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Article

The Effect of a Nonlocal Thermoelastic Model on a Thermoelastic Material under Fractional Time Derivatives

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Abstract: This article develops a novel nonlocal theory of generalized thermoelastic material based on fractional time derivatives and Eringen's nonlocal thermoelasticity. An ultra-short pulse laser heats the surface of the medium's surrounding plane. Using the Laplace transform method, the basic equations and their accompanying boundary conditions were numerically solved. The distribution of thermal stress, temperature and displacement are physical variables for which the eigenvalues approach was employed to generate the analytical solution. Visual representations were used to examine the influence of the nonlocal parameters and fractional time derivative parameters on the wave propagation distributions of the physical fields for materials. The consideration of the nonlocal thermoelasticity theory (nonlocal elasticity and heat conduction) with fractional time derivatives may lead us to conclude that the variations in physical quantities are considerably impacted.

Keywords: fractional time derivative; nonlocal thermoelastic theory; Laplace transform; eigenvalue method

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1. Introduction

The advantages of the generalized thermoelastic theory have recently been extended to a variety of study domains, including nuclear reactors, rockets, and missile technologies, shipbuilding, and huge steam turbine mechanisms, thanks to ongoing advancements in this theory. Biot was the first to put forward the idea of the linked thermoelastic theory being subjected to the conventional Fourier's law [1]. However, since heat signals travel at an unlimited speed, this idea ran into complications. Since thermal waves are hyperbolic, several nonclassical heat conduction rules [2–4] have been developed to forecast this. Eringen was the one who initially promoted the nonlocal elastic hypothesis [5]. After two years, Eringen [6] investigated the nonlocal thermoelasticity hypothesis. In continuum mechanics, he addressed constitutive equations, laws of equilibrium, governing equations, displacement equation and temperature under nonlocal elasticity theory. Under the framework of elastic theory, Eringen [7] investigated nonlocal electromagnetic solids and superconductivity. Eringen [8] has given a thorough explanation of the relationship between nonlocal theories of field elasticity and continuum mechanics.

Fractional calculus describes the calculus of derivatives and integrals of any real or complex order. The nonlocal quality of models with long memories, which is their hereditary phenomenon, may be measured using fractional-order differential equations. Due to its many applications in viscoelasticity, bioengineering, biophysics continuum mechanics, signal and image processing, heat conduction, biology, diffusion, electrochemistry, and control theory, fractional calculus has lately attracted a lot of interest. Furthermore, it is challenging to describe the thermo-mechanical characteristics under the generalization of thermoelastic models, making use of fractional-order differential equations crucial for many biopolymers and physical processes with low temperature and transient loads. Other approaches have been put forward to extend the notions of derivatives and integrals to non-integral orders, and different definitions of fractional derivatives have also been developed.

In the context of generalized thermoelastic theories, the authors of Refs. [9,10] proposed the generalized fractional-order thermoelastic of low and high thermal conductivities. Another model for a fractional order generalized thermoelastic model, based on a time-fractional order Taylor expansion, has been put out by [11–13].

Hobiny et al. [14] investigated how the fractional time derivatives of the bio-heat model affected skin tissue exposed to laser radiation. Abbas [15] applied the eigenvalue approaches to study the fractional order model of thermoelastic diffusion problems for unbounded elastic media with spherical cavities. Lotfy [16] discussed the analytical solutions of the fractional time derivative heating equation with the effects of varying thermal conductivity during the photothermoelastic excitation of a semiconducting material with spherical cavities.

A nonlocal elastic rod's thermoelastic reaction to nonlocal heat conduction was studied by Sarkar [17]. The nonlocal thermoelasticity media with voids and fractional derivative heat transmissions were researched by Bachher and Sarkar [18]. Sarkar et al. [19] applied the LS model for the propagation of the photothermal wave in semiconductor nonlocal elastic media. Sarkar [20] studied the thermoelastic response of a finite rod under nonlocal thermal conduction. Lata and Singh [21] studied the deformations in a nonlocal magneto-thermoelastic solid with hall currents caused by normal forces. The effects of changing the heating source on a magneto-thermoelastic rod were investigated by Bayones et al. [22] under Eringen's nonlocal model with TPL and memory-dependent derivatives. Biswas [23] studied the propagations of plane waves in nonlocal viscothermoelastic porous media based on a nonlocal strain gradient model. Abouelregal et al. [24] investigated the temperature-dependent physical characteristics of the rotating nonlocal nanobeam under varying thermal sources and mechanical loading. Sheoran et al. [25] studied the thermomechanical interaction in nonlocal transversely isotropic materials with rotations. Abbas et al. [26] studied the generalized thermoelastic interactions in a plane under a nonlocal thermoelastic model. Hobiny et al. [27] studied the analytical solution of nonlocal thermoelastic interactions on a thermoelastic medium with ramp-type heating. Several researchers, including [28–42], also conducted studies using the thermoelastic theory.

