



Communication Wavelength-Independent Correlation Detection of Aberrations **Based on a Single Spatial Light Modulator**

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Abstract: The cumulative achievements in the fields of science and technology have allowed us to substantially approach the solution of the phase problem in optics. Among all phasometric methods, single-beam methods are the most promising, since they are more variable and versatile. Single-beam methods are based either on the analysis of the intensity distribution, as is conducted by interferometers and wavefront sensors, or on the transformation of the phase into an intensity distribution due to spatial filtering, as is conducted by holographic methods. However, all these methods have the problem of working with polychromatic radiation and require spectral filters to process such radiation. This paper presents a new approach to the synthesis of Fourier holograms used in holographic wavefront sensors that make it possible to create achromatic elements and work with white light without the use of additional filters. The approach was numerically and experimentally verified.

Keywords: chromatic aberration; white light; wavefront sensor; computer-generated hologram; spatial light modulator



1. Introduction

The solution of image (field) analysis tasks associated with the problem of pattern recognition is greatly facilitated by using the technique of optical matched filtering and correlation analysis proposed by Vander Lugt [1]. This technique is based on the possibility of simple implementation of complex matched filters for images of any complexity in singlechannel systems with Fourier transform components [2]. With the development of devices, such as spatial light modulators (SLM) based on liquid crystals (LC-SLM) [3] and micromirrors (DMD), multichannel optical pattern recognition systems also began to develop. These systems began to use the color component of the image as an information parameter [4,5]. Such devices allow solving pattern recognition problems by spatial and spectral parameters in real time [6]. Color images are decomposed into three monochromatic channels (RGB) using various techniques (time [7] and space division [8,9], SLM region division [10,11] and spatial superposition [12,13]), which are then processed independently of each other [14].

Undoubtedly, the use of a white light source when working with such devices leads to unwanted chromatic effects [15,16]. In addition, the LC-SLM used in the majority of such systems, which operate primarily with monochromatic light, typically provides a limited range of phase modulation [16,17]. These facts led to the development and use of achromatic elements displayed on the SLM and provided the same optical power for radiation at different wavelengths [18–20].

In addition to the recognition of 2D and 3D images, optical matching filtering and correlation analysis methods are also used for wavefronts [21–23]. When solving such problems, it is necessary to constantly monitor the temporal or spatiotemporal couplings (STC) of light beams [24–26]. These STC [27] are usually inseparable chromatic aberrations,



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which can be caused by very common optical elements, such as diffraction gratings and thick lenses, or prisms created from scattered material [28].

This one-to-one relationship of characteristics is determined using standardized instruments that include intensity and phase measurements. For example, shearing interferometers, Shack–Hartmann sensors, and quantitative phase imaging techniques are actively used in phase measurement problems. The principles of reconstruction are the same [29], and each method involves tradeoffs between dynamic range, resolution, bandwidth, and usability. However, these methods are often inapplicable due to their peculiarities, for example, in tasks of laser spectroscopic characterization of dielectric materials.

In our study, we propose to link the well-known method of correlation analysis of wavefronts based on a scheme with a single SLM and a method for synthesizing a spatial achromatic filter in the form of a computer-generated Fourier hologram (CGH); as a result, the range of tasks to be solved will be significantly expanded.

2. Materials and Methods

Let us consider the synthesis of a spatial filter in the form of the CGH according to the Vander Lugt scheme for colliding beams [1]. This recording scheme implies the use of a point quasi-monochromatic radiation source, S_0 , displaced relative to the optical axis of the system. Since the radiation source is in the front focal plane of the Fourier lens, the spherical wave propagating from it is converted into a plane wave after passing through the lens. In the mathematical model, the radiation source will be represented as an ideal point, so we denote its field as a displaced two-dimensional Dirac delta function with a unit amplitude.

$$E_0(x,y) = \delta(x,y-\Delta), \tag{1}$$

where Δ is the value of the shift relative to the optical axis. The transformation of the complex amplitude of the radiation passing through the lens can be described using the Fourier transform [30]. Using the filtering properties of the Dirac delta function and the properties of the Fourier transform [30], it can be shown that the Fourier image of the source in the hologram plane will be the product of the complex spectrum from the radiation source on the optical axis of the system and the phase shift function.

$$\widetilde{E}(x',y') \sim H_E^0 \cdot \exp\left[-j\frac{2\pi\Delta}{\lambda f}y'\right],$$
(2)

where λ is the wavelength of the quasi-monochromatic radiation source, *f* is the len's focal length, H_E^0 is the amplitude coefficient of the centered radiation source. Equation (2) is the subject wave in the hologram formation plane. The distribution of the phase argument of the object wave is shown in Figure 1a.



