Multimodal Technologies and Interaction

MDPI

Article

Opportunities and Challenges of Bodily Interaction for Geometry Learning to Inform Technology Design

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Received: 23 May 2018; Accepted: 2 July 2018; Published: 9 July 2018

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Abstract: An increasing body of work provides evidence of the importance of bodily experience for cognition and the learning of mathematics. Sensor-based technologies have potential for guiding sensori-motor engagement with challenging mathematical ideas in new ways. Yet, designing environments that promote an appropriate sensori-motoric interaction that effectively supports salient foundations of mathematical concepts is challenging and requires understanding of opportunities and challenges that bodily interaction offers. This study aimed to better understand how young children can, and do, use their bodies to explore geometrical concepts of angle and shape, and what contribution the different sensori-motor experiences make to the comprehension of mathematical ideas. Twenty-nine students aged 6–10 years participated in an exploratory study, with paired and group activities designed to elicit intuitive bodily enactment of angles and shape. Our analysis, focusing on moment-by-moment bodily interactions, attended to gesture, action, facial expression, body posture and talk, illustrated the 'realms of possibilities' of bodily interaction, and highlighted challenges around 'felt' experience and egocentric vs. allocentric perception of the body during collaborative bodily enactment. These findings inform digital designs for sensory interaction to foreground salient geometric features and effectively support relevant forms of enactment to enhance the learning experience, supporting challenging aspects of interaction and exploiting the opportunities of the body.

Keywords: geometry; elementary mathematics; digital design; embodied cognition

1. Introduction

1.1. Problem Space and Motivation

Recent developments in sensor-based technologies have the potential to enhance learning experiences by foregrounding embodied, sensory and multi-modal elements of learning topics. For elementary mathematics learning, they bring new ways for children to use their bodies in exploring and engaging with mathematical ideas that can be challenging for young children, such as shape and angles. Theories of embodied cognition highlight the important role of experience, the sensory body, emotion and social interaction for cognition and learning [1], and provide sensori-motoric representations that will be used in later reasoning. The benefits of embodied cognition for mathematics learning are particularly well theorised (e.g., [2]) and evidenced (e.g., [3,4]), mathematics being grounded in the physical environment and based in perception and action [5]. Given this, research has begun to explore the role of the body in various contexts of mathematical learning (e.g., [6–8]), with some studies specifically examining bodily movement in the context of learning angles [9]. The value of 'mindful' movement is also thought to be important for thinking through or making sense of mathematical questions to find answers or create new insights not accessible through conventional means [10], in encoding those concepts [11], and in contributing to the phenomena being explored (e.g., triangle/shape) as well as providing representational meaning that is externally visible and accessible to others [12]. Furthermore, a large body of work on gesture draws on the idea that even 'off-line' cognition draws on these perceptual-action experiences, evidenced through the kinds of gestures used in communicating mathematical ideas [5]. This also suggests the importance of the specific underlying action experiences that give rise to the gestures used to explain or express or externalise mathematical ideas.

Our work seeks to inform the design of digital environments that can support appropriate 'grounding' actions for mathematical ideas by using body interaction with visuals and/or sound. However, designing effective technology is challenging. In order to inform the effective design of digital environments to support whole body interaction, we need an in-depth understanding of the ways in which the body can be used to explore shapes and angles, the opportunities this provides for students, and the challenges that this poses. By supporting the challenging aspects of interaction and exploiting the bodily opportunities, we can better inform technology designs that effectively support relevant forms of interaction and enhance the learning experience appropriately. This paper reports a study that examines children's bodily repertoires and experiences when exploring ideas of angles and shape.

1.2. Geometry Learning

Understanding two-dimensional geometric figures involves attending to their sides as well as their angles, whilst three-dimensional shapes introduce concepts such as faces and vertices. Elementary-aged children sometimes have difficulty conceptualising shapes and angles because, through visual perception alone, they do not distinguish between characteristics that are relevant and those that are not [13]. For example, paying attention to an angle as one of the attributes of the shape, and distinguishing this from measuring a side of a shape [14], or understanding that an angle is essentially a measure of rotation [15]. In the UK curriculum, students are encouraged to explore angles in terms of 'turns' or 'rotations'. However, a common misconception is students' tendency to associate the size of the angle with the length of the ray or line segment. Elementary children need to understand, for example, that for regular polygons, all angles are equal and all sides are the same length, and for irregular polygons, not all angles are equal and not all sides are the same length [15]. Beyond visual engagement, "learning processes exploit also perceptual-motor components . . . the body becomes essential in the teaching and learning processes" [13] (p. 11).

