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# **Estimation of Temperature-Dependent Band Parameters for Bi-Doped SnSe with High Thermoelectric Performance**

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**Abstract:** Recent studies have revealed the outstanding thermoelectric performance of Bi-doped n-type SnSe. In this regard, we analyzed the band parameters for  $\text{Sn}_{1-x}\text{Bi}_x\text{Se}$  (x = 0.00, 0.02, 0.04, and 0.06) using simple equations and the Single Parabolic Band model. Bi doping suppresses the carrierphonon coupling while increasing the density-of-states effective mass. The n-type SnSe is known to have two conduction bands converge near 600 K. Bi doping changes the temperature at which the band convergence occurs. When x = 0.04, its weighted mobility maximized near 500 K, which indicated the possible band convergence. The highest *zT* of the x = 0.04 sample at mid-temperatures (473–573 K) can be attributed to the engineered band convergence via Bi doping.

Keywords: thermoelectric; SnSe; B-factor; weighted mobility; single parabolic band model

## 1. Introduction

Research on various eco-friendly energy technologies has been conducted in various fields to mitigate the acceleration of global warming [1]. In particular, environmentally friendly energy harvesting technologies can play an important role in reducing greenhouse gas emissions, which are a main cause of climate change. Among many environmentally friendly technologies that can harvest energy without consuming fossil fuels, thermoelectric technology is drawing increasing attention because it can convert waste heat into electricity [2]. The conversion efficiency (from waste heat to electricity) of the device is closely coupled with the thermoelectric performance of a material used in the device. The thermoelectric performance of a material is evaluated with a figure of merit, zT, which is defined as  $zT = S^2 \sigma T / (k_e + k_l)$ , where S,  $\sigma$ , T,  $k_e$ , and  $k_l$  are the Seebeck coefficient, electrical conductivity, absolute temperature, electronic thermal conductivity, and lattice thermal conductivity, respectively. Here, the  $S^2\sigma$ , termed as the power factor, determines the electronic transport properties of the material [3]. As evident from the equation, a high zT can be achieved by using a material with a high power factor and low thermal conductivity. However, enhancing the power factor is not simple. The S and  $\sigma$  move in the opposite direction when the carrier concentration of a material is controlled with doping or alloying [4–12]. Commonly, several band engineering strategies have been introduced to decouple the trade-off relationship between the S and  $\sigma$  [13–17]. At the same time, materials with an inherently high power factor have been sought.

Kutorasinski et al. found that SnSe, a layered material, has highly anisotropic hole transport properties. Their study showed that p-type SnSe cannot transport charged carriers (holes) in the interlayer, whereas n-type SnSe can do so, thus yielding higher thermoelectric performance. Consequently, n-type SnSe is a better thermoelectric material, owing to its high power factor [4]. Recently, Duong et al. reported n-type Bi-doped SnSe with a maximum *zT* of 2.2 at 773 K with a high power factor while maintaining a low thermal conductivity [18]. They synthesized Bi-doped SnSe (Sn<sub>1-x</sub>Bi<sub>x</sub>Se; *x* = 0, 0.02, 0.04, and 0.06) and experimentally demonstrated that the power factor of the *x* = 0.04 sample improved



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). by more than a factor of eleven compared to the pristine SnSe (x = 0) at 572 K. Moreover, an exceptionally high zT of 2.2 was obtained in the x = 0.06 sample at 773 K. Because the total thermal conductivities ( $k_e + k_l$ ) of the x = 0.04 and 0.06 samples are similar, their zTs are closely related to their power factors. With a changing Bi doping content (x), the corresponding  $\sigma$  changed more significantly than their *S* did. However, the effect of Bi doping on electronic band parameters of Sn<sub>1-x</sub>Bi<sub>x</sub>Se was not provided.