Figure 1. Distribution of the phase argument of the (**a**) object wave and (**b**) reference wave in the hologram recording plane.

The reference wave $R_{ref}(x', y')$ is a laser beam with a unit amplitude and a phase argument, which is the sum of the orthogonal basis functions

$$R_{ref}(x',y') = \exp\left[j \cdot \frac{2\pi}{\lambda} \cdot f_{ref}(x',y')\right],\tag{3}$$

where $f_{ref}(x', y')$ is a spatial phase shift produced by aberration, which is independent from λ . It can be presented as:

$$f_{ref}(x',y') = \sum_{n} C_n \cdot W_n(x',y'), \qquad (4)$$

where C_n is the set of Zernike coefficients; $W_n(x', y')$ is the Zernike polynomials that have been first defined in polar coordinates and then converted into Cartesian coordinates using well known techniques [31]. Like all other basis functions describing aberrations, the weight coefficients of the Zernike polynomials can be defined in wavelengths. As a result, the coefficients can be written as $C_n = N_n \lambda$, where N_n is a real number. As a result, Equation (3) will have the form:

$$R_{ref}(x',y') = \exp\left[j \cdot \frac{2\pi}{\lambda} \cdot \sum_{n} N_n \lambda \cdot W_n(x',y')\right],$$
(5)

where N_{ln} is the aberration value presented in terms of λ_l . If we have another wavelength λ_k in the source spectrum the same aberrated beam $R_{ref}(x', y')$ can be presented as a weighted sum of the same set of Zernike functions $\{W_n(x', y')\}$ and coefficients $\{C_n\}$. The difference is in representation of Zernike coefficients in terms of number of wavelengths λ_k by the set of real valued coefficients $\{N_{kn}\}$ which are related to $\{N_{ln}\}$ as:

$$N_{kn}\lambda_k = N_{ln}\lambda_l, \tag{6}$$

Equation (5) shows that the wavelength λ can be moved outside the weighted sum. In this case, Equation (5) becomes independent of the wavelength of radiation. Consequently, the resulting holograms also become independent of the wavelength of the radiation used for reconstruction. As a consequence, they can be used to work with white light, and the spectral selectivity will be due to diffraction. Figure 1b shows the distribution of the phase argument of the reference wave. Actually, the hologram results from the interference of the object wave $\tilde{E}(x', y')$ and the reference wave $R_{ref}(x', y')$ in the plane of their interaction, and the amplitude pattern of the CGH can be calculated as follows:

$$h_{amp}(x',y') = 2\operatorname{Re}\left[R_{ref}(x',y') \cdot \exp\left(-j\frac{2\pi\Delta}{\lambda f}y'\right)\right].$$
(7)

The further hologram reconstruction model directly depends on the type of its modulation. In case of amplitude modulation, in the back focal plane of the Fourier lens, the complex field amplitude will be proportional to the Fourier transform of the product of the laser beam function and the hologram. If phase modulation is used, the CGH will be as follows:

$$h_{ph}(x',y') = \exp\left[j \cdot \mu \cdot h_{amp}(x',y')\right].$$
(8)

where $\mu = 2\pi$.

According to the Jacobi–Anger property [30], the right side of Equation (8) can be represented as the sum of weighted Bessel functions of the 1st kind of the nth order.

$$h_{ph}(x',y') = \sum_{n \in \mathbb{Z}} i^n J_n(\mu) \exp\left(j \cdot \mu \cdot n \cdot f_{ref}(x',y')\right) \cdot \exp\left(-j\frac{2\pi\Delta}{\lambda f}y'\right).$$
(9)

Amplitude and phase modulation of the hologram will lead to the appearance of higher orders of diffraction in the plane of formation of the Fraunhofer diffraction pattern.

