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Unruh Effect and Information Entropy Approach

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Abstract: The Unruh effect can be considered a source of particle production. The idea has been widely employed in order to explain multiparticle production in hadronic and heavy-ion collisions at ultrarelativistic energies. The attractive feature of the application of the Unruh effect as a possible mechanism of the multiparticle production is the thermalized spectra of newly produced particles. In the present paper, the total entropy generated by the Unruh effect is calculated within the framework of information theory. In contrast to previous studies, here the calculations are conducted for the finite time of existence of the non-inertial reference frame. In this case, only a finite number of particles are produced. The dependence on the mass of the emitted particles is taken into account. Analytic expression for the entropy of radiated boson and fermion spectra is derived. We study also its asymptotics corresponding to low- and high-acceleration limiting cases. The obtained results can be further generalized to other intrinsic degrees of freedom of the emitted particles, such as spin and electric charge.

Keywords: multiparticle production; Unruh effect; information theory

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1. Introduction

As was demonstrated by Unruh [1], the observer comoving with the non-inertial reference frame (RF) with the acceleration a will detect particles thermalized at temperature

$$T = \frac{a}{2\pi}$$

in Planck units, whereas the observer in any inertial RF will see bare vacuum. If the acceleration a equals the surface gravity of some Schwarzschild black hole (BH), when the observer is at the horizon, T coincides with the temperature $T_{\rm BH}$ of the Bekenstein–Hawking radiation [2–4] of the horizon.

This peculiar non-invariance of the vacuum has raised a lot of interest in the topic (for review see, e.g., [5] and references therein). Recall that the Unruh effect was initially derived for scalar particles. Here, the change in the ratio between the negative and positive frequency modes of scalar fields in the noninertial RF was considered [1]. Generalizations to arbitrary trajectories of the observer is discussed in [6,7], whereas the generalization to the accelerated reference frames with rotation can be found in [8,9]. The emergence of the Unruh effect in the Rindler manifold of an arbitrary dimension and its relationship to the vacuum noise and stress are investigated in [10]. Various methods and approaches have been employed. For instance, an algebraic approach was used to extend the Unruh effect to theories with arbitrary spin and with interaction [11,12], whereas the path integral approach was applied to derive the effect for fermions within the framework of quantum field theory [13]. Among the recent studies, one can mention the relativistic quantum statistical mechanics approach [14–16] based on the application of Zubarev's density operator [17,18].

Within this approach, the Unruh effect was obtained first for the scalar particles [14] and then generalized to the gas of massless fermions [16]. In the present study, we employ the approach based on the application of the information theory, which is a promising tool to study the black hole information dynamics, as one may see in [19] or reviews [20,21].

Usually, the non-inertial observer is assumed to accelerate forever. However, such an assumption implies the availability of an infinite energy supply and ever-lasting particle emission. The more sophisticated scenario, which considers the Unruh effect at finite time interval, is analyzed in papers [22–25].

There are a lot of proposals for the detection and application of the Unruh effect, see, e.g., [26–28]. The paper [29] discusses the possibility of eavesdropping in the non-inertial reference frame. The production of the entangled photon pairs from the vacuum with the help of the Unruh effect was investigated in [30], whereas, in [31], the creation of accelerated black holes by means of the Unruh effect was studied. In [32], the authors discuss the possibility of using accelerated electrons as thermometers; more on the topic can be found in Refs. [33,34]. Generated bosons and fermions were considered to be produced via the quantum tunneling mechanism at the Unruh horizon in [35,36].

The Unruh effect can be considered as a source for the creation of new particles. This idea has been widely employed [37–44] in order to explain multiparticle production in hadronic and heavy-ion collisions at ultrarelativistic energies. The attractive feature of the application of the Unruh effect as a possible mechanism of multiparticle production is the thermalized spectra of newly produced particles. Experiments with ultrarelativistic hadronic and heavy-ion collisions and their theoretical interpretations indicate that the produced matter seems to reach equilibrium extremely quickly, see, e.g., [45,46] for the present status of the field. The mechanism of this fast equilibration is still debated; therefore, the Unruh effect might be of great help. At the same time, since the Unruh source is thermal, it results in the observer-dependent entropy generation [47]. In the present paper, we also consider the Unruh horizon as a thermal source of particles. These particles are characterized by thermal distribution. Our aim is to estimate the entropy of the distribution and to define its dependence on any intrinsic degrees of freedom of the emitted particles.

In this paper, we consider Unruh radiation at some fixed energy E, which is assumed to be a parameter. It can be argued that such an analysis is incorrect because we should have taken into account the all-energy modes E_i via the product $\prod_{E_i}$ in the corresponding density matrix, see, e.g., [5]. This approach implies an independent emission of modes at different energies. In other words, in that case, one deals with the energy modes via the tensor product of the corresponding subspaces. However, such a generalization is not mandatory for the Unruh effect. For instance, in [35,36], the authors demonstrate that one can obtain the Unruh effect for some fixed energy without any need to take a product of all the modes to encompass the all-Unruh thermal bath states. This circumstance allows us to consider energy E of the mode as a parameter and, therefore, to take into account any correlations between the modes originating from the finite energy supply and restrictions imposed by energy conservation.

The paper is organized as follows. Section 2 presents the necessary basics from the probability theory and the information theory. Section 3 briefly describes the Unruh effect and the density matrix of the emitted quanta. The total entropy of the Unruh source is estimated in Section 4. Here, the general expression for the entropy of fermion and boson radiation is derived, as well as its analytic series expansion. In Section 5, one is dealing with the analysis of temperature asymptotics of the entropy. Two limiting cases corresponding to low and high temperatures, or, equivalently, the acceleration of the observer, are considered. Section 6 is devoted to the contribution of intrinsic degrees of freedom of the produced particles. Final remarks and conclusions can be found in Section 7.