Here, we estimate the band parameters, namely, the density-of-states effective mass  $(m_d^*)$ , non-degenerate mobility  $(\mu_0)$ , weighted mobility  $(\mu_W)$ , and *B*-factor of  $\text{Sn}_{1-x}\text{Bi}_x\text{Se}$  (x = 0, 0.02, 0.04, and 0.06) using the single parabolic band (SPB) model. From how the band parameters change with different Bi doping content, we evaluate the impact of Bi doping on the electronic transport properties of Bi-doped SnSe. At 300 K, Bi doping first decreases the  $\mu_W$ , which is directly proportional to the theoretical maximum power factor; however, when the x = 0.06, the  $\mu_W$  is improved by more than a factor of five compared to the pristine SnSe. A drastic increase in the 300 K power factor of the x = 0.06 sample is predicted. The 300 K  $\mu_W$  improvement is mostly due to the  $m_d^*$  increase with Bi doping, despite the substantial drop in the  $\mu_0$ .

## 2. Materials and Methods

Temperature-dependent *S* and Hall carrier concentration ( $n_H$ ) were obtained from the literature [18]. The experimental *S* and  $n_{H_i}$  both measured at 300 K, were used to plot the *S* as a function of  $n_H$ . The  $n_H$  was converted to chemical carrier concentration (n) using Equation (1) [19].

$$\frac{n}{n_H} = 1.17 - \left\lfloor \frac{0.216}{1 + \exp\left(\frac{|S| - 101}{67.1}\right)} \right\rfloor$$
(1)

The experimental *n*-dependent *S* at 300 K was converted to *S*-dependent  $\log_{10}(n)$  to estimate the  $m_d^*$ . The  $m_d^*$  of each sample was estimated by fitting Equation (2) into the experimental data expressed as *S*-dependent  $\log_{10}(n)$  [20].

$$\log_{10}\left(\frac{m_d^*T}{300}\right) = \frac{2}{3}\log_{10}(n) - \frac{2}{3}\left[20.3 - (0.00508 \times |S|) + \left(1.58 \times 0.967^{|S|}\right)\right]$$
(2)

Temperature-dependent  $\mu_0$  was estimated from the experimental  $n_H$ -dependent  $\mu_H$  at different temperatures using the Single Parabolic Band (SPB) model [21,22]. According to the SPB model under acoustic phonon scattering, the  $\mu_H$  is defined in terms of  $\mu_0$  and reduced fermi level  $\eta$  (Equation (3)).

$$\mu_H = \mu_0 \frac{F_{-1/2}(\eta)}{2F_0(\eta)} \tag{3}$$

The fermi integral of order  $i(F_i)$  in Equation (3) is defined as in Equation (4).

$$F_n(\eta) = \int_0^\infty \frac{\varepsilon^n}{1 + \exp(\varepsilon - \eta)} d\varepsilon$$
(4)

The  $n_H$  is defined in terms of  $m_d^*$  and  $\eta$  (Equation (5)).

$$n_H = \frac{16\pi}{3} \left( \frac{2m_d^* k_B T}{h^2} \right) \frac{(F_0(\eta))^2}{F_{-1/2}(\eta)}$$
(5)

Equations (3)–(5) were applied to the experimental  $n_H$ -dependent  $\mu_H$  to estimate  $\mu_0$ . The temperature-dependent  $\mu_W$  was calculated using Equation (6) [23].

$$\mu_{w} = \frac{3h^{3}\sigma}{8\pi e(2m_{e}k_{B}T)^{3/2}} \left[ \frac{\exp\left[\frac{|S|}{k_{B}/e} - 2\right]}{1 + \exp\left[-5\left(\frac{|S|}{k_{B}/e} - 1\right)\right]} + \frac{\frac{3}{\pi^{2}}\frac{|S|}{k_{B}/e}}{1 + \exp\left[5\left(\frac{|S|}{k_{B}/e} - 1\right)\right]} \right]$$
(6)

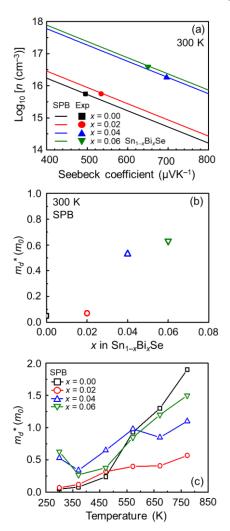
The *e*, *m*<sub>e</sub>, *h*, and *k*<sub>B</sub> are the electric charge, electron rest mass, Planck constant, and Boltzmann constant, respectively. Finally, the *B*-factor is estimated using the  $\mu_W$  and the lattice thermal conductivity ( $\kappa_l$ ), as in Equation (7).

