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Decoding-Order-Based Power Allocation (DOPA) Scheme for Non-Orthogonal Multiple Access (NOMA) Visible Light Communication Systems

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Abstract: Non-orthogonal multiple access (NOMA) is an effective multiple access scheme that can be used to improve considerably the spectral efficiency of indoor downlink visible light communication (VLC) systems. However, NOMA suffers from inevitable multi-user interference which degrades the system performance. In this paper, a NOMA scheme is applied in a downlink VLC system and the impact of the multi-user interference on the system performance is studied. A closed-form expression for the user symbol error rate (SER) is derived and a decoding-order-based power allocation (DOPA) method is proposed to reduce the multi-user interference and find the optimal power allocation that minimizes the SER. The significance of the proposed schemes is demonstrated by simulation. The results show that the proposed DOPA method is able to reduce effectively the multi-user interference and provide more sum rate in comparison with benchmarking schemes such as the gain ratio PA (GRPA) and the normalized gain difference PA (NGDPA) methods.

Keywords: non-orthogonal multiple access (NOMA); visible light communication (VLC); successive interference cancellation (SIC); interference management



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1. Introduction

Visible light communication (VLC) is one of the most promising technologies that is attracting more attention as the demand for wireless data communication continues to increase. As the radio frequency (RF) spectrum is becoming more and more saturated, RF-based technologies, such as cellular networks and Wi-Fi, will no longer keep up with the growing demand for more data rate [1]. In contrast, VLC systems occupy the license-free light spectrum which is out of the RF spectrum and can provide a much wider bandwidth spanning approximately from 400 THz to 800 THz [2–4]. VLC is an alternative technology for indoor wireless communications which is expected to be an integral part of the future wireless networks, such as networks used beyond 5G, due to its energy efficiency and its ability to achieve high data rates [5,6]. Although the most potential applications of VLC are in indoor scenarios, VLC has many promising applications in outdoor environments such as vehicular VLC (V-VLC) as an alternative vehicular access solution to RF-based vehicular communications [7–9].

In order to realize a multi-user VLC system, a multiple access (MA) scheme is required. Many MA schemes have been proposed for VLC such as time division multiple access (TDMA) [10], space division multiple access (SDMA) [11], code division multiple access (CDMA) [12], and orthogonal frequency division multiple access (OFDMA) [13]. One of the recently proposed MA schemes is the non-orthogonal multiple access (NOMA) which is characterized by its high spectral efficiency and its ability to support more users [14–17].

The essence of NOMA is to exploit the power domain to squeeze more users in the same time-frequency (TF) resources. Hence, a significant increase in the system capacity can be achieved but at the cost of increasing the multi-user interference and the complexity of the receiver. NOMA employs a multi-user detection (MUD) technique such as successive interference cancellation (SIC) to eliminate the multi-user interference and decode the desired signal.

Power allocation (PA) plays an important role in the performance of NOMA. Hence, a suitable PA method with low complexity is required to exploit the power domain optimally. Recently, several PA methods for NOMA have been proposed. For example, one of the low-complexity PA methods is the fractional transmission PA (FTPA) [18]. In FTPA, the powers are allocated to the users based on their channel gains.

1.1. Related Work and Motivation

Recently, NOMA has been proposed for VLC to enhance the spectral efficiency of the system. In [19], NOMA has been applied in a VLC system and a gain ratio PA (GRPA) strategy has been proposed. It was demonstrated that the performance of the system can be improved by tuning the transmission angle of the light emitting diode (LED) and the field of view (FOV) of the receiver. In [20], the performance of NOMA in an indoor 2×2 multiple input multiple output (MIMO) VLC system has been investigated, and a normalized gain difference PA (NGDPA) method has been proposed to improve the achievable sum rate of the system. The authors in [21] proposed an enhanced PA (EPA) method for maximizing the sum rate of an OFDM-NOMA VLC system with an arbitrary number of multiplexed users. In [22], the authors propose a low-complexity PA scheme, called simplified gain ratio power allocation (S-GRPA), for NOMA-based indoor VLC systems. In S-GRPA, the users' channel gains are obtained by a lookup-table method to reduce the complexity and GRPA is used as the PA method. In [23], a low-complexity PA called a simple fair power allocation strategy (SFPA) was proposed to ensure a fair distribution of transmission capacity in a multi-user scenario. SFPA shows robustness to channel estimation errors. The performance of SFPA was compared to other PA methods. Although SFPA offers the highest fairness among the other methods, NGDPA outperforms SFPA in terms of the average sum rate.