2. Probability and Entropy

Let us consider some distribution $\{X\}$ with the unnormalized distribution probability $d(x)$. In other words, $d(x)$ is a number of events in which x is being observed. Shannon entropy $H(X)$ may be written as

$$H(X) = -\sum_x \frac{d(x)}{\mathcal{D}_X} \ln \frac{d(x)}{\mathcal{D}_X} = \ln \mathcal{D}_X - \frac{1}{\mathcal{D}_X} \sum_x d(x) \ln d(x)\,, \tag{1}$$

where $\mathcal{D}_X = \sum_x d(x)$. $H(X)$ encodes the amount of information we need in order to completely describe $\{X\}$, i.e., this is amount of information we are lacking. Therefore, we should deal with the distribution $\{X\}$. It is scale-invariant, so it does not change under the transformations $d(x) \to \alpha d(x)$ for any $\alpha = \text{const}$.

Similarly, for joint distribution $\{X, Y\}$ with the unnormalized distribution probability $d(x, y)$, one can write down Shannon entropy $H(X, Y)$ as

$$H(X,Y) = -\sum_{x,y} \frac{d(x,y)}{\mathcal{D}_{X,Y}} \ln \frac{d(x,y)}{\mathcal{D}_{X,Y}} = \ln \mathcal{D}_{X,Y} - \frac{1}{\mathcal{D}_{X,Y}} \sum_{x,y} d(x,y) \ln d(x,y), \tag{2}$$

where $\mathcal{D}_{X,Y} = \sum_{x,y} d(x, y)$.

In the joint case, one may define the conditional probability $d(x|y)$ as

$$d(x|y) = \frac{d(x,y)}{d(y)}\,, \qquad d(y) = \sum_x d(x,y). \tag{3}$$

It defines the amount of events with x from the set of events in which y occurs. Using Equation (1), Shannon entropy $H(X|y)$ becomes

$$H(X|y) = \ln \mathcal{D}_{X|y} - \frac{1}{\mathcal{D}_{X|y}} \sum_x d(x|y) \ln d(x|y) = -\sum_x d(x|y) \ln d(x|y), \tag{4}$$

where $\mathcal{D}_{X|y} = \sum_x d(x|y) = 1$, as follows from Equation (3).

Finally, substituting Equations (3) and (4) into Equation (2), one obtains

$$H(X,Y) = H(Y) + \langle H(X|y)\rangle_Y = H(X) + \langle H(Y|x)\rangle_X, \tag{5}$$

where averaging taken over X or Y reads

$$\langle \mathcal{A} \rangle_Z = \frac{1}{\mathcal{D}_Z} \sum_z d(z)\mathcal{A}\,, \qquad Z \equiv X, Y.$$

Recall that all the formulae above are valid for the discrete distributions only. In the continuous case, one should use the probability density function (PDF) $p(x)$ instead of $d(x)$. Shannon entropy becomes dimensionally incorrect and should be re-defined, as shown in [48,49].

For the distribution $\{X\}$ with the PDF $p(x)$, the entropy given by Equation (1) is generalized to

$$H(X_p) = \ln \mathcal{D}_{X_p} - \frac{1}{\mathcal{D}_{X_p}} \int p(x) \ln p(x) \mathrm{d}x - \langle \ln \mathrm{d}x \rangle_{X_p}, \tag{6}$$

where $\mathcal{D}_{X_p} = \int p(x)\mathrm{d}x$ is the norm and

$$\langle \mathcal{A} \rangle_{X_p} = \frac{1}{\mathcal{D}_{X_p}} \int p(x)\mathcal{A}\mathrm{d}x.$$

The last term in Equation (6) is related to the limiting density of discrete points and takes into account the amount of information encoding a discrete-continuum transition (see [48,49] for details). The term originates from the fact that the PDF $p(x)$ is not dimensionally invariant compared to the discrete probability $d(x)$. The last one can be set to be dimensionless—see the explanation below Equation (1)—while $p(x)$ cannot. In any realistic computational task, the term determines the contribution of the bin widths $\mathrm{d}x$ of the distribution to the entropy. Note that one may formally reduce $H(X_p)$ to $H(X)$ by substituting $\int p(x)\mathrm{d}x$ into $\sum_x d(x)$ and setting $\langle \ln \mathrm{d}x \rangle_{X_p}$ to zero; the same procedure is valid in the opposite direction.

3. Unruh Effect

From here, we will use Planck (or natural) units, $c = G = \hbar = k_B = 1$. Furthermore, we restrict our analysis to $1 + 1$-dimensional space-time because two other spatial dimensions play no role and, therefore, can be neglected.

As was already mentioned in Section 1, vacuum is non-invariant with respect to the reference frame [1]. In the non-inertial RF determined with the acceleration a, one meets the appearance of horizon that separates space-time into the inside and outside domains. As a result, the non-inertial observer detects the radiation going out from the horizon, while the inertial one detects the Minkowski vacuum state $|0\rangle$ only. For bosons, the latter reads [5,35,36]

$$|0\rangle = \sqrt{\frac{1-\exp\left(-E/T\right)}{1-\exp\left(-NE/T\right)}} \sum_{n=0}^{N-1} \exp\left(-nE/2T\right) |n\rangle_{\text{in}} |n\rangle_{\text{out}}\,, \tag{7}$$

whereas, for fermions, one obtains

$$|0\rangle = \frac{1}{\sqrt{1+\exp\left(-E/T\right)}} \sum_{n=0}^{1} \exp\left(-nE/2T\right) |n\rangle_{\text{in}} |n\rangle_{\text{out}} \tag{8}$$

Here, E is the energy of the quanta emitted at the Unruh horizon with the temperature $T = a/(2\pi)$. The denominator for bosons stands for the normalization reasons. Parameter N, as can be seen from Equation (7), encodes the maximum amount of quanta at energy E plus 1. Loosely speaking, N is the number of dimensions of the corresponding Fock space at the given energy E and temperature T of the source. The subscripts *in* and *out* denote the components of the field (Rindler modes) with respect to the horizon.

