac{(r+r_q)^5}{r^5}.$$
(72)

Normalization of \mathcal{L}_F in the Maxwell weak field limit to $\mathcal{L}_F = 1$ at $r \to \infty$ yields $A = q^2/8\pi$, and the electromagnetic density takes the form

$$\tilde{\rho}(r) = \frac{q^2}{8\pi (r+r_q)^4} \implies \rho(r,\theta) = \frac{r^4}{\Sigma^2} \frac{q^2}{8\pi (r+r_q)^4} = \frac{r^4}{(r^2+a^2\cos^2\theta)^2} \frac{q^2}{8\pi (r+r_q)^4}.$$
 (73)

The spherical density $\tilde{\rho}(r)$ provides the proper behavior of the axially symmetric electromagnetic density $\rho(r, \theta)$, related with $\tilde{\rho}(r)$ by Equation (22), the finite self-interaction at approaching the disk, $\rho(r, \theta) = \tilde{\rho}_0 = q^2/(8\pi r_q^4)$, and the proper behavior in the Maxwell limit, $\rho \to q^2/(8\pi r^4)$, characteristic of the Coulomb law.

The spherical electromagnetic density (73) determines the mass function $\mathcal{M}(r)$, the total mass *m* and the regularization parameter r_q as

$$\mathcal{M}(r) = 4\pi \int_0^r \tilde{\rho}(x) x^2 dx = \frac{q^2}{6r_q} \frac{r^3}{(r+r_q)^3}, \quad m = \frac{q^2}{6r_q}; \quad r_q = \frac{q^2}{6m}.$$
 (74)

For $r \gg r_q$ the mass function $\mathcal{M}(r) \rightarrow m - q^2/2r$, and the metric (7) approaches the Kerr–Newman metric.

Regular solutions, describing rotating objects, are most frequently obtained by applying the Newman–Janis algorithm (1) for the spherical solutions (5) with the spherical mass function $\mathcal{M}(r)$, chosen in some physical model. Here we have shown that the original spherical mass function (74) for electrically charged rotating objects is determined uniquely in the frame of nonlinear electrodynamics minimally coupled to gravity, which describes these objects in a general setting, in the self-consistent, and model-independent way, i.e., the mass function (74) comes from the dynamical equations without any additional special assumptions. This is the key point that distinguishes it from the models of mass function presented in the literature as chosen ad hoc.

The electromagnetic density profile (73), uniquely determined by NED-GR for regular rotating electrically charged objects, allows us to obtain the explicit form of the Lagrange derivative \mathcal{L}_F , of the dielectric and magnetic permeability, and the exact general solution for the components of the electromagnetic tensor $F_{\mu\nu}$, as we shall see below.

The function $(p_{\perp} + \rho)$, calculated for the electromagnetic density (73) from the basic Equation (55), takes the form

$$p_{\perp} + \rho = \frac{q^2}{4\pi\Sigma^2} \frac{r^2(r^3 - r_q a^2 \cos^2\theta)}{(r + r_q)^5}.$$
(75)

It vanishes at infinity and goes to zero as $(p_{\perp} + \rho) \propto r/r_q^5$ at approaching the disk.

Applying Equation (75) in Equation (47) we present the Lagrange derivative for the density profile (73) as

$$\mathcal{L}_F = \frac{(r+r_q)^5}{r^2(r^3 - r_q a^2 \cos^2 \theta)}$$
(76)

which evidently has the proper asymptotic behavior: it goes to infinity on the disk, and approaches $\mathcal{L}_F = 1$ as $r \to \infty$.

By using the expression for the Lagrange derivative (76) in the general solution (46) and taking into account Equation (13), we obtain four non-zero components of the electromagnetic tensor, which describe the electromagnetic field of regular rotating electrically charged objects 2(2 - 2 - 2a)(3 - 2a)(3 - 2a)

$$F_{10} = \frac{qr^{2}(r^{2} - a^{2}\cos^{2}\theta)(r^{3} - r_{q}a^{2}\cos^{2}\theta)}{\Sigma^{2}(r + r_{q})^{5}};$$

$$F_{20} = -\frac{qa^{2}r^{3}(r^{3} - r_{q}a^{2}\cos^{2}\theta)\sin 2\theta}{\Sigma^{2}(r + r_{q})^{5}};$$

$$F_{31} = \frac{qar^{2}(r^{2} - a^{2}\cos^{2}\theta)(r^{3} - r_{q}a^{2}\cos^{2}\theta)\sin^{2}\theta}{\Sigma^{2}(r + r_{q})^{5}};$$

$$F_{23} = \frac{qar^{3}(r^{2} + a^{2})(r^{3} - r_{q}a^{2}\cos^{2}\theta)\sin 2\theta}{\Sigma^{2}(r + r_{q})^{5}}.$$
(77)

In the weak field limit $r \to \infty$ they coincide with the known solution [3,77].

