

Supplementary

PATTERNED COLOURING VIA VARIABLE SPEED SINGLE STRETCHING

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S1 VIDEOS

Video S1. Polyethylene terephthalate (PET) under uniaxial stretching at periodically varied speed

<https://www.youtube.com/watch?v=FvuM48sgIA4>

Video S2 Polyvinyl chloride (PVC) under uniaxial stretching at periodically varied speed

<https://www.youtube.com/watch?v=M2gcL37Eu-k>

S2 PVC TRANSPARENCY

An A4 sized commercial PVC transparency (0.2 mm thick) for overhead projector was used in this study. **Figure S1** is the differential scanning calorimetry (DSC) result of this PVC under two continuous cycles at a heating/cooling speed of 10 °C/min. It appears that its glass transition temperature is about 72 °C.

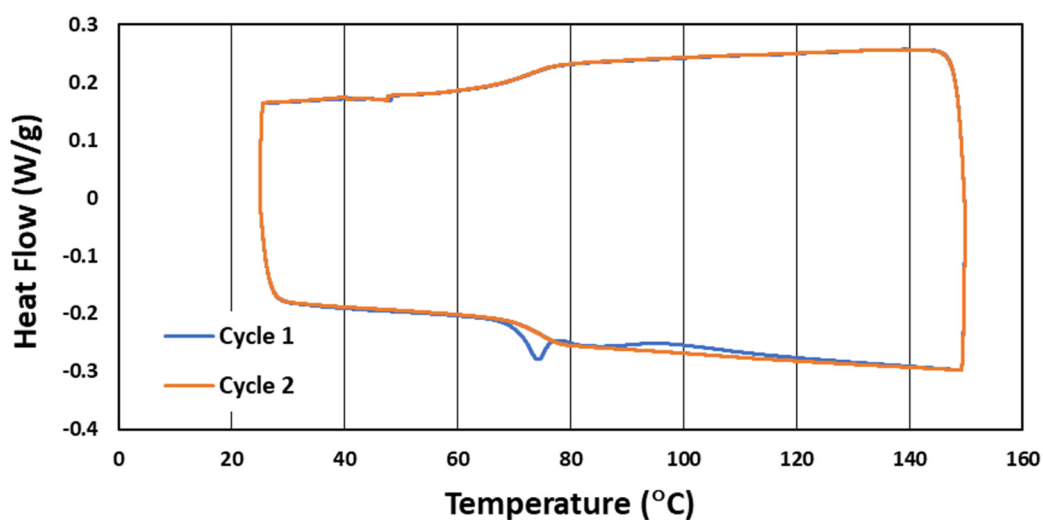


Figure S1. DSC result of PVC. (Heating/cooling speed: 10 °C/min)

Stress whitening upon folding at room temperature (about 22 °C) is observed (**Figure S2a**).

Upon heating to 100 °C, the folding lines all disappear (**Figure S2b**).