Usually, N is assumed to be infinite. One may argue that the finiteness of the parameter N in the boson case is incorrect from a mathematical point of view since one deals with the incomplete basis then. However, in any real physical situation, one is dealing with the finite number of produced particles, bosons and fermions. Taking $N \to \infty$ in Equation (7), as it is widely used in the literature on the topic, seems to be too strong of an assumption because the source produces an infinite amount of energy, $(N-1)E \to \infty$. This is valid in the case of everlasting acceleration or the non-zero probability of detecting $N \to \infty$ amount of particles at some finite time interval; both scenarios can be provided with the infinite energy supply only. This is because the infinite sum for bosonic modes—see Equation (7)—contains an arbitrary amount of particles: despite being exponentially suppressed, the probability for any $n \neq \infty$ in the sum for bosons is non-zero. Such a scenario seems to be rather unlikely from the physical point of view, especially when one considers the application of the Unruh effect for the description of particle production in relativistic hadronic or heavy ion collisions. Therefore, we assume the maximum number of particles to be finite in all calculations below.

Furthermore, let us consider only boson production in what follows because the expression for the fermions given by Equation (8) can be derived from Equation (7) by setting $N = 2$.

Expression (7) is the Schmidt decomposition [50]. The outgoing radiation is described by the density matrix

$$\rho_{\text{out}} = \text{Tr}_{\text{in}} |0\rangle\langle 0| = \frac{1 - \exp\left(-E/T\right)}{1 - \exp\left(-NE/T\right)} \sum_{n=0}^{N-1} \exp\left(-\frac{nE}{T}\right) |n\rangle_{\text{out}} \langle n|_{\text{out}}\,, \tag{9}$$

where we have traced over the inaccessible degrees of freedom (*in-* modes). Thus, the pure vacuum state from the inertial RF has transformed into the mixed one in the non-inertial RF. Here, the geometric origin of the Unruh effect appears. Namely, finiteness of the speed of light leads to the appearance of the horizon dividing the all modes in Hilbert space into the accessible (*out-*) and non-accessible (*in-*) ones. The complete state is obviously pure and follows unitary evolution. However, because one has limited access to it in the non-inertial RF, it looks like a decoherence. The eigenvalues of the density matrix ρ_{out} define the emission probability of a certain number of particles at energy E and temperature T. Therefore, Equation (9) describes the conditional multiplicity distribution $\{n|N, E, T\}$ at any given N, E and T.

One may assume that once we have the distribution, it is possible to calculate the corresponding Shannon entropy due to the formulae presented in Section 2. However, to deal with the density matrix ρ, one should use the von Neumann entropy $H(\rho)$ instead, which is defined as

$$H(\rho) = -\text{Tr}\rho \ln \rho.$$

The key difference of the von Neumann entropy from its classical analog, Shannon entropy, is related to its meaning: $H(\rho)$ defines the amount of information encoded with the correlations between the system described by ρ and the rest of the world. From this point of view, the density matrix ρ defines the projection of some larger system, which was determined in the larger Hilbert space, to the space in which the observed system is being defined. The projection might result in a loss of information encoded with the corresponding correlations between the Hilbert subspaces. The von Neumann entropy is the quantity to estimate the amount of this information. Due to its origin, it can be equal to zero for the entire space and non-zero for its subspace. This is not the case for the Shannon entropy because classical entropy of the whole system cannot be less than that of some part of it. However, the von Neumann entropy can be set as equal to its Shannon counterpart provided that the Schmidt decomposition coincides with the basis of the detector [51].

4. Unruh Entropy

For the emission probability ρ_{out} from Equation (9), the von Neumann entropy is defined as

$$H(\rho_{\text{out}}) = -\text{Tr}\rho_{\text{out}} \ln \rho_{\text{out}} = H(n|N, E, T) = \sigma(qE/T)\Big|_{q=N}^{q=1}\,, \tag{10}$$

where we use the following notations

$$\sigma(qE/T) = \frac{qE/T}{\exp\left(qE/T\right) - 1} - \ln\left[1 - \exp\left(-\frac{qE}{T}\right)\right]\,, \tag{11}$$

$$f(x)\Big|_{x=b}^{x=a} = f(a) - f(b)\,. \tag{12}$$

As one may notice, $H(n|N, E/T)$ is an even function of E/T, i.e., $H(n|N, E/T) = H(n|N, -E/T)$. The asymptotic behavior of the entropy (10) with respect to E/T is the following

$$\lim_{E/T \to 0} H(n|N, E, T) = \ln N = \max(H) \lim_{E/T \to \infty} H(n|N, E, T) = 0\,. \tag{13}$$

Expression (10) defines the entropy of the emitted quanta, as well as the quanta inside the horizon, for some mode of the radiated field only, which is determined by parameter N, energy E and temperature T. Parameter N depends on the amount of time during which the observer is being described by the non-inertial reference frame. It follows from the fact that the longer one is observing the horizon, the more particles at any fixed energy may be detected. Therefore, we conclude that N should increase with time. Temperature T is completely determined by the acceleration a, see [1]. However, E cannot be considered as a fixed parameter. The non-inertial observer is expected to detect particles at different energies. The energy range for the particles may be written as

$$m \le E \le M\,, \tag{14}$$

where m is the invariant mass of the particles, and M is the maximum energy to be observed, respectively. We assume M to be limited by the acceleration a since the observation of the high-energy particles is very unlikely due to energy conservation law: one cannot extract more energy from the vacuum than is being spent to sustain the observer's acceleration.

