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Editorial

Numerical and Analytical Methods for Differential Equations and Systems

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The theory and applications of differential equations have played an essential role both in the development of mathematics and in exploring new horizons in the applied sciences. From a theoretical viewpoint, the qualitative theory of differential equations as well as analytical methods have contributed to the development of many new mathematical ideas and techniques for solving ordinary and partial differential equations as well as systems of differential equations. From the viewpoint of applications, differential equations are crucial and necessary for modelling many kinds of dynamical systems or processes in real life. Many problems are too complex to be amenable to analytical methods, and so numerical methods are also crucial in the investigation of differential equations and approximation of their solutions [1].

Topics related to the theoretical and numerical aspects of differential equations have been undergoing tremendous development for decades. There are so many different differential equations, with applications in so many different fields, that any number of contributions can easily be made to the theory and development of differential equations. A particularly fertile field for research is in fractional differential equations [2], since these were not often investigated until recent decades, so many significant advances can still be made.

This volume gathers together some recent research contributions to the study of differential equations and systems, using both numerical and analytical methods. Various new advances were made in the contributions listed here, but the study of differential equations is such a wide field that of course many problems are still open and further new advances are expected.

Out of 61 submissions to the special issue, 20 were finally published in this volume. Submissions were open from June 2021 until June 2022. In the current piece, we summarise the published articles one by one, grouping together some that are very closely related to each other.

Kumar et al. (contribution 1) and Almutairi et al. (contribution 3) study the oscillation of solutions for certain neutral differential equations, respectively third-order and fourth-order with delay, using the Riccati transformation to achieve results on oscillation, which are illustrated with examples.

Asjad et al. (contribution 2) use the method of Laplace transforms to provide analytical solutions for certain Prabhakar fractional differential equations, which are used to model a non-Newtonian fluid containing nanoparticles between two parallel plates.

Abdelhakem et al. (contribution 4) use a pseudo-Galerkin method, with a basis of derivatives of Chebyshev polynomials, to find approximate solutions to some boundary value problems of both linear and nonlinear types. They also include error analysis to verify the mathematical correctness of the method.

Ahmadova and Mahmudov (contribution 5) study Caputo fractional stochastic multi-term differential equations, and achieve global well-posedness results as well as asymptotic separation of solutions in general.

Avci (contribution 6) and Hashim et al. (contribution 8) study Caputo fractional delay differential equations using numerical methods. They use bases of Taylor polynomials (contribution 6) or shifted Chebyshev polynomials (contribution 8) to create operational matrices for fractional integration. Avci includes pseudocode for the computer programme he used, while Hashim et al. include a theorem on uniform convergence.

Branga (contribution 7) applies concepts from fixed point theory, in metric spaces over topological modules, to prove existence–uniqueness results for some integral equations including a fractional integral equation.

Pan et al. (contribution 9) and Pan et al. (contribution 16) are two studies on the dynamics of the COVID-19 pandemic. The first one uses sensitivity analysis to show that asymptomatic infections play a major role and seriously affect the endemic equilibrium, therefore it is important to control asymptomatic infections in order to combat the pandemic. The second one makes a compartmentalised study, categorising regions into high-risk, medium-risk, and low-risk areas, and concluding that the disease-free and endemic equilibria are both asymptotically stable.

Fernandez and Fahad (contribution 10) study general classes of fractional-calculus operators, both weighted and weighted with respect to functions, emphasising their relationships with the classical fractional calculus and how these classes include well-known operators such as Hadamard-type and Erdélyi–Kober fractional calculus.

Zhang et al. (contribution 11) study a nonlinear differential equation arising from the modelling of mixed convection boundary layer flow of a viscous fluid over an elastic surface, and solve this equation numerically using the spectral local linearisation method.

Moaaz et al. (contribution 12) obtain oscillation conditions for neutral differential equations of arbitrary even order, extending some existing results from the second-order case, assuming the non-canonical case.

Lu et al. (contribution 13) consider a fractional version of the Fokas equation in $4+1$ dimensions, using Riemann–Liouville fractional calculus. They obtain symmetry properties and conservation laws and solve the equation both analytically and numerically for comparison.

Sunday et al. (contribution 14) adapt a computation method known as the variable step hybrid block method to the differential equation arising from the Kepler problem. They describe the algorithm using a flow chart and consider its stability and convergence.

Duan et al. (contribution 15) study an incommensurate Caputo fractional differential equation and apply a numerical scheme using quadratic splines to solve it. They compare this with other methods using illustrative examples.

Fokas et al. (contribution 17) attack an important open problem in integrable systems by studying an integrable extension of the famous Kadomtsev–Petviashvili equation to three spatial dimensions. They construct multi-solitons and high-order breathers and rational solutions for this new equation.

Liu et al. (contribution 18) consider a Caputo time-fractional version of the nonlinear foam drainage equation and solve it numerically using a combination of Laplace transforms and the homotopy perturbation method.

Alzaleq and Manoranjan (contribution 19) study the Klein–Gordon equation with a cubic nonlinear term and solve it using an energy-conserving numerical scheme, whose stability and convergence are also verified mathematically.

Sultana et al. (contribution 20) apply a so-called "fractional novel analytic method", which is described as actually being a numerical method, to solve some nonlinear fractional partial differential equations. They provide examples to compare these numerical solutions with exact solutions for some well-known equations.

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