The purpose of this article was to study the impacts of the fractional order derivative and nonlocal parameter in an elastic medium. The Laplace and eigenvalues methodology, which is based on numerical and analytical techniques, was used to address the governing equations. The numerical results for the relevant physical quantities were generated and visually represented.

2. Materials and Methods

Following the Eringen [6] and Lord and Shulman models [2], based on Ezzat et al. [43], the constitutive relations for a fractional-nonlocal, isotropic thermoelastic material are given as:

$$\rho\left(1-\beta^2\nabla^2\right)\frac{\partial^2 u_i}{\partial t^2} = \mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \gamma_t T_{,i}, \tag{1}$$

$$K\nabla^2 T = \left(1-\beta^2\nabla^2\right)\left(1+\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c_e\frac{\partial T}{\partial t} + \gamma_t T_o \frac{\partial u_{i,i}}{\partial t}\right),\ 0<\alpha\leq 1, \tag{2}$$

$$\left(1-\beta^2\nabla^2\right)\sigma_{ij} = (\lambda u_{k,k} - \gamma_t T)\delta_{ij} + \mu\left(u_{i,j}+u_{j,i}\right), \tag{3}$$

where ρ is the density of material, β is the parameter of nonlocal model, K is the heat conductivity, σ_{ij} are the components of stresses, $T = T^* - T_o$, T^* is the temperature variations, u_i is the displacements, λ, μ are the Lame's constants, t is the time, τ_o is the thermal relaxation time, $\gamma_t = (3\lambda+2\mu)\alpha_t$, α_t is the linear thermal expansion coefficient, c_e is the specific heat at constant strain and T_o refers to the reference temperature. Considering the previous definitions, we may say:

$$\frac{\partial^{\alpha} g(\boldsymbol{r},t)}{\partial t^{\alpha}} = \begin{cases} g(\boldsymbol{r},t) - g(\boldsymbol{r},0), & \alpha \to 0, \\ I^{\alpha-1} \frac{\partial g(\boldsymbol{r},t)}{\partial t}, & 0 < \alpha < 1, \\ \frac{\partial g(\boldsymbol{r},t)}{\partial t}, & \alpha = 1, \end{cases} \tag{4}$$

where I^{α} is the integral fraction of Riemann–Liouville, which are natural extensions of the well-known integral $I^{\alpha} g(\boldsymbol{r},t)$, for which a convolution type may be used to represent it.

$$I^{\alpha} \beta(\boldsymbol{r},t) = \int_0^t \frac{(t-s)^{\alpha}}{\Gamma(\alpha)} g(\boldsymbol{r},s) ds, \ \alpha > 0, \tag{5}$$

where $\Gamma(\alpha)$ is the gamma function and $g(\boldsymbol{r},t)$ is the Lebesgue integral function. In the case when it is unquestionably continuous, it is possible to write:

$$\lim_{\alpha \to 1} \frac{\partial^{\alpha} g(\boldsymbol{r},t)}{\partial t^{\alpha}} = \frac{\partial g(\boldsymbol{r},t)}{\partial t}, \tag{6}$$

where the different values of the fractional parameter cover two types of conductivity, $0 < \alpha \ \leq 1$, $\alpha = 1$ for normal conductivity and $0 < \alpha < 1$ for low conductivity as in Equations (4)–(6). Consider now a half-space ($x \geq 0$) in which the x-axis points into the medium. Consideration of the one-dimensional problem simplifies the analysis. The following are the displacement components for a one-dimensional medium:

$$u_x = u(x,t), u_y = 0, u_z = 0. \tag{7}$$

From Equation (7) in Equations (1)–(3), the basic equations can be expressed by

$$\rho \left(1 - \beta^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma_t \frac{\partial T}{\partial x}, \tag{8}$$

$$K \frac{\partial^2 T}{\partial x^2} = \left(1 - \beta^2 \frac{\partial^2}{\partial x^2}\right) \left(1 + \frac{\tau_o^{\alpha}}{\Gamma(\alpha+1)} \frac{\partial^{\alpha}}{\partial t^{\alpha}}\right) \left(\rho c_e \frac{\partial T}{\partial t} + \gamma_t T_o \frac{\partial^2 u}{\partial t \partial x}\right), \tag{9}$$

$$\left(1 - \beta^2 \frac{\partial^2}{\partial x^2}\right) \sigma_{xx} = \sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma_t T. \tag{10}$$

3. Initial and Boundary Conditions

Using the appropriate beginning and ending positions, the problem may be solved. The initial conditions are assumed to be:

$$T(x,0) = 0, u(x,0) = 0, \frac{\partial T(x,0)}{\partial t} = 0, \frac{\partial u(x,0)}{\partial t} = 0. \tag{11}$$

In contrast, the thermal and the mechanical boundary conditions may be expressed as:

$$\sigma_{xx} = 0. \tag{12}$$

Heat flux with an exponentially decaying pulse causes the surface $x = 0.0$ [44].

$$K \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} = -q_o \frac{t^2 e^{-\frac{t}{t_p}}}{16 t_p^2}, \tag{13}$$

in which t_p represents the characteristic time of pulse heat flux and q_o is a constant. To obtain the main fields in nondimensional formats, use the nondimensional variables listed below.

$$(x', u', \beta') = \eta c(x, u, \beta),\ T' = \frac{\gamma_t T}{\rho c^2}, \left(t', t'_p, \tau'_o\right) = \eta c^2\left(t, t_p, \tau_o\right),\ \sigma' = \frac{\sigma}{\rho c^2}, \tag{14}$$

where $c^2 = \frac{\lambda+2\mu}{\rho}$, $\eta = \frac{\rho c_e}{K}$. If the dashes are ignored and nondimensional types are used for the variables, the governing equations may be shown as follows:

$$\left(1 - \beta^2 \frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x}, \tag{15}$$

$$\frac{\partial^2 T}{\partial x^2} = \left(1 - \beta^2 \frac{\partial^2}{\partial x^2}\right)\left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial T}{\partial t} + \varepsilon \frac{\partial^2 u}{\partial t \partial x}\right), \tag{16}$$

$$\sigma = \frac{\partial u}{\partial x} - T, \tag{17}$$

$$T(x,0) = 0, \frac{\partial T(x,0)}{\partial t} = 0, u(x,0) = 0, \frac{\partial u(x,0)}{\partial t} = 0, \tag{18}$$

$$\sigma = 0\ ,\ \frac{\partial T}{\partial x} = -q_o \frac{t^2 e^{-\frac{t}{t_p}}}{16 t_p^2} \text{ on } x = 0, \tag{19}$$

where $\varepsilon = \frac{\gamma_t^2 T_o}{(\lambda+2\mu)\rho c_e}$.

4. Analytical Method

Applying the Laplace transform for Equations (15)–(19) can be expressed as:

$$\overline{f}(x,p) = L[f(x,t)] = \int_0^\infty f(x,t) e^{-pt} dt. \tag{20}$$

Hence, the following systems are obtained:

$$\left(1 - \beta^2 \frac{d^2}{dx^2}\right) p^2 \overline{u} = \frac{d^2 \overline{u}}{dx^2} - \frac{d\overline{T}}{dx}, \tag{21}$$

$$\frac{d^2 \overline{T}}{dx^2} = \left(1 - \beta^2 \frac{d^2}{dx^2}\right)\left(p + p^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\left(\overline{T} + \varepsilon \frac{d\overline{u}}{dx}\right), \tag{22}$$

$$\overline{\sigma} = \frac{d\overline{u}}{dx} - \overline{T}, \tag{23}$$

$$\overline{\sigma} = 0, \frac{d\overline{T}}{dx} = \frac{-q_o t_p}{8\left(p t_p + 1\right)^3} \text{ on } x = 0. \tag{24}$$