$$B = \left(\frac{k_B}{e}\right)^2 \frac{8\pi e (2m_e k_B)^{3/2}}{h^3} \frac{\mu_W T^{5/2}}{\kappa_l}$$
(7)

#### 3. Results and Discussion

## 3.1. Calculation of Density-of-States Effective Mass, m<sub>d</sub>\*

Figure 1 shows the calculated  $m_d^*$  for  $Sn_{1-x}Bi_xSe$  (x = 0.00, 0.02, 0.04, and 0.06). Figure 1a shows the experimental |S| and  $n_H$  for varying x at 300 K expressed in  $log_{10}(n)$  versus |S| graph (in filled symbols). Because we could not measure the n directly, the measured  $n_H$  was converted to n using Equation (1) [19]:



**Figure 1.** (a) Calculated (lines) and experimental (symbols) magnitude of Seebeck coefficient (|S|)-dependent  $\log_{10}(n)$  for  $\operatorname{Sn}_{1-x}\operatorname{Bi}_x\operatorname{Se}(x = 0.00, 0.02, 0.04, \text{ and } 0.06)$  at 300 K [18], (b) calculated density-of-states effective mass ( $m_d^*$ ) varying with x in  $\operatorname{Sn}_{1-x}\operatorname{Bi}_x\operatorname{Se}(x = 0.00, 0.02, 0.04, \text{ and } 0.06)$  at 300 K, and (c) calculated  $m_d^*$  for  $\operatorname{Sn}_{1-x}\operatorname{Bi}_x\operatorname{Se}(x = 0.00, 0.02, 0.04, \text{ and } 0.06)$  varying with temperature (300–773 K).

The ratio of *n* to  $n_H$  is a Hall factor  $(r_H)$ , and it is a function of  $\eta$  (reduced fermi level). Because the *S* is determined by the  $\eta$  mathematically according to the SPB model, the  $r_H$  can also be expressed in terms of *S* instead of  $\eta$ . That is the reason why the  $r_H$  in Equation (1) is expressed in terms of *S*. The conversion from the  $n_H$ -dependent *S* to the *S*-dependent  $\log_{10}(n)$  is necessary to accurately estimate the  $m_d^*$  from it. The simple

and accurate equation that enabled us to calculate the  $m_d^*$  without having to worry about solving the fermi integrals in the SPB model numerically (Equation (4)) was expressed in terms of *S* and  $\log_{10}(n)$ , as shown in Equation (2) [20].