1.3. Body and Geometry

The role of the body in geometry learning is increasingly being recognised. Geometry originally involved an embodied performance of measurement: formally fixing objects in spatial relations of order and measurement, which takes place through the body in multidimensional space. Through visual communication methods and abstraction into two dimensional symbols, this bodily experience is 'left behind' [16]. Recent research addresses this through evidencing the importance of different sensory experiences in understanding geometric figures (e.g., [17]). Visual perception provides "direct access to the shape and never gives a complete apprehension of it" [18] (pp. 129–130). Experiencing the difference between looking down on a geometric figure on paper compared to being inside a shape, or the tactile experience of that same figure, is shown to be beneficial to learning [17].

Engaging in mindful movement has been shown to support geometry learning (e.g., [10–12]). A study on second and third grade children who were taught about angles, either verbally by a teacher or through mindful movements, found that the latter outperformed the former in terms of learning outcomes [11]. In this case, the movements being performed by the learners allowed for the concepts to be encoded, and for their thoughts and understandings to be externalised, cementing their learning. The 'walking scale geometry' study also aimed to leverage whole body interaction to better support mathematical learning [12], introducing an interactionist's perspective of embodied cognition to mathematical activity. Ma highlights ways in which the 'body' serves a dual purpose: one in representational meaning that is externally visible and accessible to others (e.g., an angle), supporting communication and negotiation of mathematical ideas, and one that is a contribution to the phenomena being explored (e.g., triangle/shape), where students' own bodies became mathematical objects (e.g., vertices of a triangle). In the context of measurement, Davidson & Ryberg [19] also identify the bodily-material resources as a communicative and illustrative resource for showing each other their understandings and instructing each other, as well as a cognitive auxiliary tool scaffolding knowledge building.

A central component of movement or gesture is 'enactment'. Barsalou highlights the importance of re-enaction of modality specific experiences: "Just as thinking about (or recognizing) a cup might involve sensorimotor preparation for grasping or drinking, thinking about an equilateral triangle might involve covertly re-enacting modality-specific experiences of physical measurement or the construction of geometric objects (e.g., forming a triangle with rigid, same-length segments)" [1] (p. 210). Indeed, Kaur [20] found that children's gestures and movement were important in supporting their decision-making on angle comparison tasks, and showed the importance of research that facilitates learners' integration of the 'static' and 'dynamic' perspectives on angle: dynamic being indicative of the role of movement.

A substantial body of work in the embodied cognition paradigm highlights the importance of gesture as evidence of the role of bodily experience in supporting children's thinking and communication of mathematical ideas (e.g., [21–23]). This work shows the significance of recognising how mathematical ideas are expressed through children's bodies, both in gesture and movement, providing evidence for the embodiment of cognitive processes [6,24,25]. In particular, the work demonstrates how children's use of gesture in describing or explaining mathematical ideas is grounded in bodily experience of interaction. This body of work primarily focuses on gesture as a body based action that provides evidence for the embodiment of cognitive processes with young adults. "In effect, one way of knowing a mathematical relationship is by being the relationship. In particular, learners can enact and therefore become mathematical relations by using gestures, an important type of body-based action" [6] (p. 479). The work suggests the importance of dynamic gestures, and given that gestures are thought to be grounded on bodily action and that specific actions and movement may be instrumental in shaping understanding [26], then effective dynamic engagement with geometrical concepts that underpins this is critical to cognitive development. Gesture and embodied interaction are also seen as important ways of having 'felt' experiences [27] and of 'feeling' meaning [28]. Our work speaks to this by examining ways in which children use their bodies to explore angles and shape and to understand the repertoires of action, as well as examining the degree to which salient mathematical concepts are foregrounded.

1.4. Digital Design and Embodied Interaction

The growth of ubiquitous technologies brings new possibilities for digitally enhancing physical environments and bodily forms of interaction. A growing body of research illustrates ways in which digital environments can foster embodied engagement in various learning contexts, including science, history, art and mathematics (e.g., [3,29–31]). While a number of technology applications have been developed to support geometry learning, typically these are designed for desktop computers (GeoGebra) or touch-based mobile devices (e.g., sketchpad) which rely on interaction with visual graphics and dynamic visualisation. Few digital applications have supported bodily interaction in the context of understanding shape and angles. Smith et al. [9] explored 8–10 year old children's understanding of angles through a body-based angle task in a game designed using Kinect for Windows. The software showed visualisations of angles based on the arm movements of participants in the game. While improved understanding of angles was found in post testing, suggesting that the game facilitated connection between physical and abstract representations, qualitative analysis also showed the difficulty some students experienced in making a connection between their body activity and a software visualisation of it. A key question here then is the degree to which it is the visualisations or the sensori-motor experience that underpins the concepts students are developing around angles. Sinclair et al. [13] highlight the importance of further research on technology design and its role in teaching and learning of geometry, while Núñez-Pacheco presses for "technology that takes into account the experiential and smart body that expands its semiotic world, providing new channels for self-expression and learning" [32] (p. 951). A further key question is what contribution different sensory experiences can make to the comprehension of mathematical ideas (such as shapes in geometry) and how can we augment sensory interaction to support effective mathematical thinking.