Equations (21) and (22) can be rewritten in the following forms:

$$\frac{d^2 \overline{u}}{dx^2} = a_{31}\overline{u} + a_{34}\frac{d\overline{T}}{dx}, \tag{25}$$

$$\frac{d^2 \overline{T}}{dx^2} = a_{41}\overline{u} + a_{42}\overline{T} + a_{43}\frac{d\overline{u}}{dx}, \tag{26}$$

where $a_{31} = \frac{p^2}{g}, a_{34} = \frac{1}{g}, a_{41} = \frac{-\beta^2\left(p + p^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)a_{31}}{h}, a_{42} = \frac{\left(p + p^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)}{h}$,
$a_{43} = \frac{\left(p + p^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\varepsilon}{h}, g = 1 + \beta^2 p^2,\ h = 1 + \beta^2\left(p + p^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)(1 + \varepsilon a_{34})$.

Now, the suggested eigenvalues techniques may be applied to obtain the solutions of coupled differential Equations (25) and (26) as in [45–48]:

$$\frac{dV}{dx} = AV, \tag{27}$$

where $V = \begin{bmatrix} \overline{u} & \overline{T} & \frac{d\overline{u}}{dx} & \frac{d\overline{T}}{dx} \end{bmatrix}^T$ and $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix}$.

As a result, the characteristic relationship of matrix A is assumed to be:

$$m^4 - m^2(a_{34}a_{43} + a_{31} + a_{42}) + ma_{34}a_{41} + a_{42}a_{31} = 0. \tag{28}$$

The four roots of Equation (28) that are designated here as $m_1,\ m_2,\ m_3$ and m_4 are the eigenvalues of matrix A. Thus, the corresponding eigenvector $X = [X_1, X_2, X_3, X_4]$ can be calculated as:

$$X_1 = ma_{34}, X_2 = m^2 - a_{31}, X_3 = m^2 a_{34}, X_4 = m\left(m^2 - a_{31}\right). \tag{29}$$

So, the analytical solution of Equation (27) can be given by:

$$V(x, p) = \sum_{i=1}^{4} B_i X_i e^{m_i x}, \tag{30}$$

where $B_1,\ B_2,\ B_3$ and B_4 are constants which are determined using the problem boundary conditions. The numerical inversion approaches used the final solutions of the temperature, displacement and stress distributions. The Stehfest method [49] may be provided by

$$f(x,t) = \frac{ln(2)}{t} \sum_{n=1}^{G} V_n \overline{f}\left(x, n\frac{ln(2)}{t}\right), \tag{31}$$

with

$$V_n = (-1)^{\left(\frac{G}{2}+1\right)} \sum_{p=\frac{n+1}{2}}^{min\left(n,\frac{G}{2}\right)} \frac{(2p)!p^{\left(\frac{G}{2}+1\right)}}{p!(n-p)!\left(\frac{G}{2}-p\right)!(2n-1)!}, \tag{32}$$

where G is the number of terms.

5. Results

The numerical data on the physical properties of copper-like materials were used to illustrate the behaviors of nondimensional field variables, viz. the displacement variation, the temperature increment and the stress distributions through several graphical representations. The physical statistics for the copper material, whose material qualities were taken into consideration by the writers, are supplied below.

$$\alpha_t = 1.78 \times 10^{-5}\left(\mathrm{k}^{-1}\right), \lambda = 7.760 \times 10^{10}(\mathrm{N})\left(\mathrm{m}^{-2}\right), t = 0.5,\ \beta = 0.3$$

$$\mu = 3.86 \times 10^{10}(\mathrm{N})\left(\mathrm{m}^{-2}\right),\ \rho = 8954(\mathrm{kg})\left(\mathrm{m}^{-3}\right),\ T_o = 293(\mathrm{k})$$

$$\tau_o = 0.05, t_p = 0.3, K = 386(\mathrm{N})\left(\mathrm{k}^{-1}\right)\left(\mathrm{s}^{-1}\right),\ c_e = 383.1\left(\mathrm{m}^2\right)\left(\mathrm{k}^{-1}\right).$$

Numerical calculations were carried out for two different cases: the impacts of nonlocal parameter and the fractional time derivative parameter. In the first case, as in Figures 1–3, we considered four different values of nonlocal parameter β, when the fractional time derivative $\alpha = 0.5$ and the thermal relaxation time $\tau_o = 0.05$. In the second case, as in Figures 4–6, we considered three different values for fractional time derivative parameter α under nonlocal thermoelastic model $\beta = 0.3$ and the thermal relaxation time $\tau_o = 0.05$.