In the equatorial plane, the field components (77) take the form

$$F_{10} = \frac{qr^3}{(r+r_q)^5}; \quad F_{20} = -\frac{qa^2r^2}{(r+r_q)^5}; \quad F_{31} = \frac{aqr^3}{(r+r_q)^5}; \quad F_{23} = \frac{qar^2(r^2+a^2)}{(r+r_q)^5}.$$
 (78)

On approaching the disk F_{10} and F_{31} decrease as r^3 , and F_{20} and F_{23} decrease as r^2 .

Applying the expression for the Lagrange derivative (76) in (21), we obtain the explicit expressions for the eigenvalues of the tensors of the dielectric and magnetic permeability given by

$$\epsilon_r^r = \frac{(r^2 + a^2)(r + r_q)^5}{\Delta r^2 (r^3 - r_q a^2 \cos^2 \theta)}, \quad \epsilon_\theta^\theta = \frac{(r + r_q)^5}{r^2 (r^3 - r_q a^2 \cos^2 \theta)};$$
$$\mu_r^r = \frac{r^2 (r^2 + a^2)(r^3 - r_q a^2 \cos^2 \theta)}{\Delta (r + r_q)^5}, \quad \mu_\theta^\theta = \frac{r^2 (r^3 - r_q a^2 \cos^2 \theta)}{(r + r_q)^5}$$
(79)

We see that the electromagnetic density profile (73) provides the proper behavior of the Lagrange derivative (76) and of dielectric and magnetic permeability, the regularity of electromagnetic fields in the strongly nonlinear regime on the disk, as well as the proper asymptotic behavior in the weak field limit, and makes sure that the density profile (73) guarantees the proper behavior of geometry and of the electromagnetic field.

4. Conclusions

Nonlinear electrodynamics minimally coupled to gravity describes the electromagnetic field of regular rotating electrically charged objects by the system of four dynamical equations for two independent components of the electromagnetic field $F_{\mu\nu}$. The dynamical system is restricted by two constraints, which guarantee the fulfillment of the full system of dynamical equations. The Lagrange derivative \mathcal{L}_F , which depends on the spherical electromagnetic density $\tilde{\rho}(r)$ and its derivative, satisfies the necessary and sufficient compatibility condition for the dynamical system of four equations for two field functions. It follows that two independent components of the electromagnetic field and the spherical electromagnetic density $\tilde{\rho}(r)$ are uniquely determined by three independent dynamical equations. In consequence, additionally, the mass function $\mathcal{M}(r) = 4\pi \int_0^r \tilde{\rho}(x) x^2 dx$ is determined uniquely from the compatibility condition, not leaving any freedom in choosing it ad hoc.

The density profile $\tilde{\rho}(r)$ allows us to obtain the explicit form of the Lagrange derivative, the exact form of the general solution for the electromagnetic field in all regions, as well as the explicit form of the dielectric and magnetic permeability.

The basic constituent of all rotating compact objects in the axially symmetric geometry is the interior equatorial disk (10). In regular geometry the disk (10) is filled with the de Sitter vacuum $p = -\rho$, which is the fundamental generic feature of all regular rotating electrically charged NED-GR objects. Regularity is provided by the behavior of the Lagrange derivative on the disk, $\mathcal{L}_F \to \infty$, which leads to zero magnetic permeability, $\mu = \mathcal{L}_F^{-1} = 0$, and the infinite dielectric permeability, $\epsilon = \mathcal{L}_F \to \infty$, and identifies the disk as the perfect conductor and ideal diamagnetic.

The strongly nonlinear behavior of the electromagnetic field on the disk represents the realization ([71] and references therein) of the underlying hypothesis of non-linearity replacing a singularity [57].