The bit error rate (BER) performance of a downlink VLC with NOMA has been analyzed in [24] considering both perfect and imperfect channel state information (CSI). NOMA with OFDM has been applied to an indoor VLC system and its superior performance over OFDMA was demonstrated in [25]. In [26], the authors have derived closed-form expressions for the system coverage probability and the ergodic sum rate. Moreover, the probability that NOMA has higher individual rates than OMA has also been derived. In [27], it has been shown that the LED driving power, which ensures the same quality of service for each user, is lower for NOMA than OMA. In [28], a simple user pairing scheme for NOMA-based VLC systems has been proposed. The superiority of NOMA over conventional OMA has been shown. A flexible-rate SIC-free NOMA technique is considered in [29] for downlink VLC systems, using uneven constellation demapping (UCD) and constellation partitioning coding (CPC).

In [30], a convex optimization is applied to NOMA-based VLC system for downlink transmission in terms of the bit error rate (BER) and the sum-rate. The authors in [31] have proposed and demonstrated a real-time software reconfigurable dynamic power-and-subcarrier allocation scheme for OFDM-NOMA in VLC systems. In [32], an offset QAM/OFDM combined with NOMA (OQAM/OFDM-NOMA) modulation scheme is employed in an asynchronous multi-user multi-cell VLC system with experimental demonstration. In [33], inter-cell interference mitigation is considered in a dimming-aware way for multi-cell NOMA-VLC systems through efficient time-scheduling, scaling, and coordination of NOMA transmissions at the access points. In [34], error analysis is presented for a downlink power-domain NOMA-based VLC system with high-order square quadrature amplitude modulation (QAM) schemes where imperfect SIC is considered.

Most of the existing NOMA VLC studies assume perfect SIC and ignore the serious effect of the multi-user interference on the system performance. This could lead to an inaccurate and impractical analysis on such systems, as multi-user interference is inevitable in NOMA-based VLC systems. Reducing the multi-user interference can be achieved by a careful PA. In this paper, we provide a theoretical analysis of the multi-user interference and user symbol error rate (SER) in NOMA-based VLC systems. Furthermore, we propose a low-complexity PA method that meets NOMA requirements for VLC systems and reduces the multi-user interference.

1.2. Contributions

In this paper, a NOMA scheme is applied in a downlink VLC system. The performance of the system is analyzed and evaluated in terms of the user SER. The main contributions of this paper are summarized as follows:

- (1) *SER Analysis*: An exact closed-form expression for the user SER in a NOMA-based VLC is derived with imperfect SIC. The pulse amplitude modulation (PAM) is considered. The derived expressions are validated through computer simulations.
- (2) *Power Allocation*: A decoding-order-based PA (DOPA) method is proposed which reduces the multi-user interference and satisfies all the PA constraints of NOMA-based VLC. In addition, the optimal PA, which mitigates the multi-user interference and minimizes the error during SIC, is found. The performance of DOPA is compared with GRPA and NGDPA.
- (3) *Multi-user interference Effect*: The impact of the multi-user interference on the performance of NOMA-based systems and the role of PA in controlling the multi-user interference are shown.
- (4) *Sum-rate Improvement*: The contribution of DOPA in the sum rate performance is illustrated compared with GRPA and NGDPA.

1.3. Paper Organization

The rest of this paper is organized as follows. In Section 2, the system model of VLC propagation is described. Section 3 discusses management of inter-group interference for multiple users in NOMA VLC system. In Section 4, the concept of NOMA-based VLC system is illustrated. Then, Section 5 introduces the SER performance analysis. The proposed DOPA is introduced in Section 6. The performance evaluation of the proposed scheme with simulation results are presented and discussed in Section 7. Finally, the conclusion is given in Section 8.

2. System Model

We consider a downlink VLC network composed of multiple optical APs given by $L, l = \{1, \dots, L\}$, serving $K, k = \{1, \dots, K\}$, users distributed on the receiving plane as shown in Figure 1. In this paper, NOMA is proposed as a multiple access scheme, and therefore, the K users must be divided into multiple groups given by $G, g = \{1, \dots, G\}$, each with multiple users given by $K_g, k_g = \{1, \dots, K_g\}$, grouped based on the difference in the channel gain. In this context, the received signal of user k_g served by L optical APs regardless of the transmission scheme, i.e., NOMA, is