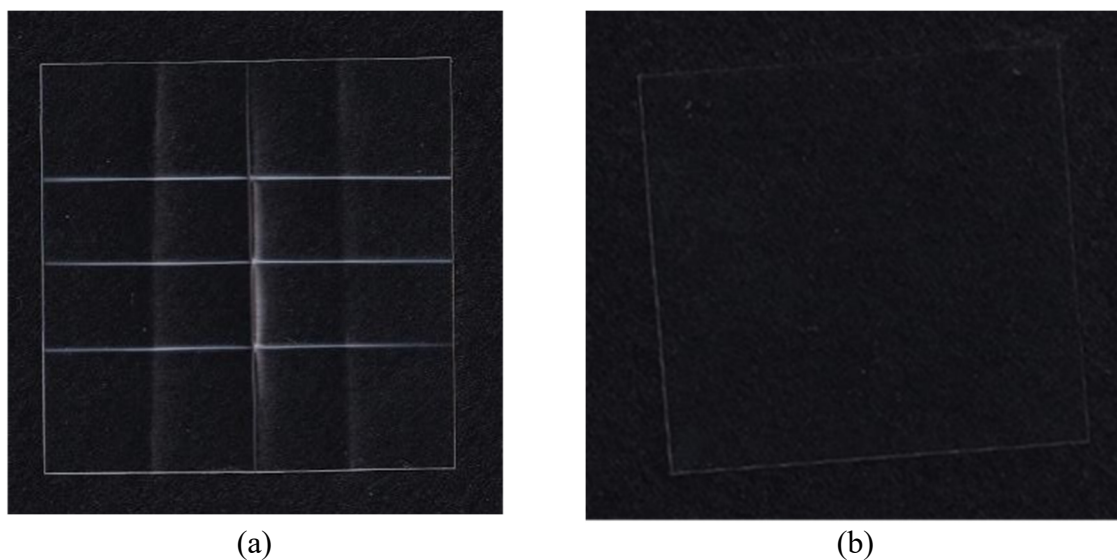


Figure S2. Stress whitening in PVC transparency after folding at room temperature (a) and after heating to 100 °C for recovery.

Dog-bone shaped samples (ASTM D638 Standard Type V) were used for uniaxial stretching at periodically varied strain rates. Refer to Video 2 (above) for an infrared video of two typical tests. Two typical samples after stretching are presented in **Figure S3**. Whitening effect is observed at one speed, but the colour pattern is apparently irregular.

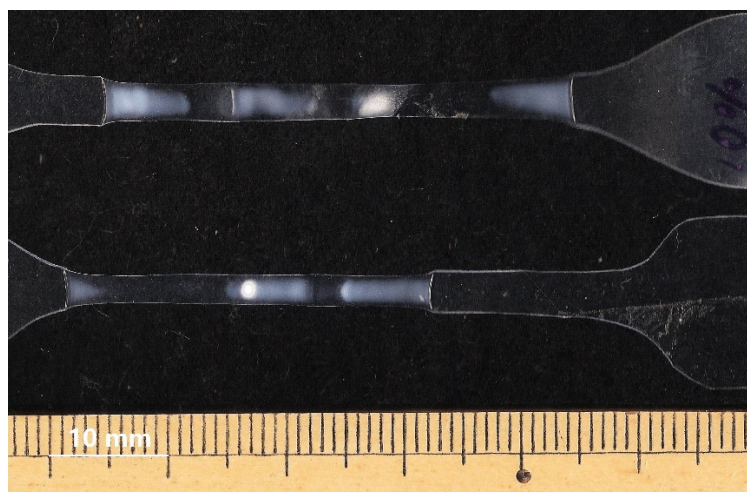


Figure S3. Typical PVC samples after uniaxial stretching at a periodically varied speed.
(Refer to Video 2 above for infrared video of stretching tests)

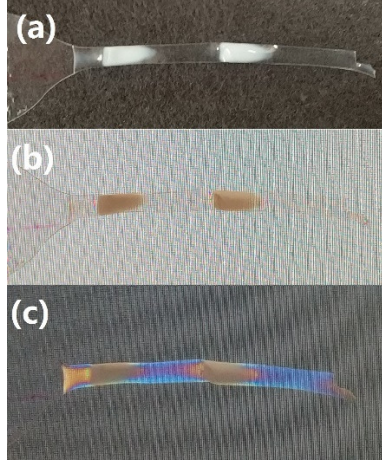


Figure S4. Comparison of normal optical (a) and photoelasticity (b and c, polarized lens with different angles with the monitor) images.

S3 CLOSED FORM SOLUTIONS OF MOVING HEAT SOURCE PROBLEM

Refer to [1]. A piece of long shape memory alloy (SMA) wire is stretched within a medium (e.g., water or air) [2]. Two situations are considered. One is two-directional, in which the martensite nucleates and moves in two opposite directions at the same speed; while the other is that the front moves along only one direction. Suppose that the cross-section of SMA is small (e.g., a thin wire), so that the temperature within the same cross-section is about a constant.

3.1 Two-directional case [1]

Refer to Figure S5(a). The martensite (M)-austenite (A) fronts move at a constant speed of v along both $\pm X$ directions. The governing equation may be expressed as,

$$\rho c \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = \rho q v \delta(x - vt) - \frac{hl}{A} (T - T_0) \quad (S1)$$

