

# Article H<sub>2</sub>O·HF@C<sub>70</sub>: Encapsulation Energetics and Thermodynamics

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**Abstract:** This report deals with the quantum-chemical evaluation of the energetics and thermodynamics of the simultaneous encapsulation of HF and H<sub>2</sub>O by the IPR (isolated pentagon rule)  $C_{70}$ fullerene cage, yielding H<sub>2</sub>O·HF@C<sub>70</sub> species which were synthesized and characterized recently, thus further expanding the family of fullerene endohedrals with non-metallic encapsulates. The structures were optimized at the DFT (density functional theory) M06-2X/6-31++G<sup>\*\*</sup> level. The encapsulation energetics were further refined by the advanced B2PLYPD/6-31++G<sup>\*\*</sup> and B2PLYPD/6-311++G<sup>\*\*</sup> methods. After enhancement of the B2PLYPD/6-311++G<sup>\*\*</sup> encapsulation energy for the BSSE and steric corrections, the encapsulation energy gain was obtained, as 26 kcal/mol. The equilibrium encapsulation thermodynamics were described using the M06-2X/6-31++G<sup>\*\*</sup> partition functions. The results correspond to our previous evaluations for the water dimer encapsulation by C<sub>84</sub> cages.

Keywords: endohedral; fullerene; metallofullerene; stability

## 1. Introduction

In addition to metal encapsulations yielding metallofullerenes, it is also possible for non-metals, and even small non-metallic molecules, to be encapsulated by fullerene cages. Metallofullerenes themselves are stabilized via charge transfer of up to four electrons from the metal to the cage. Such strong charge transfer and stabilization, primarily via Coulombic interactions, does not occur with non-metal encapsulation. The stabilization of non-metallic molecules is based [1] on non-bonding, in particular van der Waals interactions.  $N_2@C_{60}$  and  $N_2@C_{70}$  primarily represent such endohedrals that encapsulate non-metal molecules, prepared [2] by heating under high pressure. N<sub>2</sub>@C<sub>60</sub> has also been observed [3] in chromatographic separation after nitrogen ion implantation, otherwise primarily yielding N@C<sub>60</sub> [4–9]. Complexes of fullerenes with rare gas atoms [10-14] can also be prepared using [10] high temperatures, high pressures, and a catalyst [13]. A new, elegant encapsulation method for non-metallic molecules—such as molecular hydrogen molecules [15] and water molecules [16]—places the species inside open-cage fullerenes, and then closes the cage window synthetically [17,18]. Multi-step synthesis has even produced [19] (H<sub>2</sub>O)<sub>2</sub>@C<sub>70</sub>. Carbon monoxide [20,21] and H<sub>2</sub>O<sub>2</sub> [22,23] have also been placed inside open-cage  $C_{60}$  derivatives.

Fullerene encapsulations of non-metal species have also been computed [24–42]: for example, the lowest-energy N<sub>2</sub>@C<sub>60</sub> structure that has been identified [28] is the N<sub>2</sub> unit, which is oriented towards a pair of parallel pentagons, so that the complex exhibits  $D_{5d}$  symmetry. This type of minimum energy structure has also been computed [26] for NH<sub>3</sub>@C<sub>60</sub>. With N<sub>2</sub>@C<sub>60</sub> [28], the MP2 = FC/6-31G\* encapsulation energies, before and after the so-called basis set superposition error (BSSE) correction, are equal to -17.5 kcal/mol and -9.28 kcal/mol, respectively. The BSSE-corrected MP2 = FC/6-31G\* value for NH<sub>3</sub>@C<sub>60</sub> is [26] -5.23 kcal/mol. Once the corresponding entropy change  $\Delta S_T^o$  is evaluated, one can