The fluctuations in temperature, the displacement and the stress distributions for the first case are shown in Figures 1–3. From these figures, it was obtained that the temperature decreased with the increase in distances x, as in Figure 1. Figure 2 shows the variations of displacement along distances x. The magnitudes of displacement reduced with increasing distance until they were near to zero. Figure 3 depicts the stress fluctuations along the x-axis. It is evident that the magnitudes of stress began with zero values that fulfilled the boundary conditions of the problem, then dropped with increasing x to achieve maximum values, then rose again to reach zero values. Figures 4–6 show the effects of the fractional time derivative on the variation of temperature, the variation of displacement and the stress variations with respect to the distance x. In Figures 4–6, it is seen that the dotted line (. . .) refers to the Lord and Shulman nonlocal thermoelastic model when $\alpha = 1$, the dashed line (—) points to the fractional nonlocal thermoelastic model when $\alpha = 0.5$, while the solid line (—) points to the fractional nonlocal thermoelastic model when $\alpha = 0.1$. As expected, the fractional time derivative parameter had a great effect on the distributions of all field quantities.

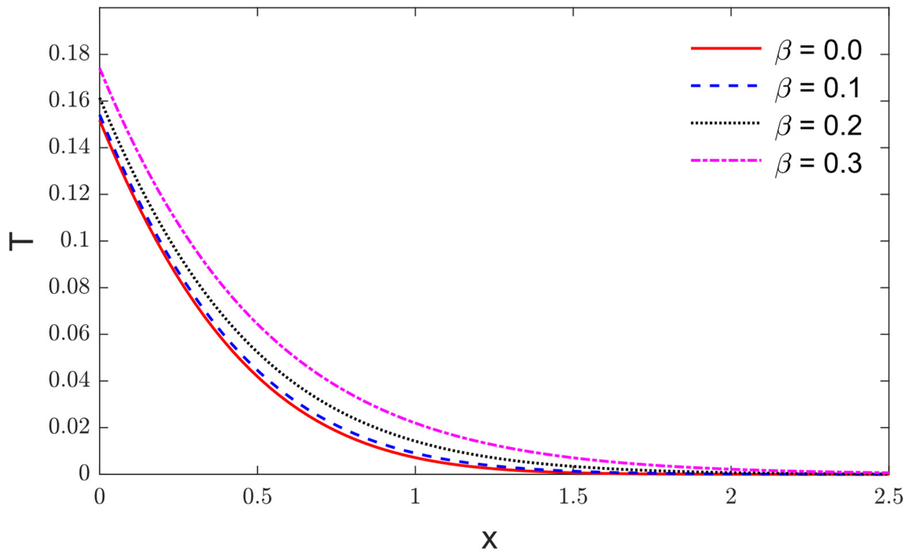

Figure 1. The impact of nonlocal parameter in the temperature variations along the distance x when $\alpha = 0.3$ and $\tau_o = 0.05$.