1.5. Study Rationale/Research Objectives

Given the potential of technology to support mindful movement interaction with geometrical concepts, our study seeks to inform the design of a digital serious game environment that encourages bodily movement using sensor-based technologies, such as Kinect or virtual reality. In order to do this, our work focuses on the kinds of bodily engagement that it is possible for young children to engage in, with respect to geometrical concepts of shape and angles. In this way, we can better understand the benefits and limitations of bodily exploration in this context—what movements children can and do make, and how they interpret/use them during interaction. These sensori-motor interactions provide the basis for gesturing that may act as evidence of embodiment of cognitive processes but is also valuable in informing the design of technology to augment/address these limitations in fruitful ways. This study seeks to understand what contribution different bodily experiences can make to the experience of mathematical ideas (such as shapes in geometry) and how we can augment bodily interaction to support effective mathematical thinking, specifically in relation to the following:

- Exploration of the 'bringing back in' of bodily experience [16];
- Identification of the role of enactment in engaging in mathematical ideas of shape;
- Understanding the role of the explorer in contributing to the phenomena being explored, and the role of the observer in interpreting bodily information made externally accessible to others;
- Identifying the opportunities and challenges of using the body to explore shape and angles.

1.6. Research Approach

The study proposes an embodied and enactive approach to learning. Enactive knowledge is not simply multisensory mediated knowledge, but knowledge stored in the form of motor responses and acquired by the act of doing. The approach is inspired by Nemirovsky et al.'s [33] study with undergraduate students' geometric problem solving, where the focus is on 'experiential, non-causal/mechanistic, descriptive, lived experience' to explore what children did, what they perceived, and how this shaped their engagement with the mathematical ideas. Like them, we also draw on the idea of 'realms of possibilities' taken from Husserl's notion of the 'horizon' and the role of perception in bodily awareness, in terms of peripersonal, extrapersonal space and proprioceptive space, and the 'possibilities' that arise within interaction or engagement with that space/objects. Poincaré (1952) discussed an example relevant to the present study: "How is it that we perceive, say, that something has been rotated 90 degrees? Poincaré pointed out that it is possible (but not necessary) for us to reposition our body at a 90-degree angle in such a way that we can view the object from its original vantage, and that envisioning such a possibility of restoring the original view is what makes us "see" that it has been rotated 90 degrees. He proposed that the same integration of perception and body motion is at the root of all geometric transformations. From the perspective of perceptuo-motor activity as an enactment of a realm of possibilities, the distinction between concrete and abstract is fluid and situated." [33] (p. 292). Thus, 'seeing' is not just related to vision, but also a perception of what something would like from a different perspective [34].

Davidsen & Ryberg [19] also highlight the importance of paying attention to bodily action/interaction in gaining insight into how children's understanding of scale is anchored in their bodily-material resources. By taking this approach, we aim to gain insight into the role of the body in expression and in communication, and the degree to which it supports enactment of shape and angles. In so doing, we can identify opportunities and challenges experienced by students and observed in their interaction to inform digital design that effectively enhances sensori-motor interaction.

2. Materials and Methods

The overall aim of this work is to better understand the role of the body in children's experience of mathematical ideas (such as shapes in geometry) and use this to inform a digital game design that aims to effectively foster bodily enactment for young children's (aged 6–10) learning of geometric concepts. In order to achieve this, as part of the study presented here, we collected data on children's spontaneous, intuitive body actions and bodily representation for illustrating angles and shape, to explore what contribution whole body sensory experiences can make to the engagement with mathematical ideas.

2.1. Participants

A total of 29 students with mixed ability in maths participated. Three one-hour sessions took place in a large room in two urban-based schools: one session comprised seven Year 3 children, aged between 7–8 years; another comprised twelve Year 5 students aged 9–10 years; and the third comprised ten Year 6 students aged 10–11 years. Students were selected on the basis of age (school class) and provision of informed parental and student consent.