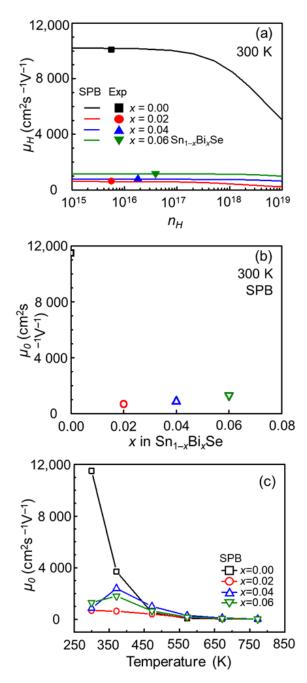
When the *T*, *n*, and *S* in units of K, cm<sup>-3</sup>, and  $\mu$ V K<sup>-1</sup> are inserted into Equation (2), the  $m_d^*$  in the unit of  $m_e$  is obtained. While varying S from 400 to 800  $\mu$ V K<sup>-1</sup>, the  $m_d^*$  in Equation (2) is fitted to describe the experimental data in the filled symbols. The results of fitting  $m_d^*$  using Equation (2) are presented in lines in Figure 1a. From the fact that the filled symbols are on top of the lines of the same color, we can conclude that the fitted  $m_d^*$ accurately represents the electronic band structure of the samples. From the experimental data (filled symbols), we can see that the n increases with increasing Bi doping content (x). However, the *S* only increases up to x = 0.04. The *S* of the x = 0.06 sample is lower than that of the x = 0.04 sample. It is to be noted that the x = 0.00 sample is p-type, while Bi-doped SnSe samples are all n-type. Figure 1b shows the 300 K  $m_d^*$  for different *x*. In general, the  $m_d^*$  increases with an increasing x. The rate of  $m_d^*$  increase is the most abrupt when the x increases from 0.02 to 0.04, but the  $m_d^*$  increases from x = 0 to 0.02 or from x = 0.04 to 0.06 are not substantial. As the x increases from 0 to 0.06, the calculated  $m_d^*$  increases from 0.05 to 0.63  $m_e$  (increasing by more than a factor of twelve). When we assume that there is only one band contributing to the electronic transport properties of  $Sn_{1-x}Bi_xSe$  (x = 0.00, 0.02, 0.04, and 0.06), the curvature of the band substantially increases with Bi doping at 300 K. How the  $m_d^*$  changes with temperature may also be affected by the amount of x. Figure 1c shows the temperature-dependent  $m_d^*$  for different values of x. Although the  $m_d^*$  of the x = 0.00 sample is the lowest among other x values at 300 K, it becomes the highest at 773 K. The rate of  $m_d$ \* change with temperature is different for different x values. Except for the x = 0.00 sample, the rate of  $m_d^*$  increase increases with an increasing x. Although varying in degree, local peaks are observed near 550–650 K for all x. This can be attributed to the two conduction bands converging near 600 K [24]. Because the two conduction bands converging near 600 K are calculated for n-type SnSe, Bi doping can alter the temperature at which the two conduction bands converge and even the curvature of the two bands. Because we employed the SPB model (only one band contributing to transport), if band convergence occurs, it will be reflected as an increase in the  $m_d^*$  of the assumed single band.

#### 3.2. Calculation of Non-Degenerate Mobility, $\mu_0$

Figure 2a shows the calculated (lines) and experimental (filled symbols)  $n_H$ -dependent  $\mu_H$  for Sn<sub>1-x</sub>Bi<sub>x</sub>Se (x = 0.00, 0.02, 0.04, and 0.06) at 300 K [18]. The  $n_H$ -dependent  $\mu_H$  calculated using the SPB model (denoted by lines) are in agreement with the experimental data (filled symbols) when the  $\mu_0$  is fitted accurately. According to the SPB model, the  $\mu_H$  and  $n_H$  are defined in Equations (3)–(5) [21,22].

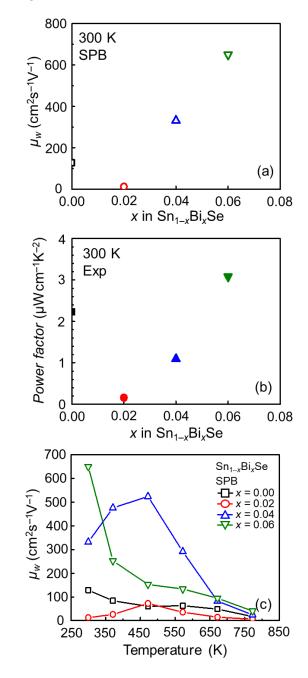
To calculate  $n_H$ -dependent  $\mu_H$ , two band parameters needed to be determined:  $\mu_0$ (Equation (3)) and  $m_d^*$  (Equation (5)). By substituting the  $m_d^*$  we obtained in Figure 1c into Equation 5, only the  $\mu_0$  could be estimated by fitting Equations (3)–(5) to the experimental data (filled symbols). The  $\mu_H$  of the x = 0.00 sample is approximately a factor of 10 greater than those of the Bi-doped samples. When Bi is doped to the x = 0.00 sample, the type of the sample changes from p- to n-type. Once Bi is doped, increasing x also increases the  $\mu_H$  and  $n_H$ , at the same time. The estimated  $\mu_0$  at 300 K is shown in Figure 2b. Similar to  $\mu_H$ , the  $\mu_0$ rapidly decreases as Bi doping commences, and subsequently increases with an increase in x. As the Bi-doping concentration, x, increases from 0 to 0.06 in steps of 0.02,  $\mu_0$  is calculated to be 11,500, 680, 860, and 1300 cm<sup>2</sup> s<sup>-1</sup> V<sup>-1</sup>, respectively. The  $\mu_0$  is always higher than  $\mu_H$ , because it represents the  $\mu_H$  without defects at low degeneracy. The  $\mu_0$ , in turn, depends on single band mass (related to  $m_d^*$ ) and deformation potential (related to carrier-phonon interaction). The heavier the band mass, the lower the  $\mu_0$ . If the deformation potential is high, corresponding  $\mu_0$  will be low. Because Bi doping increases the  $m_d^*$  at 300 K (Figure 1b), from the fact that Bi doping also increases the  $\mu_0$  at 300 K (Figure 2b) we can suggest that the interaction between the charged carriers and the phonons decreases with Bi doping at 300 K (favorable to electronic transport). The estimated  $\mu_0$  with temperature is plotted