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). deal with the thermodynamics-controlling Gibbs energy term  $\Delta G_{T}^{0}$ . Using the partition functions from the DFT calculations, and the enthalpy terms derived from the BSSEcorrected MP2 = FC/6-31\* stabilization energy, the  $\Delta G_T^o$  standard changes for productions of  $N_2@C_{60}$  and  $NH_3@C_{60}$  at room temperature [26,28] read -2.64 and 1.53 kcal/mol, respectively. Such stability evaluations have also been performed for water dimer and trimer encapsulations, in particular by the  $D_2(22)$ - $C_{84}$  cage [36–39]. For example, when the encapsulation energy for the cyclic water-trimer encapsulation by  $D_2(22)$ - $C_{84}$  was computed at the M06-2X/6-31++G\*\* level with the BSSE correction, it was found that the trimer storage in  $C_{84}$  yielded a potential-energy gain of 10.4 kcal/mol. The encapsulated trimer could have two different forms: either the conformation known for the free-gas-phase water trimer (*trans*,  $C_1$  symmetry) or the arrangement with the three non-hydrogen-bonded H atoms on the same side of the OOO plane (*cis*, *C*<sub>3</sub> symmetry). The latter endohedral isomer was calculated [41] as lower in potential energy, by 0.071 kcal/mol, and formed about 57% of the equilibrium mixture at room temperature. The mentioned examples show that quantum-chemical calculations can productively complement observations of the non-metallic fullerene endohedrals.

This report continues the computational research line, and deals with quantumchemical evaluation of the energetics and thermodynamics of the encapsulation of HF and H<sub>2</sub>O into the IPR (isolated pentagon rule) C<sub>70</sub> fullerene cage, yielding H<sub>2</sub>O·HF@C<sub>70</sub> species synthesized and characterized [43] recently. The calculations could possibly be applied in the organization of direct high-pressure preparation of the species used for other systems [10–14] (where a temporary cage window is created by a catalytic action).

## 2. Calculations

The calculations began with geometry optimizations, performed using the density functional theory (DFT) approach—namely, the M06-2X functional, tested recently [44]—as it is the most reliable approximation for numerous application situations, including longrange interactions, hydrogen bonds, thermochemistry, and kinetics. The M06-2X functional was applied here with the standard 6-31++G\*\* basis set [45] (i.e., the M06-2X/6-31++G\*\* treatment). In order to check the geometrical or physical nature of the stationary points localized on the M06-2X/6-31++G\*\* potential hypersurface, harmonic vibrational analysis was carried out, thus confirming that the local energy minima had been found. An ultrafine grid in numerical integrations of the DFT functional (or superfine grid for the endohedral, to improve the reliability of low frequencies), and a tight SCF convergency criterion, were used.

The encapsulation energetics were refined beyond the DFT level, in order to reliably describe the electron-correlation effects, namely using the B2PLYPD treatment [46] with a dispersion (D) correction, and considering all electrons (B2PLYPD = FU). The B2PLYPD approach is a relatively new method, representing the application of the original second-order Møller–Plesset (MP2) perturbation treatment [47] to DFT wavefunctions. The B2PLYPD treatment was performed here, in the the optimized M06-2X/6-31++G\*\* structures, using the 6-31++G\*\*, and also 6-311++G\*\* basis sets, i.e., B2PLYPD/6-31++G\*\* and B2PLYPD/6-311++G\*\* quantum-chemical levels. Moreover, the basis set superposition error (BSSE) was estimated by the Boys–Bernardi counterpoise (CP) method [48] (for a more detailed description, see Appendix A). The CP correction is only rarely considered [49–52] with fullerene species, though it can bring about significant energy changes. The BSSE-corrected values were still further improved here by the recently suggested [37,53] steric correction.

All the computations were carried out with the Gaussian 09 program package [54]. The computations were performed in a parallel regime, with up to 24 processors (up to 3 GHz each).

#### 3. Results and Discussion

The M06-2X/6-31++G<sup>\*\*</sup> optimized structure of  $H_2O \cdot HF@C_{70}$  agreed with the observed results [43]. In particular, the observed hydrogen-bond length is 1.39 Å, while the

value calculated here was 1.481 Å. Similarly, the observed F-O distance is 2.438 Å, while the calculated one was 2.447 Å. The calculations treated a free, gas-phase H<sub>2</sub>O·HF@C<sub>70</sub> species, while the X-ray experiment [43] dealt with a porphyrin cocrystal.