with the boundary conditions,

$$\left(\frac{\partial T}{\partial x} \right)_{x=0} = 0 \quad (S2)$$

$$\left(\frac{\partial T}{\partial x} \right)_{x=\infty} = 0 \quad (S3)$$

$$T(\infty, t) = T_0 \quad (S4)$$

and the initial condition,

$$T(x, 0) = T_0 \quad (S5)$$

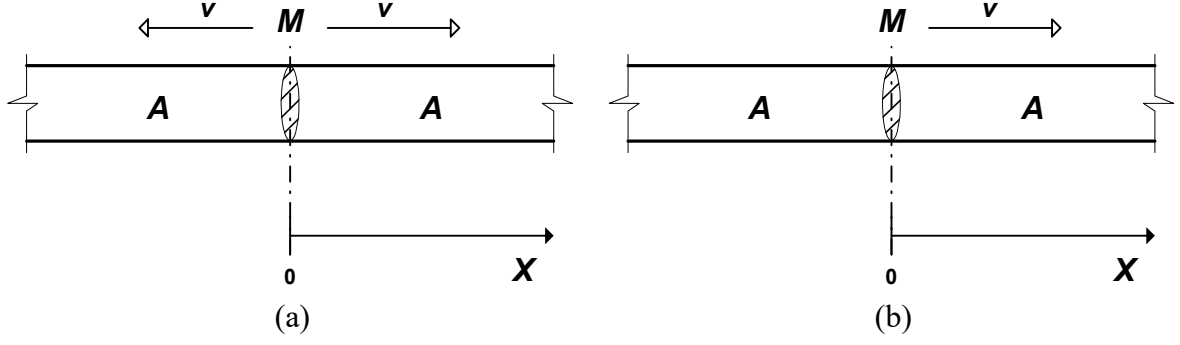


Figure S5. Illustration of propagation of the phase transformation front(s) along a long wire
(a) in two directions and (b) in one direction.

Here, A is the cross-sectional area of the SMA wire, l is the perimeter of the cross-section, T_0 is ambient temperature, h is the convection heat-transfer coefficient, ρ is the density of the SMA, q is the latent heat generated during the phase transformation, c is the heat capacity of SMA, and k is the thermal conductivity of SMA.

This is the case that martensite nucleates in the middle part of a SMA wire during stretching and then propagates along the wire in two opposite directions at the same speed.

The closed form solution of Eqn. (S1) with initial and boundary conditions (Eqns. 2-5) is obtained as,

$$T(x, t) = (\Theta_{11} + \Theta_{12})e^{-\alpha t} + \Theta_2 + T_0 \quad (\text{S6})$$

where

$$\Theta_{11} = \frac{ab}{2v\sqrt{4a^2c + v^2}} \left\{ \sum_{i=1}^2 \left[-\beta_i e^{\alpha\beta_i + \beta_i^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_i t^{\frac{1}{2}} \right) \right] + \sum_{i=3}^4 \left[\beta_i e^{\alpha\beta_i + \beta_i^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_i t^{\frac{1}{2}} \right) \right] \right\} \quad (\text{S7})$$

$$\begin{aligned} \Theta_{12} = \frac{abc}{2v\sqrt{4a^2c + v^2}} & \left\{ \frac{1}{y_1} \left[\beta_1 e^{\alpha\beta_1 + \beta_1^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_1 t^{\frac{1}{2}} \right) + \beta_2 e^{\alpha\beta_2 + \beta_2^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_2 t^{\frac{1}{2}} \right) \right] \right. \\ & \left. - \frac{1}{y_2} \left[\beta_3 e^{\alpha\beta_3 + \beta_3^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_3 t^{\frac{1}{2}} \right) + \beta_4 e^{\alpha\beta_4 + \beta_4^2 t} \text{Erfc} \left(\frac{1}{2} \alpha t^{\frac{1}{2}} + \beta_4 t^{\frac{1}{2}} \right) \right] \right\} \quad (\text{S8}) \end{aligned}$$

$$\Theta_2 = \frac{b}{\sqrt{4a^2c + v^2}} H\left(t - \frac{x}{v}\right) \text{Exp}\left(\frac{v(vt - x)}{2a^2}\right) \left[\text{Exp}\left(\frac{\sqrt{4a^2c + v^2}}{2a^2}(vt - x)\right) - \text{Exp}\left(-\frac{\sqrt{4a^2c + v^2}}{2a^2}(vt - x)\right) \right] \quad (\text{S9})$$