in Figure 2c. The increasing carrier-phonon scattering at high temperatures and the  $m_d^*$  (Figure 1c) are responsible for the drop in  $\mu_0$  with temperatures. Among Bi-doped samples, unlike the x = 0.02 sample whose  $\mu_0$  decreases with temperatures, the  $\mu_0$  of x = 0.04 and 0.06 samples peak near 373 K. With the exception of 300 and 773 K, the  $\mu_0$  of x = 0.04 are higher than those of the x = 0.06 sample at all temperatures. Different trends observed in the temperature-dependent  $\mu_0$  can be understood in terms of how the  $m_d^*$  and deformation potential of the samples change with temperatures. At 773 K, SnSe has the lowest  $\mu_0$  of 6.4 cm<sup>2</sup> s<sup>-1</sup> V<sup>-1</sup>, whereas Sn<sub>1-x</sub>Bi<sub>x</sub>Se, with x = 0.02, 0.04, and 0.06, have  $\mu_0$  of 13, 21, and 22 cm<sup>2</sup> s<sup>-1</sup> V<sup>-1</sup>, respectively.



**Figure 2.** (a) Calculated (lines) and experimental (symbols)  $n_H$ -dependent Hall mobility ( $\mu_H$ ) for Sn<sub>1-x</sub>Bi<sub>x</sub>Se (x = 0.00, 0.02, 0.04, and 0.06) at 300 K [18], (b) calculated non-degenerate mobility ( $\mu_0$ ) varying with x in Sn<sub>1-x</sub>Bi<sub>x</sub>Se (x = 0.00, 0.02, 0.04, and 0.06) at 300 K, and (c) calculated  $\mu_0$  for Sn<sub>1-x</sub>Bi<sub>x</sub>Se (x = 0.00, 0.02, 0.04, and 0.06) varying with temperature (300–773 K).

Figure 3 presents  $\mu_W$  calculated from experimental  $\sigma$ , S, and T using Equation (6) [23]. Figure 3a shows  $\mu_W$  calculated at 300 K.



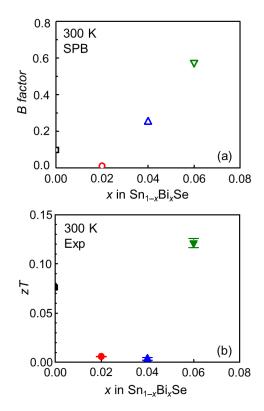
**Figure 3.** (a) Calculated weighted mobility ( $\mu_W$ ) varying with *x* in Sn<sub>1-*x*</sub>Bi<sub>*x*</sub>Se (*x* = 0.00, 0.02, 0.04, and 0.06) at 300 K, (b) experimental power factor varying with *x* in Sn<sub>1-*x*</sub>Bi<sub>*x*</sub>Se (*x* = 0.00, 0.02, 0.04, and 0.06) at 300 K [18], and (c) calculated  $\mu_W$  for Sn<sub>1-*x*</sub>Bi<sub>*x*</sub>Se (*x* = 0.00, 0.02, 0.04, and 0.06) varying with temperature (300–773 K).