The presence of higher orders leads to a decrease in the diffraction efficiency of the method and can cause a decrease in the detection accuracy. However, there are methods based on computer holography that generally allow one to increase the diffraction efficiency of a hologram by forming only one diffraction order in the Fraunhofer diffraction pattern [30]. This effect can be obtained only with phase modulation, if we consider Equation (9) at n = 1.

$$h_{ph}(x',y') = \exp\left(j \cdot 2\pi \cdot \left[f_{ref}(x',y') - \frac{\Delta}{\lambda f} y' \right] \right)$$
(10)

When implementing this hologram with a phase SLM (Figure 2a), such a hologram (Figure 2b) will form only one order of diffraction (+1). However, an important condition for this is the requirement for the depth of the phase modulation. This is due to the fact that the complex field amplitude of the object wave in Equation (2) is a one-dimensional blazed grating. Theoretically, its diffraction efficiency can be 100%, provided that the generated phase shift is equivalent to $\sim N\lambda$, where *N* is a natural number [16].



Figure 2. (a) Principle of operation CGH; (b) distribution of the phase argument of the CGH.

From the intensity and size of a given order of diffraction, one can judge the presence and magnitude of the aberration contained in the beam incident on the hologram. If the distortion amplitudes in the light beam incident on the hologram and in the function describing the aberrations in the CGH coincide, an autocorrelation impulse response will be observed in the rear focal plane of the Fourier transform objective.

3. Experimental Demonstration

Experimental demonstration of the method was carried out using three independent coherent quasi-monochromatic radiation sources with operating wavelengths $\lambda_1 = 473$ nm, $\lambda_2 = 532$ nm, and $\lambda_3 = 561$ nm. The laser beams propagating from the sources were expanded and collimated independently of each other using the Kepler system (at the output of these systems, the aperture of the laser beams was 4 mm). The convergence of laser beams on one optical axis was carried out using several beam splitting cubes (BS). Since this part of the optical scheme does not carry any information, we will not show it further when illustrating the scheme of the experiment.

According to the simplified experimental scheme shown in Figure 3, chromatic aberration was introduced using a biconvex lens (L1). The laser radiation incident on the lens was focused on the focal plane at a distance ~100 mm. Due to the dispersion properties of the lens, the focal plane has an extended size along the axis of radiation propagation when it is illuminated by a white light source. As a result, in the plane with the phase SLM, the values of the radii of curvature of the wavefronts do not match for laser beams with different wavelengths. In the SLM plane (Holoeye PLUTO-2-VIS-016, 1920 \times 1080, pixel



size 8 μ m), the incident radiation interacts with the CGH displayed on the SLM (the phase function of the CGH is described by Equation (10), while its size is 1920 \times 1080 pixels).

Figure 3. Simplified experimental scheme.

Detecting the presence of aberrations, determining their type and value is carried out using an achromatic (in the range of 0.3–0.8 μ m) Fourier lens (L2). In the focal plane of L2, there is a matrix photodetector, which is connected to the SLM via a personal computer using a feedback line. In this case, monochrome sCMOS Camera (CMOS, Thorlabs CS2100M-USB, 1920 × 1080, pixel size 5.04 μ m) was used. The process of forming the correlation response and finding the main maximum of the correlation function is carried out using the algorithm based on the method of gradient descent adapted for this problem [21]. Previously, this algorithm was demonstrated in [22], where the optimal parameters of its operation for this problem were determined. It should be noted that the synthesis of Fourier holograms is carried out in real time and is completely determined by the algorithm.

The laser beam passes through a nonpolarizing BS (50:50 (R:T) split ratio) before falling on the phase reflective SLM. The scheme uses a Shack–Hartmann wavefront sensor (WFS, Thorlabs WFS300-14AR, Newton, NJ, USA), which allows for monitoring wavefronts at a frequency up to 15 Hz with an accuracy up to λ /50 and sensitivity up to λ /150. Since the WFS is not capable of handling polychromatic radiation, bandpass filters were placed in front of it, each corresponding to a different source.

The first experimental approbation was carried out using only one quasi-monochromatic radiation source (532 nm). In this case, the SLM gamma curve provided modulation in the [0, pi] range at a wavelength of $\lambda_2 = 532$ nm. Then, using the previously developed algorithm based on the gradient descent method, the search for the main maximum of the correlation function was performed automatically. The algorithm [21] required an average of 10 iterations to determine the true value of the aberrations.