Unfortunately, the definition of the energy range does not mean we know the spectrum distribution $\{E\}$. It is determined by the unnormalized PDF $p(E)$ of the emission of a particle from the vacuum at energy E.

In order to figure out $p(E)$ somehow, we use the following procedure. As can be noticed from Equation (9), for any particle number $n > 0$, the emission probability is proportional to the factor $\exp(-E/T)$. The case with $n = 0$ means no emission at all. Therefore, one should expect exponential behavior for $p(E)$

$$p(E) = C\exp(-E/T)\,, \tag{15}$$

where prefactor C is responsible for any corrections that might depend on the particle type and its quantum numbers. For the sake of simplicity, we assume $C = \text{const}$ and, therefore, drop it due to normalization reasons (see Section 2) in what follows. It is worth noting that such assumption results in Schwinger-like mechanism of particle production [52]. Thus, we recovered Schwinger-like particle production from the properties of Hilbert space and space-time only. Recall, however, that this result is generated by the Unruh effect after neglecting all possible corrections.

Now, we have the spectrum distribution $\{E\}$ as given by Equation (15). Without any loss of generality, we assume energy to be defined within the range $m \le E \le M$ (Equation (14)). From Equations (5) and (6), one obtains

$$\begin{aligned} H(n,E|N,T) &= -\langle \ln \mathrm{d}E\rangle_{E_p} + \ln\mathcal{D}_{E_p} - \frac{1}{\mathcal{D}_{E_p}}\int_m^M p(E)\ln p(E)\mathrm{d}E \\ &+ \frac{1}{\mathcal{D}_{E_p}}\int_m^M p(E)H(n|N,E,T)\mathrm{d}E\,, \end{aligned} \tag{16}$$

where the subscript E_p implies that the energy distribution is not discrete but rather a continuous one, i.e., it is defined with some PDF—see the text concerning Equation (6). In order to obtain the analytic expression, we substitute Equations (15) and (10) into Equation (16) and obtain, after the straightforward calculations, the total Unruh entropy $H(n,E|N,T)$ in a form

$$\begin{aligned} H(n,E|N,T) &= -\langle \ln \mathrm{d}E\rangle_{E_p} + 1 + \ln\mathcal{D}_{E_p} + \frac{m\exp(-m/T) - M\exp(-M/T)}{\mathcal{D}_{E_p}} \\ &+ \frac{T}{\mathcal{D}_{E_p}}\sum_{k=1}^{\infty}\left\{\left[\frac{2kq+1}{k(kq+1)} + q\frac{E}{T}\right]\times\frac{\exp[-(kq+1)E/T]}{kq+1}\Bigg|_{E=M}^{E=m}\right\}\Bigg|_{q=N}^{q=1}\,, \end{aligned} \tag{17}$$

where

$$\mathcal{D}_{E_p} = \int_m^M p(E)\mathrm{d}E = T\left[\exp\left(-\frac{m}{T}\right) - \exp\left(-\frac{M}{T}\right)\right] \tag{18}$$

and $\sigma(qE/T)$ from Equation (10) is represented by the following series

$$\sigma(qE/T) = \sum_{k=1}^{\infty} \left(\frac{1}{k} + q\frac{E}{T} \right) \exp\left(-\frac{kqE}{T} \right). \tag{19}$$

The first term in Equation (17) is responsible for encoding the discrete-continuum transition, see [48,49]. It is expected to depend neither on any quantum numbers of outgoing particles nor on the reference frame. Therefore, we assume $\langle \ln \mathrm{d}E \rangle_{E_p}$ to be constant.

Expression (17) defines entropy for the distribution $\{n, E|N, T\}$ of the particles being detected by the observer associated with non-inertial RF moving with acceleration $a = 2\pi T$. Recall that in the case of fermions, one should use $N = 2$. For the bosons, N may take any positive integer value obeying the energy conservation law. The entropy calculated for the Unruh radiation of fermions and bosons is presented in Figures 1 and 2, respectively. One can see the distinct maximum in the region of small values of the m/T ratio. The maximum increases with rising the M/T ratio and becomes more pronounced with the increase in radiated particles (see Figure 2).