Here,

$$a^2 = \frac{k}{\rho c}$$

$$b = \frac{qv}{c}$$

$$c = \frac{hl}{A\rho c}$$

$$H\left(t - \frac{x}{v}\right) = \begin{cases} 1 & \left(t - \frac{x}{v} \geq 0\right) \\ 0 & \left(t - \frac{x}{v} < 0\right) \end{cases}$$

$$\text{Erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-w^2} dw$$

$$y_1 = \frac{1}{2} \left[2c + \left(\frac{v}{a}\right)^2 + \frac{v}{a} \sqrt{4c + \left(\frac{v}{a}\right)^2} \right]$$

$$y_2 = \frac{1}{2} \left[2c + \left(\frac{v}{a}\right)^2 - \frac{v}{a} \sqrt{4c + \left(\frac{v}{a}\right)^2} \right]$$

$$\beta_1 = -\sqrt{y_1}$$

$$\beta_2 = \sqrt{y_1}$$

$$\beta_3 = -\sqrt{y_2}$$

$$\beta_4 = \sqrt{y_2}$$

$$\alpha = \frac{x}{a}$$

3.2 One-directional case [1]

Refer to Figure S5(b). The martensite (M)-austenite (A) front moves at a constant speed of v along one direction only. The governing equation and initial condition are the same as those in above two-directional case. But the boundary conditions are,

$$\left(\frac{\partial T}{\partial x}\right)_{x=\infty} = 0 \quad (\text{S10})$$

$$T(\infty, t) = T(0, t) = T_0 \quad (\text{S11})$$

This is the situation, in which martensite nucleates at the clamping point during stretching. Subsequently, the phase transformation front moves toward the middle portion of the wire. Clamps serve as an effective heat sink, so that at that point, the temperature may be assumed to be always T_0 .

The closed-form solution of this problem is,

$$T(x, t) = \Theta_1 - \Theta_2 + T_0 \quad (\text{S12})$$

where,

$$\begin{aligned} \Theta_1 = \frac{bv}{2a^2(S_1 - S_2)} e^{-ct} & \left[e^{-\alpha\beta_5 + \beta_5^2 t} \text{Erfc}\left(\frac{1}{2} \frac{a}{t} - \beta_5 \sqrt{t}\right) + e^{\alpha\beta_5 + \beta_5^2 t} \text{Erfc}\left(\frac{1}{2} \frac{a}{t} + \beta_5 \sqrt{t}\right) \right. \\ & \left. - e^{-\alpha\beta_6 + \beta_6^2 t} \text{Erfc}\left(\frac{1}{2} \frac{a}{t} - \beta_6 \sqrt{t}\right) + e^{\alpha\beta_6 + \beta_6^2 t} \text{Erfc}\left(\frac{1}{2} \frac{a}{t} + \beta_6 \sqrt{t}\right) \right] \end{aligned} \quad (\text{S13})$$

$$\Theta_2 = \frac{bv}{a^2(S_3 - S_4)} \text{H}\left(t - \frac{x}{v}\right) \left\{ \text{Exp}\left[S_3\left(t - \frac{x}{v}\right)\right] - \text{Exp}\left[S_4\left(t - \frac{x}{v}\right)\right] \right\} \quad (\text{S14})$$

Here,

$$\begin{aligned} S_1 &= \frac{1}{2} \left\{ 2c + \left(\frac{v}{a}\right)^2 + \sqrt{\left[2c + \left(\frac{v}{a}\right)^2\right]^2 - ac^2} \right\} \\ S_2 &= \frac{1}{2} \left\{ 2c + \left(\frac{v}{a}\right)^2 - \sqrt{\left[2c + \left(\frac{v}{a}\right)^2\right]^2 - ac^2} \right\} \\ S_3 &= \frac{1}{2} \left[\left(\frac{v}{a}\right)^2 + \sqrt{\left(\frac{v}{a}\right)^4 + 4\left(\frac{v}{a}\right)^2 c} \right] \\ S_4 &= \frac{1}{2} \left[\left(\frac{v}{a}\right)^2 - \sqrt{\left(\frac{v}{a}\right)^4 + 4\left(\frac{v}{a}\right)^2 c} \right] \\ \beta_5 &= -\sqrt{S_1} \\ \beta_6 &= \sqrt{S_2} \end{aligned}$$

All the other symbols are the same as that in the two-directional case.

Reference

[1] Huang WM. Shape memory alloys and their application to actuators for deployable

structures [PhD Dissertation]. UK: Cambridge University; 1998.

[2] Huang WM. Transformation front in shape memory alloys. *Materials Science and Engineering a-Structural Materials Properties Microstructure and Processing*. 2005;392:121-9.