At 300 K, the calculated  $\mu_W$  is highest when x = 0.06 (~650 cm<sup>2</sup> s<sup>-1</sup> V<sup>-1</sup>). Physically, the  $\mu_W$  is defined as a product between  $\mu_0$  and  $(m_d^*/m_e)^{3/2}$ , and it is directly proportional to the maximum power factor of a sample. The theoretically maximum power factor can be obtained when the  $n_H$  of the sample is optimally tuned. Hence, when the band parameter,  $\mu_W$ , of different samples are compared, a more promising sample (or composition) can be easily chosen. Therefore, the x = 0.06 sample with the highest  $\mu_W$  at 300 K tells us that the

x = 0.06 sample will exhibit the highest power factor among other samples once its  $n_H$  is at its optimum. The experimental power factor of  $Sn_{1-x}Bi_xSe$  (x = 0.00, 0.02, 0.04, and 0.06) at 300 K is provided in Figure 3b [18]. The plotted values in Figure 3a,b form similar V-shaped graphs. As the x in  $Sn_{1-x}Bi_xSe$  varies from 0 to 0.02,  $\mu_W$  and the power factor decrease. Subsequently, as x varies from 0.02 to 0.06,  $\mu_W$  and the power factor increase. The trends in  $\mu_W$  and power factor can be different from each other. The fact that both trends in  $\mu_W$ (Figure 3a) and power factor (Figure 3b) are comparable tells us that the  $n_H$  of all samples are not far from their optimum  $n_H$ . Figure 3c shows the variation in  $\mu_W$  with temperature, which is also obtained by Equation (6). The  $\mu_W$  can also be used to see if there is any band convergence benefitting the electronic transport properties. The  $\mu_W$  of the x = 0.02 and 0.04 samples peaks near 473 K. The highest  $\mu_W$  of x = 0.06 is estimated to be near 300 K, but the rate of  $\mu_W$  drop with temperature also decreases near 473 K. We believe that the electronic transport properties' improvement in Bi-doped SnSe due to band convergence is the strongest near 473 K. However, experimentally, the power factor of the x = 0.04 sample is the highest near 573 K [18]. However, if the  $n_H$  at 473 K is optimized, the power factor of the x = 0.04 sample at 473 K will be higher than that at 573 K.

## 3.4. Calculation of B-Factor

Figure 4 shows the calculated *B*-factor and the corresponding measured *zT* at 300 K [18]. The *B*-factor values shown in Figure 4a are calculated based on the  $\mu_W$  in Figure 3 and the experimental  $\kappa_I$  [18] using Equation (7) [25–30]:

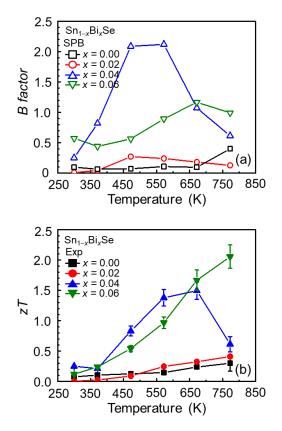


**Figure 4.** (a) Calculated *B*-factor varying with *x* in  $Sn_{1-x}Bi_xSe$  (*x* = 0.00, 0.02, 0.04, and 0.06) at 300 K, (b) experimental *zT* varying with *x* in  $Sn_{1-x}Bi_xSe$  (*x* = 0.00, 0.02, 0.04, and 0.06) at 300 K [18].