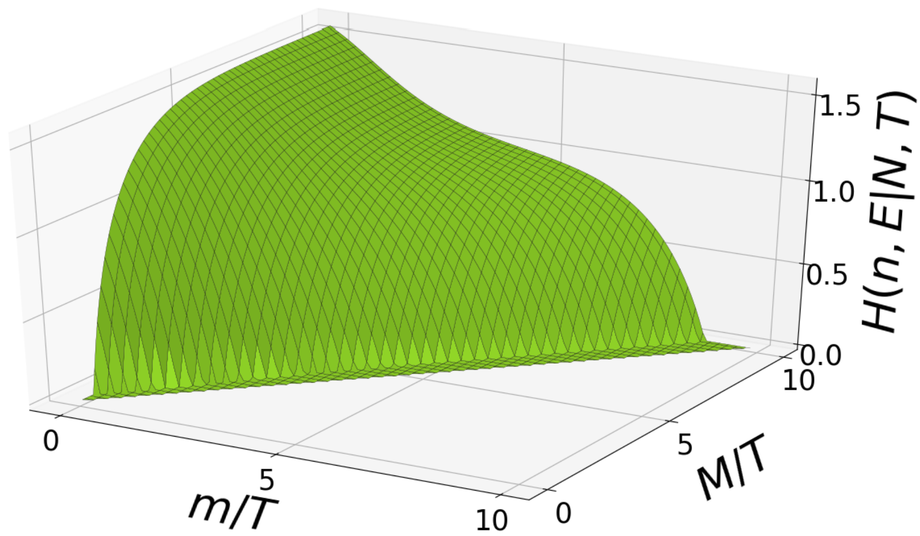

Figure 1. (Color online) The entropy $H(n, E|N, T)$ of Unruh radiation given by Equation (17) for fermions ($N = 2$) as function of m/T and M/T.

The considered example seems to be straightforward. However, one should keep in mind that the whole analysis above is valid for $1+1$-dimensional space-time. Other spatial dimensions do not contribute to the density matrix ρ_{out} or to its von Neumann entropy because the corresponding subspaces of the Hilbert space contribute to ρ_{out} via the direct tensor product and, therefore, can be traced out with no consequences to the analysis above. This simple direct extension to additional spatial dimensions for the Unruh effect may lead to the widely spread conclusion that the Unruh effect results in the appearance of thermal bath all over the space. In our opinion, this conclusion needs to be clarified. Namely, in the last case, the non-inertial observer, as well as the horizon itself, should be considered as an infinite plane in the additional spatial dimensions being accelerated alongside the normal to the plane. However, the observer should be finite and, therefore, cannot detect particles from the half-space defined by the horizon. Otherwise, it would lead to faster-than-light speed communication and causality violation because the transition to inertial RF cannot cause the immediate disappearance of the Unruh radiation from the horizon occupying the half-space.

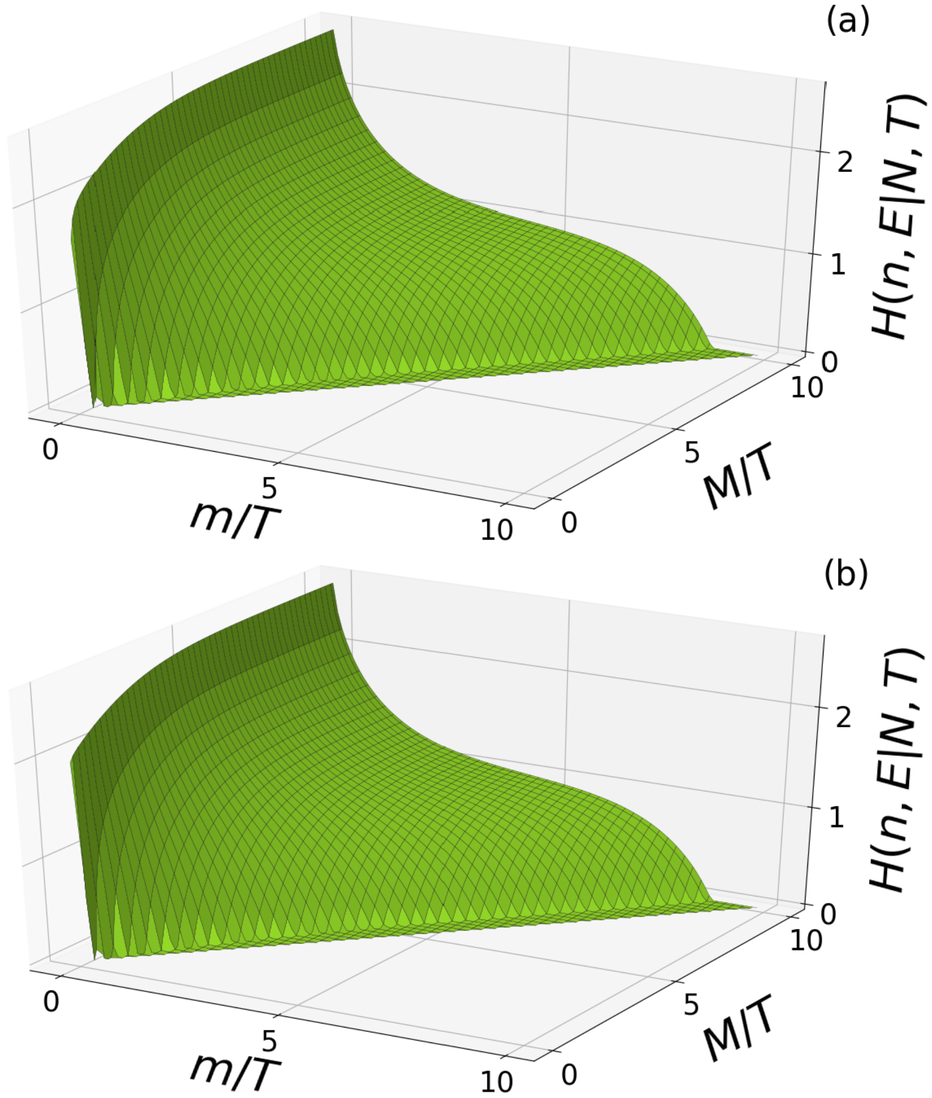

Figure 2. (Color online) The same as Figure 1 but for bosons. The spectrum of bosons contains (**a**) $N = 100$ and (**b**) $N = 1000$ particles.

To overcome the difficulties, we have to assume that

- In order to obey, the energy conservation law N should be finite;
- In the case of (2 + 1) or (3 + 1)-dimensional space-time, the Unruh horizon should be considered as a radiation source of finite size.