Table A1 reports the calculated encapsulation energetics. The presented potentialenergy changes describe the gas-phase formation of  $H_2O \cdot HF@C_{70}$ , i.e., the equilibrium encapsulation processes,

$$H_2O(g) + HF(g) + C_{70}(g) = H_2O \cdot HF@C_{70}(g),$$
(1)

connected with an encapsulation potential-energy change  $\Delta E_{enc}$ . The energy changes were always negative (Table A1), i.e., there was a gain in energy; the encapsulation process (1) is exothermic and, thus, convenient from the thermodynamic point of view. The calculated terms were rather similar in both considered basis sets. On the other hand, the M06-2X values differed significantly from the B2PLYPD ones. This situation can be interpreted as the M06-2X functional not having described sufficiently the electron-correlation effects in this type of system. Thus, the B2PLYPD terms should be preferred for the endohedrals. Hence, the B2PLYPD/6-311++G<sup>\*\*</sup> encapsulation energy value of -26.02 kcal/mol was used for the thermodynamic treatment.

The energy terms presented in Table A1 include the BSSE correction evaluated in the so-called CP3 scheme [37,55], i.e., the association of three species (1). The Boys–Bernardi CP method [48] is still an approximative approach, introduced in order to ensure that each component of a chemical process is formally treated with the same number of basis-set functions. This formal unified description is achieved via so-called ghost atoms with no electrons. The BSSE problem originates in the finiteness of basis sets, and it should disappear in the rather hypothetical case of an infinite basis set. The BSSE correction is an important term—oligomerization energy gain would otherwise be overestimated [37,55] by several kcal/mol.

There is still another computational aspect related to the CP3 estimation of the BSSE term. The original Boys–Bernardi counterpoise method was suggested [48] for dimers handled with a fixed geometry (though the structures of the monomeric units differ in free and dimeric form). Although a fully BSSE-respecting geometry optimization would, in principle, be possible [56], it is feasible only for relatively simple systems. Nevertheless, in order to reflect the geometry distortion, a simpler, straightforward steric-corrected BSSE approach has recently been suggested [37,53] (for details, see Appendix A). In the conventional CP3 treatment, the geometries of the three sub-units ( $H_2O$ , HF,  $C_{70}$  in our case) are taken to be the same as in the whole complex ( $H_2O \cdot HF@C_{70}$ ), so that only four energy calculations are required, without any structure re-optimization. The steric-corrected BSSE treatment [37,53] goes a step further, as it includes the difference between the energy of the carbon-cage geometry simply taken from  $H_2O \cdot HF@C_{70}$  and the energy of the related fullyoptimized empty  $C_{70}$  cage (which has to be slightly lower). Similar steric corrections are also computed for the H<sub>2</sub>O and HF components. For simplicity, the steric corrections in this work were evaluated only at the  $M06-2X/6-31++G^{**}$  level. The gain in the encapsulation energy was in fact reduced by the steric correction at the  $M06-2X/6-31++G^{**}$  computational level, by some 1.49 kcal/mol (i.e., a somewhat larger reduction than that, for example, found [37] for the CP3 steric corrections with  $(H_2O)_2@D_2(22)-C_{84}$  and  $(H_2O)_2@D_{2d}(23)-C_{84})$ .

The encapsulation energies  $\Delta E_{enc}$ , with the inclusion of the BSSE and steric corrections, are presented in Table A1. The B2PLYPD/6-311++G\*\* terms should be preferred in further considerations, as they represent the most sophisticated of the approaches considered here. Interestingly, the observed [57] dissociation energy of a free H<sub>2</sub>O·HF complex was also reproduced well by the B2PLYPD/6-311++G\*\* method. The encapsulation-energy gain for H<sub>2</sub>O·HF@C<sub>70</sub> was somewhat larger than previously found, for example, with (H<sub>2</sub>O)<sub>2</sub>@D<sub>2</sub>(22)-C<sub>84</sub> [37,38]. With future developments in computer technology, the B2PLYPD/6-311++G\*\* approach should, however, be tested at a still higher level of quantum-chemical methodology.