2.2. Activities/Tasks

Activities were purposefully designed to ensure that a clear maths concept was explored in such a way that whole body movement was integral to the task. This drew on Henderson and Taimina's work [35] which sets out three ways of defining angles: an angle as a geometric figure (a pair of rays with a common endpoint), an angle as a dynamic figure (a turn or rotation), and an angle as a measure. Activities were mapped to the curriculum, and tailored to the curriculum requirements specific to the age group of each set of participants. The basis of the activities included the following tasks.

2.2.1. Activity 1: Using the Body to Make Angles

Children worked in groups of 3 or 4, each group having one facilitator. One child chose an angle depicted numerically on a piece of paper at random from an envelope (options were 30°, 45°, 60°, 90°, 135° and 180°) without the others seeing what it was. The facilitator checked that they understood the nature of the angle and then asked them to make the shape with their body, in whatever way they liked. The two other children had to describe the angle their peer was demonstrating, and guess what the angle was. Then, the other two children were asked to copy the angle with their bodies, and explore other ways they could represent it. Another child in the group of three was then asked to select a second angle from the envelope. They were asked to explore adding this angle to the original one, again using their bodies. Facilitators encouraged the children to 'think aloud' as they worked, using questions such as "what is the new angle?", "is it bigger or smaller than the first angle?" and "what is the combined angle in total?".

2.2.2. Activity 2: Using the Body to Make Shapes

Children again worked in groups of 3 or 4, each group having one facilitator. This time, the children worked together, and they chose a set of shape angles at random from an envelope. For example, groups of three chose from a selection of triangles e.g., 30, 30, 120 (isosceles triangle, two sides of equal length), or 60, 60, 60 (equilateral triangle) and were asked to make the shape together in their group, using their bodies. Once they had made the shape, they directed the facilitator to 'draw' their shape on the floor using coloured tape. Then, they picked a new shape with the same number of sides, and were asked to make this shape over the top of the first one, providing the basis for comparing different kinds of triangles. Then they were asked to direct the facilitator to 'draw' their new shape on the floor, on top of the old one, using different coloured tape.

2.2.3. Activity 3: Using the Body to Create Symmetry of Shapes

For this activity, all of the children worked together as one group. One child volunteered to select a shape at random from an envelope (e.g., a ⋆ or ⬟ or ⬢ or ■). They had to decide how many children they needed to make the shape, arrange themselves into the shape, and direct a facilitator to make the shape on the floor with tape. Children not in the shape were asked to describe it (e.g., Are the corners all the same? Or are some bigger than the others? What angles do the edges form? Which corners are acute? Are there any obtuse angles? Are there any right angles? Sides of equal length?) The facilitator then added a 'reflection' line with tape outside the figure, and asked the remaining children to make a reflection of the shape with reference to the line, thus creating two separate figures.

2.3. Procedure

Each session took one hour and comprised two of the activities outlined above. Children were divided into appropriate groups for the activities in which they participated (see Table 1).

Table 1. Activities undertaken by each year group.

	Making Angles	Making Shapes	Reflection
Year 3	×	×	
Year 5	×	×	
Year 6		×	×

Each group had one facilitator who implemented the activity for that group, prompted discussion and laid tape on the floor. An additional facilitator orchestrated the session as a whole. Once the activities were completed the facilitator elicited group responses to the activities, but due to time limitations in depth interviews were not possible.

2.4. Data Collection

Data was recorded with four video cameras positioned at each corner around the room. Each group was encouraged by the facilitator to work within the range of a camera, making full use of the space and ensuring at least one good audio source for each separate group. A sync clap was made at the beginning of each of the sessions to facilitate subsequent concurrent analysis of the different camera footage. All children wore different coloured baseball caps in order to facilitate tracking during video analysis, and to manage video consent requirements regarding subsequent use of images. The caps also each had a unique shape on them, to facilitate making filed notes without having to use the name of each child. All facilitators made observation field notes during and after each of the sessions.

2.5. Analysis

Our analytical approach used techniques drawn from qualitative video analysis, as described in Video In Qualitative Research [36]. Footage was analysed using an open source software tool called ELAN [37] Video data is mounted within an annotation frame and displayed alongside an area where the user can add their own text, linked to the time code of the film. The footage can be slowed down or stepped through frame by frame, and multiple video sources can be analysed synchronously [38]. This technique has previously been applied to the analysis of social interaction during other educational settings such as the use of space and tools by participants in music lessons [39], to explore children's collaborative interaction with technology [40] and to inform the design of serious games [41].

Here, the techniques were used to examine moment-by-moment bodily interactions, where modalities including gesture, action, facial expression, body posture and talk were considered (e.g., [42–44]). The analysis focused on perceptuo-motor activity related to mathematical ideas of shape and angles, specifically to identify: 'realms of possibilities' explored by the students (how this prompted questions, led to further actions or changes e.g., looking for alternative ways of solving the problem); ways in which understanding angles or shape were made visible through body movement; how the body was used for communicative purposes or elicitation of reasoning; and/cognitive tool; any gestures or actions that revealed children's reasoning.