Due to the axial symmetry of the non-inertial reference frame, the horizon should be a disk shape with some radius r. The radius can be determined by the observer's size and causality, i.e., the finiteness of light speed. Such an assumption leads to an observer-dependent size of r. The problem may be cured, e.g., if one considers the observer's acceleration a as a surface gravity of the corresponding black hole and obtain some efficient scale $r = (4\pi T)^{-1}$.

One might be confused by the fact that since the Unruh effect describes the thermal bath, its entropy should be maximal. As can be easily noticed from the eigenvalues of the density matrix (9), all of them exponentially depend on the total energy of the emitted number of particles and thus generate a well-known partition function. Note, however, that $\rho_{\rm out}$ is defined for some *fixed* value of energy. Therefore, E can be considered a parameter of the conditional distribution $\{n|N, E, T\}$. Dealing with the joint distribution $\{n, E|N, T\}$ over multiplicity n and energy E of the emitted quanta, one should take into account energy conservation. It results in some correlations between the possible number of emitted particles and their energy. Thus, the entropy $H(n, E|N, T)$ describes not a completely thermal source but some other one.

5. Asymptotics of Unruh Entropy

Let us analyze the asymptotic behavior of the total Unruh entropy in Equation (16) for (i) small and (ii) large acceleration of the observer. The case of small acceleration is analogous to $T \to 0$; therefore, we will drop all but the leading term in Equation (16). At small temperatures, Equation (18) transforms into

$$\mathcal{D}_{E_p}\Big|_{T\to 0} \approx T \exp\left(-m/T\right), \tag{20}$$

where we have neglected the term $\exp\left(-M/T\right)$ since M is the upper bound for the energy spectrum; therefore, $M > m$. The Unruh entropy becomes

$$H(E)\Big|_{T\to 0} = \ln \mathcal{D}_{E_p} - \frac{1}{\mathcal{D}_{E_p}} \int_m^M p(E) \ln p(E) \mathrm{d}E \approx \ln T - \frac{m}{T} + 1 + \frac{m}{T} = \ln T + 1\,. \tag{21}$$

Because the entropy $H(n|N, E, T)$ equals zero when $N = 1$, we consider the case with $N > 1$ for $T \to 0$. Neglecting all the higher-order exponents, one obtains from Equation (10) that

$$H(n|N, E, T)\Big|_{T\to 0} \approx \frac{E}{T} \exp\left(-E/T\right). \tag{22}$$

Substituting Equations (21) and (22) into Equation (16), we obtain

$$H(n, E|N, T)\Big|_{T\to 0} \approx -\left\langle \ln \frac{\mathrm{d}E}{T} \right\rangle_{E_p} + 1 + \frac{1}{4}\left(1 + \frac{2m}{T}\right) \exp\left(-m/T\right), \tag{23}$$

where all the higher-order exponents are omitted. This distribution is displayed in Figure 3. The entropy reaches a quite distinct maximum at $m/T \approx 0.5$ and quickly drops to unity at larger values of this ratio.

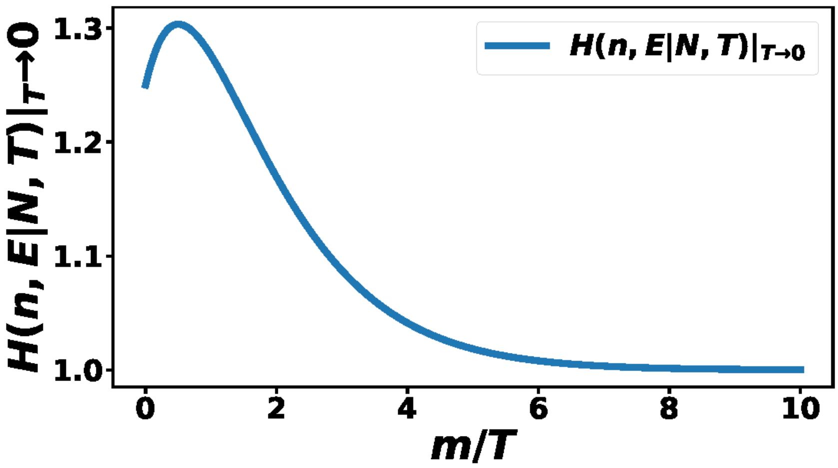

Figure 3. (Color online) Asymptotic behavior of entropy $H(n, E|N, T)$ given by Equation (23) at $T \to 0$ as function of m/T.

In the case of large acceleration $a \to \infty \Leftrightarrow T \to \infty$, one obtains from Equation (18)

$$\begin{aligned} \int_m^M p(E)\Big|_{T\to\infty} \mathrm{d}E &= \int_m^M \left(1 - \frac{E}{T} + \frac{E^2}{2T^2}\right) \mathrm{d}E + \mathcal{O}\left(1/T^3\right) \\ &= (M - m)\left(1 - \frac{M + m}{2T} + \frac{M^2 + Mm + m^2}{6T^2}\right) + \mathcal{O}\left(1/T^3\right), \end{aligned} \tag{24}$$

and, therefore,

$$H(E) = \ln \mathcal{D}_{E_p} - \frac{1}{\mathcal{D}_{E_p}} \int_m^M p(E) \ln p(E) \mathrm{d}E = \ln (M - m) - \frac{(M-m)^2}{24T^2} + \mathcal{O}\left(1/T^3\right) . \quad (25)$$