Let us note for completeness that, in addition to the CP3 scheme considered here for BSSE corrections, a simpler CP2 scheme was previously applied [37] to water-dimer encapsulation. While the CP3 scheme deals with three sub-units, as in the above reaction (1), the CP2 approach dealt simply with the encapsulation of the whole complex  $H_2O$ ·HF by the C<sub>70</sub> cage. Generally speaking, the CP3 scheme should produce larger energy gains compared to the CP2 decomposition, owing to the additional stabilization energy originating in the encapsulate formation from the monomeric units.

The encapsulation potential energy change,  $\Delta E_{enc}$ , was enhanced, for the thermodynamic treatment, by the vibrational zero-point energy ZPE, leading to encapsulation enthalpy at absolute zero temperature  $\Delta H_{0,enc}^o$ :

$$\Delta H_{0,enc}^o = \Delta E_{enc} + \Delta Z P E_{enc}.$$
 (2)

Application of the heat content functions evaluated with the partition function treatment yielded the standard encapsulation enthalpy change, at temperature  $T: \Delta H^o_{T,enc}$ . The partition function treatment also produced the standard encapsulation entropy change, at temperature  $T: \Delta S^o_{T,enc}$ . Thus, we arrived at the standard encapsulation Gibbs energy change  $\Delta G^o_{T,enc}$ :

$$\Delta G^o_{T,enc} = \Delta H^o_{T,enc} - \Delta T S^o_{T,enc}.$$
(3)

The encapsulation equilibrium constant  $K_{p,enc}$  for reaction (1), expressed in the partial pressures p of the reaction components,

$$K_{p,enc} = \frac{p_{\rm H_2O\cdot HF@C_{70}}}{p_{\rm H_2O}p_{\rm HF}p_{C_{70}}}$$
(4)

is related to the standard encapsulation Gibbs energy change  $\Delta G^o_{Tenc}$  by

$$\Delta G^o_{T,enc} = -RT \ln K_{p,enc} \tag{5}$$

where *R* denotes the gas constant.

Table A2 presents the thermodynamic characteristics for the equilibrium process (1) at room temperature. Both terms,  $\Delta H_{T,enc}^o$  and  $\Delta G_{T,enc}^o$ , remained negative. As already noted with the simpler  $\Delta E_{enc}$  term, the encapsulation process (1) was exothermic and, thus, convenient from the thermodynamic point of view. The calculated  $\Delta G_{T,enc}^o$  value of -5.63 kcal/mol (Table A2) was comparable to findings for water encapsulations by the C<sub>84</sub> fullerene cages [36–39,41]. As in our previous computational evaluations of nonmetallic fullerene endohedrals [26,28,36–41], the partition functions  $q_i$  were basically of the usual rigid rotor and harmonic oscillator (RRHO) quality [58] (as only was feasible with the presently available computer resources). In terms of the partition functions  $q_i$  and the encapsulation enthalpy at the absolute zero temperature  $\Delta H_{0,enc}^o$ , the encapsulation equilibrium constant  $K_{p,enc}$  (4) was given by a formula [58],

$$K_{p,enc} = \frac{\frac{q_{H_2O-HF@C_{70}}}{N_A}}{\frac{q_{H_2O}^o}{N_A} \frac{q_{H_F}^o}{N_A} \frac{q_{C_{70}}^o}{N_A}} \exp\left(-\frac{\Delta H_{0,enc}^o}{RT}\right),$$
(6)

where  $N_A$  denoted the Avogadro number. The form of relation (6) allowed for some convenient cancellation of the higher contributions [59] beyond the RRHO approximation. However, future efforts should deal with further improvements of the RRHO partition functions commonly employed [36–39,60] for encapsulation thermodynamics. Such developments should, in particular, deal with encapsulate motions, important not only for stability predictions but also for the cage symmetries effectively observed [18,61,62] as a consequence of the related observational time scales [58]. The symmetry issue is also closely related to the effective, dynamic symmetry numbers [38] in the rotational partition functions [63], an aspect that is important for all endohedrals, regardless of the encapsu-

late type: one option is to work with the so-called FEM approach [60], instead of RRHO partition functions (the two treatments can possibly suggest bounds for thermodynamic terms).