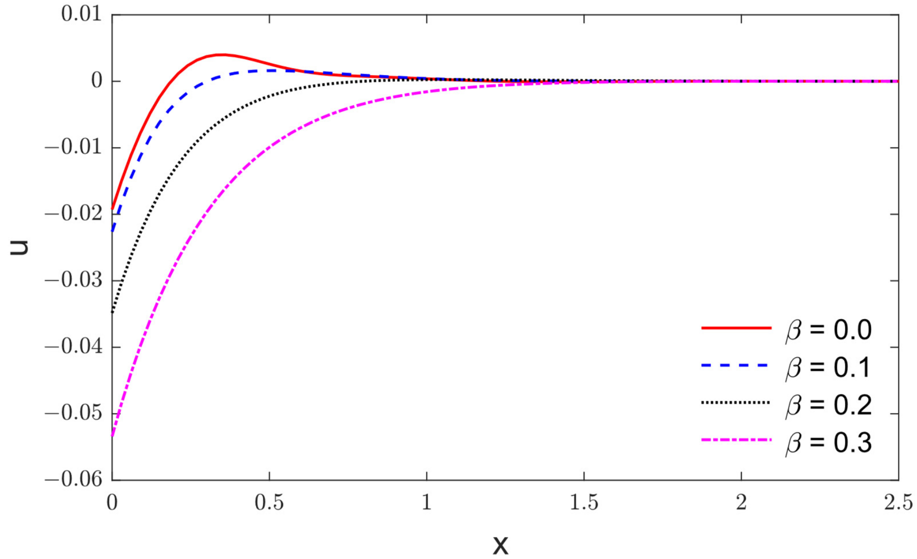

Figure 2. The impact of nonlocal parameter in the displacement variations along the distance x when $\alpha = 0.3$ and $\tau_o = 0.05$.

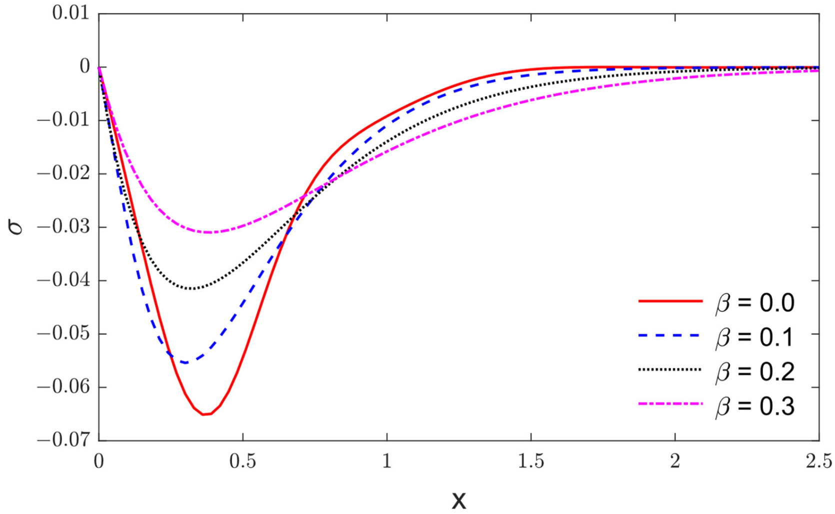

Figure 3. The impact of nonlocal parameter in the stress variations along the distance x when $\alpha = 0.3$ and $\tau_o = 0.05$.

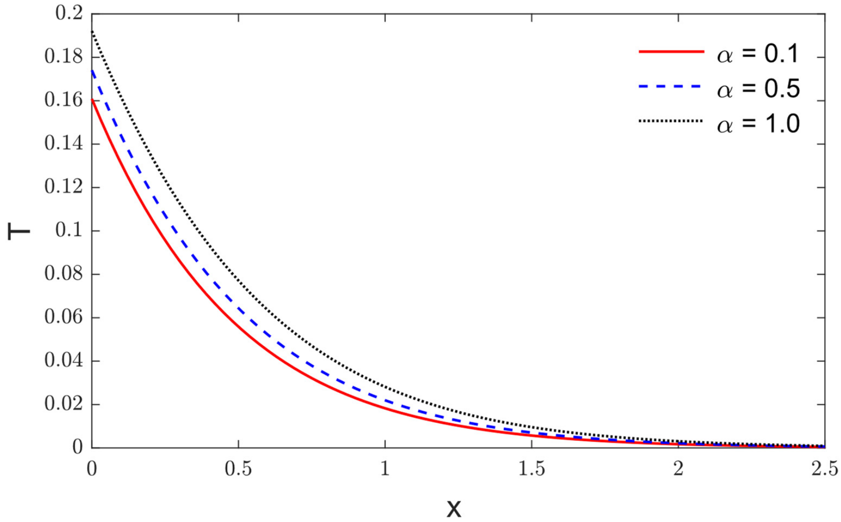

Figure 4. The impact of fractional parameter in the temperature variations along the distance x when $\beta = 0.3$ and $\tau_o = 0.05$.