3. Results and Discussion

The findings illustrate the complex relationships that emerge around a 'body in space', highlighting various important aspects that come into play when using the body as a resource. Three key themes emerged from the analysis across all groups: realms of possibilities of bodily positioning and action, egocentric vs. allocentric perspectives, and mapping of body parts to geometrical features. The results of this analysis are presented and discussed here, followed by a section exploring the implications for design of digital environments related to these mathematical activities, which aim to encourage the use of bodily interaction in space.

3.1. Realms of Possibilities

The students across all years used their arms, legs and whole bodies in a variety of ways to create the specified angles, add further angles, and to make shapes in groups, thus exploring alternative bodily representations (realms of possibility). A number of opportunities for engaging with these geometrical ideas were evident along with some significant constraints.

To enact specific angles, children across the age groups took a variety of body stances. They used their arms like a clock starting with one arm down and the other level with body (at 6 and 9 on clock) or with one arm up and the other level with body (at 12 and 3 on the clock), or out in front of the body like scissors (e.g., to demonstrate a 45-degree angle). They lay on the floor with legs in the air, the bend between trunk and body being the point of the angle, sat on the floor with their legs astride, used their feet (as in ballet positions), or stood bending at the waist, bending closer to the floor to make acute angles. This gave them a variety of bodily experiences that they could relate to different angles, and observe in one another's representations. Year 6 students also used their hands by joining the base of their palms to make angles (Figure 1). However, as the angles got bigger, children found it harder to represent them with the body: one's legs or hands cannot stretch beyond a certain point, thus larger angles became physically more difficult to enact, and progressing beyond 180 degrees was physically impossible.

However, bodily constraints also elicited more explicit engagement with the angle concept, prompting children to actively work out how they could enact larger angles. For example, when one Year 3 pair were asked to add a 135-degree angle to their initial 60-degree angle, they first tried to use their legs, but found they could not stretch far enough. This led to both children using their arms outstretched to make a horizontal 180-degree angle, then moving one arm down—adding "a bit more". Children's use of the body, thus, showed insightful ways into how they mapped their physical characteristics to the mathematical ideas. When making the angles as individuals, children sometimes started with a reference (0 degree or 90 degrees in this case) and either then made something smaller (e.g., in the case of 60 degrees, starting from reference of 90 degrees) or something bigger (e.g., in the case of 30 degrees, starting from a 0-degree reference). When representing an initial angle of 45 degrees, a child from a Year 5 pair used their arms like a clock, but in order to enact the 45 degrees, they put their arms in a 90 degree position to start and work down from, providing themselves with a base line from which to work. These examples also illustrate how children identify and make use of a reference point to begin thinking about the angle they were tasked to create, aligning with [10], and demonstrating the use of their bodies to enact the notion of a turn or 'rotation from here to there' [10].

Figure 1. Different bodily positioning and posture used to enact angles.

Children also used different body representations when mimicking or copying a peer enacting an angle, as part of Activity 1. Mimicking frequently led to children making an alternative representation of the shape with their own body as they thought through a mathematics problem, extending engagement in sensorimotor activity. For example, one child made a shape with their legs; the other two sat in front of her and mimicked the angle with their hands. However, mimicking also highlighted a key complexity around children's own body perception in space and provided insight into the 'felt' experience in relation to mathematical constructs. This is illustrated in the following example from a Year 3 pair. One child selected a right angle and used their arms like a clock to make the angle. Their peer interpreted the enacted representation as an 'obtuse' angle, so the facilitator helped the child to enact a more 'accurate' right angle, by suggesting she move her lower arm up a bit, enabling her peer to recognise the enactment as a right angle. However, when the second child was asked to copy this angle with their own body, they too adopted a similar body positioning, i.e., with their lower arm too low, making an obtuse angle. This indicates the difference between the children's felt experience of a right angle and their perception of their own body positioning in space, which is different from their actual or observed positioning.

3.2. Allocentric vs. Egocentric Perspectives

In this section, we focus on further insights that emerged during group work around children's felt experience and perception of their own bodies in space (egocentric), versus others' perceptions of them (allocentric), highlighting the opportunities and challenges of enacting and representing angles and shapes in group work. Across the various activities and year groups, there was evidence of significant differences between how children perceived the positioning of their own bodies in space and how this looked to others in the group, as well as differences that arose in terms of which body parts were being used to represent which features of a shape. This highlighted important constraints and challenges for how the body works as a tool in experiencing geometrical ideas. Here we illustrate examples of this from each of the sessions.