Thus, the conditional entropy $H(n|N, E, T)$ from Equation (10) becomes

$$H(n|N, E, T)\Big|_{T\to\infty} = \ln N - \frac{N^2 - 1}{24T^2} E^2 + \mathcal{O}\left(1/T^4\right) , \quad (26)$$

which, together with Equation (24), gives us

$$\frac{1}{\mathcal{D}_{E_p}} \int_m^M p(E) H(n|N, E, T) \mathrm{d}E = \ln N - \frac{M^2 + Mm + m^2}{72T^2} \left(N^2 - 1\right) + \mathcal{O}\left(1/T^3\right). \quad (27)$$

Finally, substituting Equations (25) and (27) into Equation (16), we obtain the desired asymptotics at high acceleration (or temperature)

$$\begin{aligned} H(n, E|N, T)\Big|_{T\to\infty} &= - \langle \ln \mathrm{d}E \rangle_{E_p} + 1 + \ln (M - m) + \ln N \\ &- \frac{(N^2 + 2)(M^2 + m^2) + (N^2 - 7) Mm}{72T^2} + \mathcal{O}\left(1/T^3\right) . \quad (28) \end{aligned}$$

The entropy asymptotics at $T \to \infty$ calculated according to Equation (28) is presented in Figure 4 for fermions $(N = 2)$ and in Figure 5 for the boson spectra with $N = 100$ and 1000 particles, respectively. At high temperatures, the entropy weakly depends on m and quickly increases with an increase in the value of M. The larger the number of particles, the steeper the rising slope. For $N = 1000$, the entropy seems to saturate at $M \geq 5$.

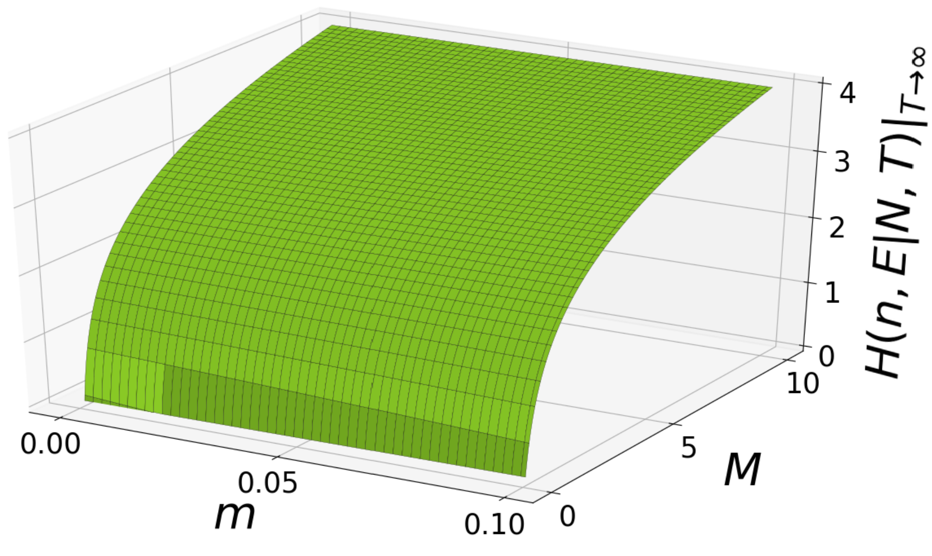

Figure 4. (Color online) High-temperature asymptotics of the entropy $H(n, E|N, T)$ of Unruh radiation given by Equation (28) for fermions $(N = 2)$ as a function of m and M.