The encapsulation equilibrium constant in Table A2 was sensitive to the encapsulation enthalpy term  $\Delta H^o_{T,enc}$ , as its related encapsulation enthalpy at the absolute zero temperature  $\Delta H^o_{0,enc}$  appeared in relation [6] in the exponential function: this aspect, in turn, highlights the importance of precise energy calculations.

#### 4. Conclusions

Our quantum-chemical evaluation of the energetics and thermodynamics of the simultaneous encapsulation of HF and H<sub>2</sub>O by the IPR C<sub>70</sub> fullerene cage further expands characterization of the relatively new family of fullerene endohedrals containing non-metallic encapsulates. The evaluations were carried out at the most advanced level presently applicable, yielding to the encapsulation-energy gain of 26 kcal/mol. Nevertheless, the results should in future be tested at still higher computational levels, such as the quadratic configuration interaction method, QCISD, or even the Gn theory [64], when allowed by computer resources. Further developments are also needed in the construction of the partition functions for thermodynamic evaluations, in order to somehow respect the anharmonic and non-rigid features of the endohedrals, though this step is also at present limited by the available computational power. The obtained estimate of the encapsulation equilibrium constant corresponded to the values previously derived [36–39,41] for the encapsulations of the water dimer by  $C_{84}$  cages, and to other computed encapsulations [60]: the possibility is thus not excluded that even  $H_2O \cdot HF@C_{70}$  could be prepared by direct catalytic high-pressure treatment [10–14]. Similar computational treatments of other fullerene systems with nonmetallic encapsulates will offer a further insight into this newly established endohedral class, even having some application potential as a different approach to modifications of fullerene properties.

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#### Appendix A

Let us consider a general association process (regardless of the nature of the bonding types involved):

$$A + B = C. \tag{A1}$$

In the tradional approach, i.e., without the Boys–Bernardi counterpoise (CP) correction [48] (also called the BSSE correction), the reaction potential-energy change  $\Delta E_r$  is taken as the difference of the potential energies of the reaction components, straightforwardly evaluated in their own basis sets (indicated in the upper indexes):

$$\Delta E_{r,noCP} = E_C^{(C)} - E_A^{(A)} - E_B^{(B)}.$$
(A2)

As the basis sets used in relation [8] are different, the three energy terms are not calculated at the same level; therefore, they are not consistent: thus, they are not strictly comparable or directly applicable.

In the CP-corrected treatment, the three reaction components are described by the same basis set, namely by the basis set of the product *C*:

$$\Delta E_{r,CP} = E_C^{(C)} - E_A^{(C)} - E_B^{(C)}.$$
(A3)

As the basis set of *C* is larger than that of either *A* or *B*, there has to be energy decrease:  $E_A^{(C)} < E_A^{(A)}$ , and similarly  $E_B^{(C)} < E_B^{(B)}$  (potential energy decreases with increasing basis set). In other words, the absolute value of the reaction energy will also be reduced:  $|\Delta E_{r,CP}| < |\Delta E_{r,noCP}|$ . For example [37], for the water-dimer encapsulation by the  $D_2(22)$ -C<sub>84</sub> cage, the M06-2X/6-31++G<sup>\*\*</sup> potential-energy reaction change, without the CP2 correction, is -23.4 kcal/mol, while with the CP2 correction the term changes to -19.2 kcal/mol. Similarly [37], for the  $D_{2d}(23)$ -C<sub>84</sub> cage, the reaction change before the BSSE correction is -21.8 kcal/mol, while after the BSSE correction the term amounts to -17.8 kcal/mol.