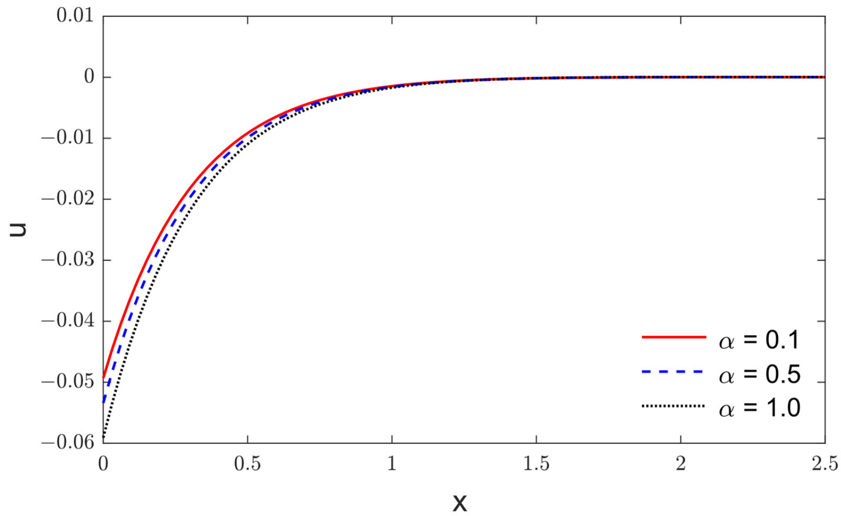

Figure 5. The impact of fractional parameter in the displacement variations along the distance x when $\beta = 0.3$ and $\tau_o = 0.05$.

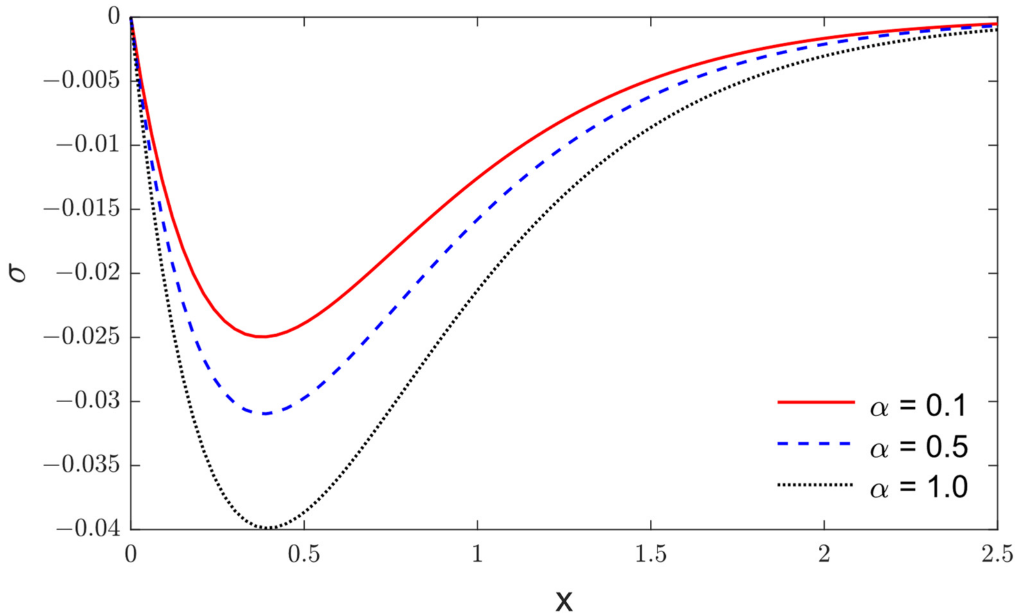

Figure 6. The impact of fractional parameter in the stress variations along the distance x when $\beta = 0.3$ and $\tau_o = 0.05$.

6. Conclusions

This present work explored the effects of fractional time derivatives in a thermoelastic material under a nonlocal thermoelastic model. The basic equations were applied under the model of Lord and Shulman with one relaxation time. The analytical expressions for the material's displacement, temperature, and stress were derived. We can conclude that the variations in the physical quantities are significantly influenced by considering the nonlocal thermoelasticity theory (nonlocal heat conduction and elasticity) with fractional time derivatives.

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