After obtaining the correlation response and determining the position of the main correlation maximum for the laser beam at $\lambda_2 = 532$ nm, the SLM was reconfigured to work with the source at $\lambda_1 = 473$ nm and $\lambda_3 = 561$ nm, respectively. Then, the experiment was repeated according to the abovementioned procedure. It is worth noting that the same CGHs were used in all these experimental series.

After successive detection of aberrations for different wavelengths, the experimental setup was rebuilt to detect the chromatic aberration (longitudinal). The chromatic aberration longitudinal was obtained by simultaneously illuminating lens L1 with laser beams with the wavelengths described above.

4. Results and Discussion

Based on the first three experimental tests, correlation responses were obtained for each of the radiation sources. The normalized dependences of the amplitude of the maximum correlation response offset are shown in Figure 4.



Figure 4. Normalized dependences of the amplitude of the maximum correlation response offset for different wavelength sources: (a) $\lambda_1 = 473$ nm; (b) $\lambda_2 = 532$ nm; and (c) $\lambda_3 = 561$ nm.

As can be seen from Figure 4, the dependencies have the same profile, but have different normalized peak parameters. For example, the height of the correlation peak or signalto-noise ratio (*SNR*) is different for all three cases: $SNR_{\lambda_1=473} = 3.34$, $SNR_{\lambda_2=532} = 2.94$, and $SNR_{\lambda_3=561} = 2.33$. The results shown in Figure 4 demonstrate the feasibility of the purposed method to be used on different wavelengths with the same set of CGHs.

Table 1 shows the values of the obtained aberrations and their corresponding errors. As the true value was taken from the data from the WFS, the stated error of measurement was $\lambda/50$. The data presented indicated that this method of CGH synthesis is reliable, and the assumption that a hologram is invariant to a change in the wavelength of the radiation incident on it is correct.

able 1. Values of the obtained aberrations.	

Methods	$\lambda = 473 \text{ nm}$	$\lambda = 532 \text{ nm}$	λ = 561 nm
WFS	0.22λ	0.45λ	0.52λ
Proposed method	0.23λ	0.43λ	0.5λ
Error	$\lambda/100$	$\lambda/50$	$\lambda/50$

The fourth experimental approbation included all three coherent radiation sources to measure the chromatic aberration (longitudinal). The two-dimensional distributions of the recorded correlation responses are shown in Figure 5a–c for offset values of 0.23λ , 0.43λ , and 0.5λ and for wavelengths $\lambda_1 = 473$ nm, $\lambda_2 = 532$ nm, and $\lambda_3 = 561$ nm, respectively. Figure 5d–f show the maximum values of the intensity distribution along the 0*y* axis for these cases. By measuring the distance between the maxima of the normalized dependence on the values of the obtained aberration values, we can determine the value of the chromatic aberration of the complex field amplitude, which in this case is $-\lambda/3$ relative to the central wavelength of 532 nm.



Figure 5. Two-dimensional distribution of the intensity: (**a**) for value offset 0.23λ and for wavelength $\lambda_1 = 473$ nm; (**b**) for value offset 0.43λ and for wavelength $\lambda_2 = 532$ nm; (**c**) for value offset 0.5λ and for wavelength $\lambda_3 = 561$ nm. Maximum values of the intensity distribution along the 0*y* axis: (**d**) for case (**a**); (**e**) for case (**b**); (**f**) for case (**c**).

5. Conclusions

The peculiarity of the wavefront aberration detection problem is that it is phase only, slowly changing the function of the wave surface of the investigated light beams. In this paper, an approach to synthesizing the spatial filters in the form of CGHs on the basis of computer holography methods was demonstrated. This allows only one correlation response in the analysis plane. Moreover, these holograms are invariant to changes in the wavelength of the analyzed laser beam, which allows them to be used to detect chromatic aberrations in white light. When working with white light, the spatial separation of the responses is due to the basic principles and laws of diffraction.

Due to the chosen scheme for the synthesis of the CGH Fourier (scheme according to the Vander Lugt method), the generated correlation responses are, in essence, an image of the point-spread function of the radiation source. Therefore, in the case of simultaneous work with several quasi-monochromatic radiation sources, it is necessary that their spatiotemporal couplings be close. In the case of working with a natural source of white light (e.g., supercontinuum), there are no such restrictions.

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