Let us move to yet another correction. In the previous paragraph, the geometries of the two reactants *A* and *B* were simply taken from the optimized structure *C*. Now, the geometries of the free reactants will also be optimized. The additional step brings new energies for the reactants *A* and *B* (the new energies are somewhat lower, as geometry optimization means searching for a local energy minimum), denoted by *o* in the lower index,  $E_{A,o}^{(A)}$  and  $E_{B,o}^{(B)}$ . Hence, we can move to a steric-corrected term,

$$\Delta E_{r,noCP,o} = E_C^{(C)} - E_{A,o}^{(A)} - E_{B,o}^{(B)}$$
(A4)

and subsequently to the steric correction  $\Delta E_{ster}$  itself:

$$\Delta E_{r,noCP,o} = \Delta E_{r,noCP} + \Delta E_{ster}.$$
(A5)

In an approximation, the steric correction  $\Delta E_{ster}$  from relation (A5) is then straightforwardly used also for the improvement of the  $\Delta E_{r,CP}$  term. In the above example [37] of the water-dimer encapsulation by the  $D_2(22)$ -C<sub>84</sub> cage, the M06-2X/6-31++G\*\* potentialenergy reaction change, with the CP2 correction of -19.2 kcal/mol, amounts, after the steric correction, to -16.9 kcal/mol. Similarly [37], for the  $D_{2d}(23)$ -C<sub>84</sub> cage, the reaction term is changed from -17.8 kcal/mol to the final value of -14.4 kcal/mol. The steric correction is positive, as the geometry optimizations still lower the energies of reactants *A* and *B*.

**Table A1.** The encapsulation energy  $\Delta E_{enc}$  for H<sub>2</sub>O·HF@C<sub>70</sub>, calculated by selected approaches <sup>*a*</sup> and inclusion of the CP3 BSSE and steric corrections.

	$\Delta E_{enc}/kcal\cdot mol^{-1}$		
Calc. Level	6-31++G**	6-311++G**	
M06-2X	-31.29	-31.79	
B2PLYPD	-25.75	-26.02	

<sup>*a*</sup> In the M062X/6-31++G<sup>\*\*</sup> optimized geometry, see Figure A1.

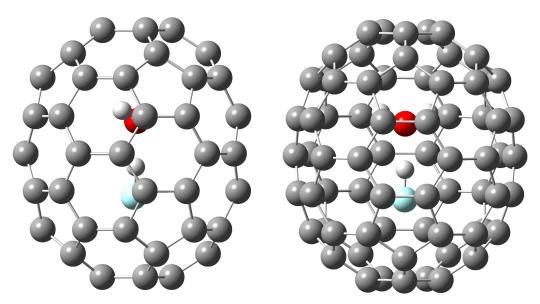


Figure A1. Two views of the M06-2X/6-31++G\*\* optimized structure of H<sub>2</sub>O·HF@C<sub>70</sub>.

**Table A2.** The standard <sup>*a*</sup> enthalpy  $\Delta H^o_{T,enc}$ , entropy  $T\Delta S^o_{T,enc}$ , Gibbs energy  $\Delta G^o_{T,enc}$  changes, and the equilibrium constants  $K_{p,enc}$  for the gas-phase equilibrium formation (1) of H<sub>2</sub>O·HF@C<sub>70</sub>, evaluated <sup>*b*</sup> at room temperature T = 298.15 K.

	$\Delta H^o_{T,enc}$ (kcal/mol)	TΔS <sup>o</sup> <sub>T,enc</sub> (kcal/mol)	ΔG <sup>o</sup> <sub>T,enc</sub> (kcal/mol)	$K_{p,enc}$ (atm <sup>-2</sup> )
B2PLYPD/6-311++G**	-23.24	-17.60	-5.63	$1.346  imes 10^4$

<sup>*a*</sup> The standard state—ideal gas phase at 1 atm = 101,325 Pa pressure. <sup>*b*</sup> The partition functions based on the  $M062X/6-31++G^{**}$  molecular characteristics.

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