The current on a surface layer is defined as $4\pi j_k = [e^{\alpha}_{(k)}]F_{\alpha\beta}n^{\beta}$, where $e^{\alpha}_{(k)}$ are the base vectors of the intrinsic coordinate system on the layer, n_{α} is the unit normal to it, directed upward, [...] denotes the jump across the layer, and $F_{\alpha\beta}$ is the electromagnetic tensor. On the equatorial disk the intrinsic coordinate system is $t, \phi, 0 \le \xi \le \pi/2, e^{\alpha}_{(\phi)} = \delta^{\alpha}_{3}$, and the vector $n_{\alpha} = \varepsilon \delta^{1}_{\alpha} (1 + q^{2}/a^{2})^{-1/2} \cos \xi$, where $\varepsilon = +1$ and $\varepsilon = -1$ on the upper and lower faces, respectively [78]. On the equatorial disk $4\pi j_{\phi} = [F^{1}_{3}\varepsilon(1 + q^{2}/a^{2})^{-1/2} \cos \xi]$.

Applying the expression for F_{31} given in (46) to explicitly introduce the magnetic permeability, we obtain the surface current [79]

1

$$T_{\phi} = \frac{qc\sin^2\xi}{2\pi a\sqrt{1+q^2/a^2}} \frac{\mu}{\cos^3\xi}.$$
 (80)

Due to $\mu = 0$ this current vanishes over the disk, except the ring $\xi = \pi/2$, where each term in the second fraction takes zero value independently, and the current (80) can take any non-zero value, which satisfies the basic criterion for transition to a superconducting state [75].

The superconducting current (80) flows within the perfect conductor region without resistance, and thus represents a non-dissipative electromagnetic source, which powers a regular rotating electrically charged compact object and provides its, in principle, an unlimited lifetime [79].

In the Kerr–Newman geometry, the ring (11), confining the equatorial disk (10), comprises the ring singularity. In the regular geometry, its place takes the superconducting current (80), which presents the source of the electromagnetic field, and, in consequence, of the gravitational field generated by the stress-energy tensor of this electromagnetic field. This source originates from its own nonlinear electromagnetic field because the dynamical Equations (31) and (32) are source-free.

As any circular current, the current (80) produces the magnetic momentum $\mu_{in} = c^{-1}j_{\phi}S$ where *S* is the disk area, which is, in essence, *intrinsic*, because the dynamical Equations (31) and (32) are source-free [80]. Introducing in (80) an uncertain coefficient *U*, we write j_{ϕ} as $j_{\phi} = -(qc/2\pi a)\sqrt{1+q^2/a^2}U$, and obtain for the magnetic momentum

$$\mu_{in}=-(qS/2\pi a\sqrt{1+q^2/a^2}U)$$

In the case, when the intrinsic magnetic moment μ_{in} of an object is known, we can restore the uncertain coefficient *U*.

For the electron, visualized as a spinning electromagnetic soliton, this gives $j_{\phi} = 79.277$ A [80]. This current powers the electron for, in principle, an unlimited lifetime

and generates its electromagnetic field, which, for a distant observer, $r \gg \lambda_e = \hbar/m_e c$, reads [79]

$$E_{r} = -\frac{e}{r^{2}} \left(1 - \frac{\hbar^{2}}{m_{e}^{2}c^{2}} \frac{3\cos^{2}\theta}{4r^{2}} \right); \quad E_{\theta} = \frac{e\hbar^{2}}{m_{e}^{2}c^{2}} \frac{\sin 2\theta}{4r^{3}}; \quad B^{r} = -\frac{e\hbar}{m_{e}c} \frac{\cos \theta}{r^{3}}; \quad B_{\theta} = -\frac{e\hbar}{m_{e}c} \frac{\sin \theta}{2r^{4}}$$

Nonlinear electrodynamics, minimally coupled to gravity, describes the regular rotating electrically charged objects in a self-consistent and model-independent way since minimal coupling implies no additional assumptions. Obtained here, spherical electromagnetic density is generic, as uniquely defined by the NED-GR dynamical equations. It allowed us to obtain the exact form of a general solution for the electromagnetic field and the explicit generic form of the dielectric and magnetic permeability. We plan to continue our series of works on regular rotating electrically charged compact objects, including an investigation into their stability with respect to external perturbations. Currently we are working on restoring the Lagrangian $\mathcal{L}(F)$ from its derivative \mathcal{L}_F as the function of the spatial variables.

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