The first example involved three children from Year 5 forming a collaborative triangle. One child lay on the floor and assumed the role of the base of the triangle; the other two children enacted the edges in the vertical plane by standing at either end (one at head, one at feet) and joining hands over the top. However, one child was smaller than the other two, and being one of the standing pair meant that they could not easily link arms over the base child because of the disparity in their height. The children recognized this issue, and changed their positions so that the shortest child was the base (Figure 2). However, in order to maintain their balance, the standing children had to bend at the waist, so whilst their felt experience was that of making a triangle shape, they were actually creating a stretched pentagon. Of interest here is the juxtaposition of the value of the embodied experience of 'feeling' key features of shape, while at the same time exposing other key features of geometry that are not at the forefront of that 'feeling'.

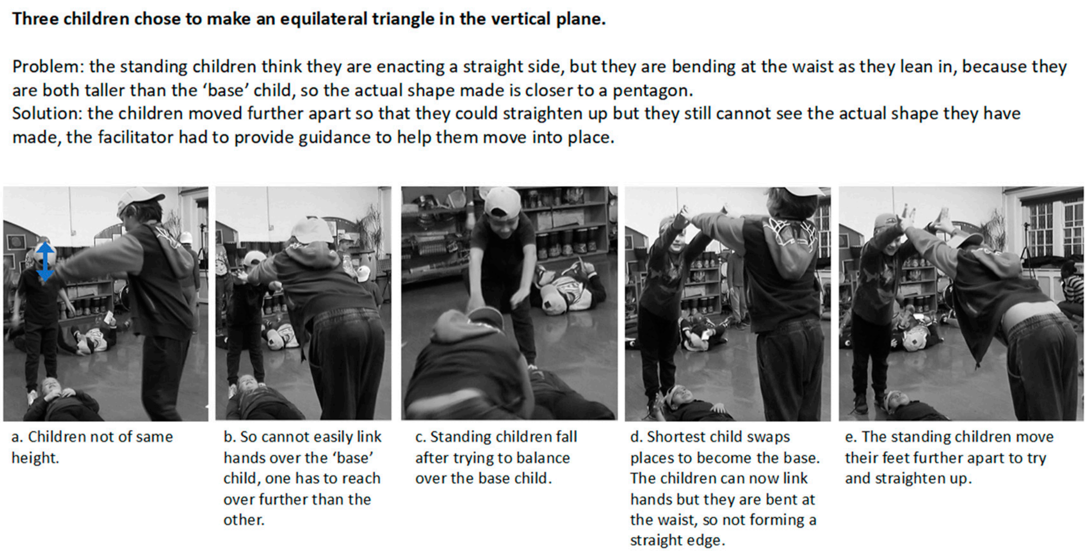

Figure 2. Three children making an equilateral triangle with their bodies.

The second example involved a Year 6 group making a triangle shape with three people, followed by another three children forming another triangle that was a reflection of the first. While the second group created a well formed reflective triangle, they were positioned too close to the (sticky taped) mirror line to enact a true reflection. It was the observing children who identified this, rather than the 'enactors', resulting in the observers moving the mirror line to create a more accurate reflection. Again, this illustrates the challenges of positioning bodies from an egocentric perspective, and differences between 'felt' experience and representational enactment. However, this example also highlights the importance of the socially shared experience of collaboration, in this case, the observer's contribution in being able to identify these differences. This in turn provided more opportunities for exploration and reflection on the properties of shape and of reflection/symmetry.

These examples demonstrate ways in which children's felt experience and internal perception of the shape or angle they are making can be different from the actual shape their bodies form. Yet, they also illustrate how bodily constraints in collaborative contexts can prompt reflection about key mathematical features of shape: children recognised inaccuracies of angle or shape from their bodily representations and sought to reconfigure their body positioning to try to accommodate different, yet salient, features of the shape. This finding highlights the value of collaborative activity, where both actors and observers participate.

Mapping Body Parts to Angles and Shape

Our analysis showed how children differently used parts of their bodies and limbs to enact angles and shapes, giving rise to challenges in both enactment and in perception of mapping. For example, where groups of children are of a similar size, their arms tend to be roughly the same length, raising challenges when enacting triangles with sides of different lengths, such as isosceles. However, it did raise opportunities to engage children in explicitly thinking about the critical features of shape and angles. A repeated observation was that children often joined hands with bent elbows, rather than straight arms, so the concept of a straight edge was less well enacted. Despite the facilitator suggesting that they straighten their arms so that their joined hands and arms made one straight line, the children did not intuitively adopt this representation of triangle sides. While this concurs with the proposal in [45] that in child development, topological features are salient prior to Euclidean, other research shows that topological features do not necessarily take this 'primacy position' [46]. This may suggest that children need support in attending to and integrating all of these geometric features.