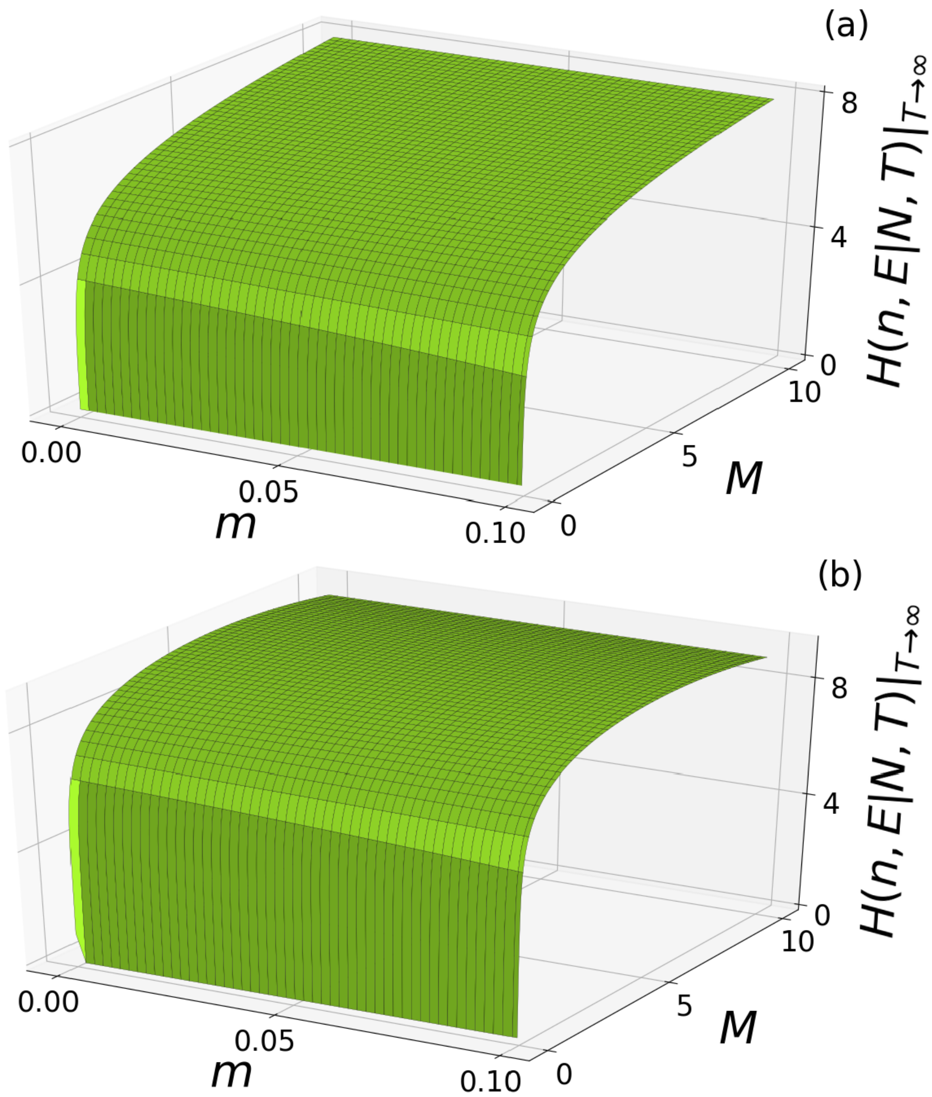

Figure 5. (Color online) The same as Figure 4 but for bosons with (**a**) $N = 100$ and (**b**) $N = 1000$ particles in the spectrum.

6. Generalization to Intrinsic Degrees of Freedom

Expression (17) is valid for some mode of the radiated field only, which is defined by the joint multiplicity-energy distribution $\{n, E\}$, temperature T and parameter N. However, since the emitted particles may have additional degrees of freedom $\{\lambda\}$, such as electric charge, spin, polarization, etc., they have to be taken into account too. This is equivalent to the following modification of the total distribution

$$\{n, E|N, T\} \to \{\lambda, n, E|N, T\} .$$

Using Equation (5), we then obtain

$$H(\lambda, n, E|N, T) = H(\lambda) + \langle H(n, E|N, T, \lambda)\rangle_\lambda . \tag{29}$$

However, such a generalization is not an easy task at all. Let us consider a simple example, while detecting a particle at some E, one should measure its energy. Such a process results in the consumption of the particle's momentum. One may argue that calorimetry is not required. The observer can build some source of similar particles and carry out interference experiments to determine the energy of the particle to be detected. However, any such interference will result in the re-distribution of the momenta during the interference and therefore will change the observer's momentum as well. Thus, one concludes that measuring the particle's energy E leads to a change in the observer's acceleration a. It implies a change in the Unruh temperature $T = a/(2\pi)$ of the source the observer is dealing with.

One may also note that the Unruh effect is being considered within the quasi-classical approach. It means that the density matrix ρ_{out} in Equation (9) is obtained under the assumption that the outgoing radiation has no influence on the background metric (see [5,35,36]). Such a remark is correct, but what about other degrees of freedom λ? For instance, taking into account the spin of the particles emitted by the Unruh horizon may lead to a change in the observer's angular momentum. In this case, the observer's acceleration a can not be constant due to the conservation of the total angular momentum anyway and thus implies a change in T in Equation (29) during particle identification.

Thus, the situation seems to be simple only if one neglects *any* influence of the outgoing particles during the Unruh effect. In this case the entropy $H(n, E|N, T, \lambda)$ does not depend on $\{\lambda\}$, and Expression (29) is reduced to the sum

$$H(\lambda, n, E|N, T) = H(\lambda) + H(n, E|N, T). \tag{30}$$

7. Conclusions

The Unruh effect is considered from the point of view of the information theory. We estimated the total entropy of the radiation generated by the Unruh horizon in the non-inertial reference frame for the state verified as vacuum by any inertial observer. Usually such a case is treated as von Neumann entropy of the corresponding density matrix. However, this is just the starting point of our study because the density matrix of the outgoing radiation describes the conditional multiplicity distribution at the given energy and Unruh temperature. As a result, it allows one to estimate the *total* entropy of the Unruh source by taking into account both the multiplicity and energy distribution of the outgoing quanta. We show how it can be calculated even without the exact knowledge of the corresponding Hamiltonian. In particular, such a lack of information results in the Schwinger-like spectrum of the emission (see Equation (15)).

The case of a finite amount of particle emission is considered. It allows us to utilize the results for realistic particle emission spectra. The asymptotics of the general expression for entropy with respect to low and high values of the Unruh temperature are also investigated. We found that in the case of small acceleration corresponding to a low temperature, the entropy of the radiation does not depend on the maximal amount of emitted particles in the leading order (see Equation (23)). The dependence on N is recovered for large accelerations when $T \to \infty$ (see Equation (28)). It can be explained by the abundant emission of particles from the hot Unruh horizon when the amount of the emitted quanta may be considered as an extra degree of freedom contributing to the total entropy.

Another interesting point is that the total entropy $H(n, E|N, T)$ quickly drops to zero with the increase in the mass m of the quanta. It can be explained by the energy conservation law: the more energy is being spent on the creation of particle's mass, the less of it may be used to generate the total distribution. At the same time, total entropy of the Unruh source slightly increases with the maximum allowed energy M because the distribution widens with the increase in M, thus leading to the total entropy increase.

The obtained results can be applied to the analysis of particle distributions in inelastic scattering processes at high energies. Furthermore, they may be generalized to other degrees of freedom of the emitted particles, such as spin, charges, etc. However, such a generalization may significantly complicate the analysis. For instance, additional conservation laws originating from the other degrees of freedom might change the metric. Therefore, one may be forced to take a distribution $\{T\}$ into account too.

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