The trend observed in the *B*-factor (Figure 4a) is similar to that observed in  $\mu_W$  in Figure 3a. The *B*-factor first decreases with Bi doping, but with increasing *x* (Bi doping content), the corresponding *B*-factor increases. The *B*-factor when x = 0.02, which is almost 0, improves to ~0.58 when x = 0.06. The fact that the trend in the *x*-dependent *B*-factor is similar to that observed in the *x*-dependent  $\mu_W$  tells us that the 300 K  $\kappa_l$  of the Sn<sub>1-x</sub>Bi<sub>x</sub>Se are also similar to each other. If the discrepancy in  $\kappa_l$  among the samples with a different

*x* is large, the *x*-dependent  $\mu_W$  must have been changed in the *x*-dependent *B*-factor. The 300 K *B*-factor when x = 0.06 is almost a factor of six greater than that of the undoped SnSe (p-type). Although the  $\mu_W$  is directly proportional to the theoretical maximum power factor, the *B*-factor is not exactly proportional to the theoretical maximum *zT*. However, the *B*-factor is related to the theoretical maximum *zT*. Because the *B*-factor of the x = 0.06 sample has the highest *B*-factor, we also expect the highest 300 K *zT* in the x = 0.06 sample if its  $n_H$  is not too far from the optimum  $n_H$ . Figure 4b shows the experimental 300 K *zT* for different *x* values, and as predicted in Figure 4a, the *zT* of the x = 0.06 sample exhibits the highest *zT* of 0.12 [18]. Experimentally, the 300 K *zT* of the x = 0.02 and 0.04 are comparable (*zT*~0.005). However, the 300 K *B*-factor of the  $n_H$  is appropriately tuned, the 300 K *zT* when x = 0.04 can be improved to be much higher than *zT* of 0.05.

Figure 5 shows the temperature-dependent *B*-factor and corresponding measured *zT*. Although, the *B*-factor when x = 0.04 is lower than that when x = 0.06 at 300 K. For temperatures from 373 to 573 K, the *B*-factor of the x = 0.04 sample is the highest among other samples. However, the *B*-factor of the x = 0.06 sample is highest at temperatures higher than 673 K. Initially, the  $\mu_W$  of the x = 0.04 sample was the highest for temperatures from 373 to 573 K, with a peak  $\mu_W$  at 473 K (Figure 3c). In terms of the *B*-factor, the peak *B*-factor is observed at 573 K. This shift of the peak temperature when comparing the temperature  $\mu_W$  and *B*-factor originates from the  $\kappa_l$  decreasing with temperature. Only the *zT* of the x = 0.04 sample at 473 and 573 K are the highest, while its *B*-factor is the highest from 373 to 573 K. Upon  $n_H$  tuning, the *zT* of the x = 0.04 sample at 373 K can be much higher than that observed for the x = 0.06 sample at 373 K. Bi doping in SnSe is an effective strategy to improve its mid-temperature *zT*, as Bi doping can engineer band convergence to maximize its thermoelectric performance.



**Figure 5.** (a) Calculated temperature-dependent *B*-factor varying with *x* in  $Sn_{1-x}Bi_xSe$  (x = 0.00, 0.02, 0.04, and 0.06), (b) experimental temperature-dependent *zT* varying with *x* in  $Sn_{1-x}Bi_xSe$  (x = 0.00, 0.02, 0.04, and 0.06) [18].

## 4. Conclusions

Bi doping transforms SnSe from p-type to n-type, resulting in its high thermoelectric performance. In this study, we calculated four band parameters (density-of-states effective mass  $(m_d^*)$ , non-degenerate mobility  $(\mu_0)$ , weighted mobility  $(\mu_W)$ , and the *B*-factor) for Sn<sub>1-x</sub>Bi<sub>x</sub>Se (x = 0.00, 0.02, 0.04, and 0.06) using simple equations and the Single Parabolic Band model. Bi doping can increase both the  $m_d^*$  and the  $\mu_0$  at the same time to improve the electronic transport properties. The physical reason behind the simultaneous improvement in  $m_d^*$  and the  $\mu_0$  with Bi doping can be attributed to suppressed carrier-phonon interaction. Bi doping also changes how the band parameters change with temperature. The two conduction bands in n-type SnSe known to converge near 600 K now converge near 500 K when x = 0.04. The existence of the band convergence is confirmed by the temperature at which the  $\mu_W$  peaks. Due to the two conduction bands converging near 500 K in the x = 0.04 sample, the zT of the x = 0.04 sample is highest for temperatures between 473 and 573 K.

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