Year 6 children often used their bodies to create collaborative shapes by lying on the floor or by bending their bodies in middle. Both of these formations led to representational difficulties, for example, the latter automatically resulted in the body creating two sides, rather than one. However, this did lead them to explore ways to achieve appropriate lengths, for example, one child lying on the floor stretching their arms out above their head, to make one side of an isosceles triangle longer than other sides enacted with body length (without stretched arms).

Group shape formation elicited differences in perception of body mapping. One example occurred during Activity 2 when Year 5 children were making a triangle from a standing position and holding hands to create the shape. Children did not naturally consider their upright bodies as vertical 'points' at vertices; instead, they often had their feet spread apart, the lower body not being aligned with the upper body, and their arms representing the sides. It was unclear where the vertices of their triangle should be: their bodies or where their hands met. One child gestured with her hands to explain how to position themselves indicating the body as the vertices, while another indicated that where their hands join should be the points of the triangle. When they moved on to represent the next shape, some children continued to use their bodies as the vertices while others positioned their joined hands as vertices, illustrating the differences in perception of mapping body parts to geometrical figure features (Figure 3).

Figure 3. Children on the far left and right of the image use their bodies as vertices, while children between them use hands as vertices.

Similar issues occurred when the Year 3 groups were asked to make a collaborative pentagon, followed by the floor taping exercise. They decided they needed five people to make the shape, leaving two children to act as observers, who were tasked with putting the five children into a pentagon shape. Here, we have subjective and objective views of the shape. From both perspectives, there was disagreement about what part of the body should act as the side and which part as the vertices of the shape. With facilitator support, the children made their pentagon; then, the two observers placed tape along the sides of where the children were standing. In so doing, they revealed discrepancies between the shape the group thought they had made and the shape they had actually made. This physical representation of the discrepancy allowed the observers and the group enacting the pentagon to identify the important features needed to fully represent the shape. For example, they realised that they had too many vertices when the tape was placed on the floor—6 instead of 5—and noted that the sides of the tape on the floor were not the same length, illustrating their recognition of how their representation deviated from their understanding of the necessary features of the shape. They pulled the tape from the floor and checked the lengths against one another, rectifying the lengths by tearing pieces from some strips and adding them to others, to make them similar in length. They then used the five adjusted lengths of tape to make another pentagon on the floor.

This also demonstrates the difference in perception of the body and the representative idea it is aiming to convey. Interestingly, when they were asked to make a reflection of this shape using their bodies again, the children enacted the shape with their bodies as the vertices, and their joined arms as sides of the shape. This formation was easier to map by taping on the floor, as someone on the ground used the children's feet as a reference point for the vertices. When joined hands are vertices, the feet become the central point in a side rather than a point in a vertex, and this is harder to map. This illustrates that the body parts themselves convey different information, and are instrumental in their use in terms of mapping and interpretation. The taped representation on the floor again providing an important opportunity for the children to reflect on the similarities and differences of the two shapes, and discuss the salient features of the shape and its reflection in symmetry. From one perspective, the children's joined hands can form corners, for example, from a bird's eye view; from another, the feet are more salient in creating a corner rather than a side, for example when taping on the floor at ground level.

4. Implications for Digital Design

This study explored primary school children's use of their bodies to engage with mathematical ideas of angles and shape, and highlights important considerations for how the body works as a tool in experiencing geometrical ideas. The findings show a complex relationship between bodies in space and socially shared constructions of instantiations of mathematical figures, highlighting opportunities for productive collaborative exploration and reflection, as well as significant constraints in terms of physical interaction and sensory perception.

While it is perhaps not surprising that our bodies cannot be positioned to enact every kind of angle or shape, given our physiological make-up, the specific limitations are important findings given the focus on embodiment in the current landscape of human–computer interaction, digital design and learning. Since sensori-motor experiences are considered the foundation for cognition [2], and action experiences are precursors to enactment and gestural abstraction of ideas [1], the kind of experiences we design for children's interaction with mathematical ideas is critical. Given the potential of digital technology design to encourage particular action experiences, it is important to know the limitations and possibilities of bodily enactment, to inform designs that provide an experience that is rich enough to be provide an effective learning experience, yet sensitive enough to acknowledge the limitations of body enactments, and aware of the nuances in order to exploit appropriate movement, yet facilitate reflection on inaccuracies due to limitations that encourage bodily reconfiguration.

One of the difficulties encountered in using bodies to illustrate or represent angles was the limitations of stretching beyond acute angles to create obtuse or reflex angles. This points to designs that exploit the angles that bodies can easily form, without limiting the digital experience to acute angles. One way to achieve might be to engage several children in contributing a body position to the construction of an angle, for example, a 135 angle made up of 90 and 45 degrees, where an external visualization of some kind shows the combined angle, alongside their respective individual angles. In this way, the subparts of the whole can be also be explored.

Differences in children's physical make-up—the lengths of their arms, legs, fingers or bodies—also influenced their collaborative shape making, and has the potential to lead to misconceptions of the properties of regular polygons. These findings suggest that digital design and external representation may offer ways to support reflection about the children's constructed shapes. For example, digital designs utilizing wearable sensors which can stream data in real-time to provide live representations of the children's formations. This could be used to provide the alternative view point, mapped against a formal representation of the shape the children have been asked to enact. This leads to considerations such as when such it might be appropriate to provide such an external visualization; for example, providing it too early may reduce the opportunities for exploration. Another consideration is whether the 'visualisation' should be provided through other modalities such as haptic or audio feedback. Using visual representations of lines and vertices may support children in creating shapes, but perhaps more importantly is the ability for them to identify discrepancies, which is an important elicitation of their understanding of shape. The use of haptic feedback in highlighting boundaries of the shape through vibration sensors may offer alternative ways of shaping the children's positions relative to one another and the reference shape, as well as making discrepancies explicit. The introduction of digital data representation of the children's forms relative to a reference could enable a form of scoring system, to introduce elements of gaming and competition, which can motivate children to explore and experiment [47].

Across the various activities and year groups, there was evidence of significant differences in how children perceived the positioning of their own bodies in space and how this looked to others in the group. Overall, this finding suggests the need for designs to promote awareness or foster the development of awareness of the body in space. An important consideration is access to different perspectives, as highlighted previously in the different interpretations made by an observer compared to those children in the enacting group. A bird's eye view, from above, is less available to the participants enacting the shape without some kind of external visualization. It is also less available to an observer unless they have a high point to view from (for example, standing on a chair). An additional representation of the group's movements and relative positions in space may be a useful addition when translating the activity to a digital domain, and feedback would not necessarily have to be visual, allowing the introduction of multimodal feedback. The use of sound or haptic feedback could indicate when the children's physical representation meets the key criteria for significant angles, such as 90, 180 and 45 degrees, and may be productive in enabling children to develop body awareness, and support enactment. An alternative could be to link to visual representations, as in Smith et al. where physical arm movements represented angles which were connected with abstract representations (i.e., a visualisation), an underlying assumption being that the perceived body position is a 'good' mapping to the concrete representation of an angle. Our study suggests that the perceived body perception can be challenging, and some students make strong associations between spontaneously generated movement and their reasoning of the concept [6]. Thus, in cases where children's movement is not properly aligned with the concept, this may be problematic. Similar events may have contributed to the difficulties experienced by children shown through qualitative analysis in [9].

As we have seen above, bodies are hard to position from an egocentric perspective, yet observers can see discrepancies in others' actions and positioning. When making angles with their bodies, as individuals or in groups, as part of forming a shape, children were less aware of how their own felt experience shaped their enactment, but could see when another child's 'felt' enactment did not map to an expected representational stance. Digital designs provide important opportunities for making explicit the links between 'felt' and 'overt' bodily experience, fostering new felt experiences of the properties of angles, shape and of reflection/symmetry. Eliciting digital reflections of group shapes would make explicit how children's shapes differ from the image of one given to them, providing opportunities for them to identify key features of the figures and new opportunities for using their bodies to support their understanding of the interrelationship of properties of shape.

Author Contributions: Conceptualization, S.D. and S.P.; Methodology, S.P. and S.D.; Formal Analysis, S.P. and S.D.; Investigation, S.P. and S.D.; Resources, S.D.; Writing-Original Draft Preparation, S.P.; Writing-Review & Editing, S.D. and S.P.; Supervision, S.P.; Funding Acquisition, WeDRAW project team.

Funding: This publication is part of weDRAW project that has received funding from the European Union's Horizon 2020 research and innovation programme under Grant Agreement No. 732391.

Acknowledgments: We would like to thank all of the pupils from the London schools who participated in the study, their teachers for helping to organize the study, and to Temi Olugbade, Rose Johnson, Joe Newbold and Nadia Berthouze from UCL for their help in facilitating the sessions and data collection.

Conflicts of Interest: The authors declare no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

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