$$y^{[k_g, g]} = \mathbf{h}^{[k_g, g]} \mathbf{x}^{[k_g, g]} + \underbrace{\sum_{k'_g=1, k'_g \neq k_g}^{K_g} \mathbf{h}^{[k_g, g]} \mathbf{x}^{[k'_g, g]}}_{\text{Intra-group-interference}} + \underbrace{\sum_{g'=1, g' \neq g}^G \mathbf{h}^{[k_g, g]} \sum_{k_{g'}=1}^{K_{g'}} \mathbf{x}^{[k_{g'}, g']}}_{\text{Inter-group-interference}} + \mathbf{z}^{[k_g, g]}, \quad (1)$$

where $\mathbf{h}^{[k_g, g]} \in \mathbb{R}_+^{L \times 1}$ is the channel vector between the L optical APs and user k_g belonging to group $g, g \in G$, and $\mathbf{x}^{[k_g, g]}$ is the signal transmitted to user k_g . Moreover, $\mathbf{x}^{[k'_g, g]}$ and $\mathbf{x}^{[k_g, g']}$ are interference signals received due to transmission to user k'_g belonging to the same group g and to user k_g belonging to the adjacent group g' , respectively. Finally, $\mathbf{z}^{[k_g, g]}$ is defined as background noise given by the sum of thermal and shot noise [35].

In the work, we propose the implementation of NOMA to manage the interference among users of the same group, while inter-group interference is avoided by considering orthogonal or blind interference alignment (BIA) transmission schemes. It is worth mentioning that, the distribution of users and the channel coherence time are known to the VLC network through the uplink transmission, which can be established using frequency other than the optical one. In the following the optical channel is derived.

Indoor VLC Propagation Model

The main parameters of the indoor VLC attocell with a single LED are shown in Figure 1. The vertical distance from the LED to the receiving plane is denoted by L , the horizontal distance from the center point to user k is denoted by r_k , and the direct distance from the LED to user k is denoted by d_k . The semi-angle of the LED is $\Phi_{1/2}$ and the FOV of user k is Ψ_k . The irradiance and incidence angles are denoted by ϕ_k and ψ_k , respectively.

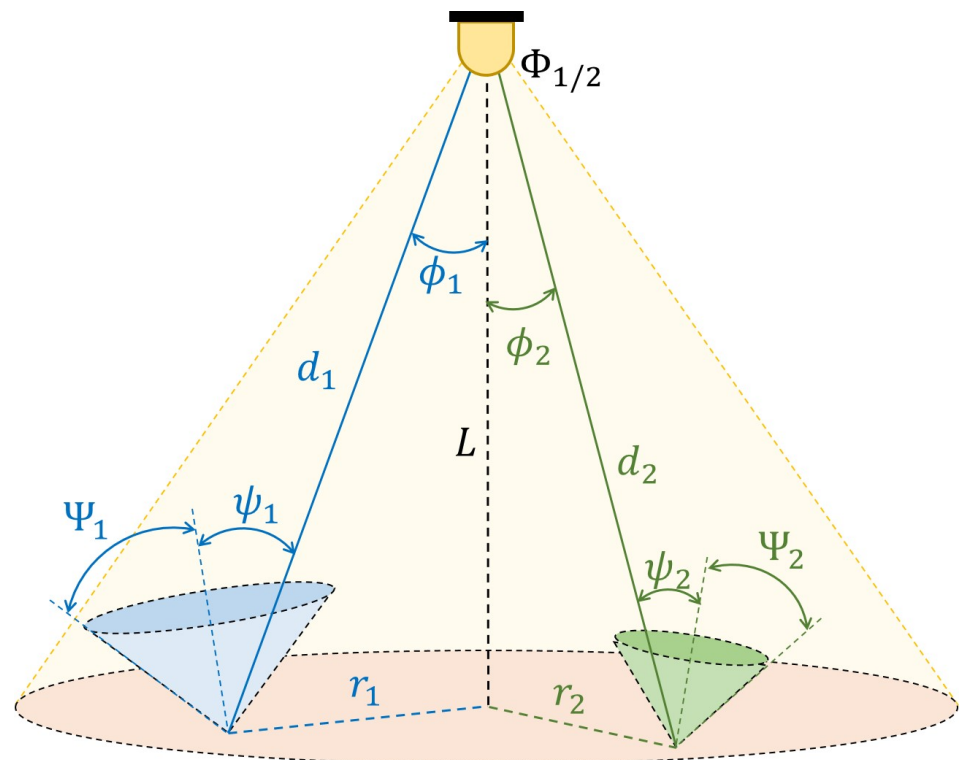


Figure 1. A VLC attocell with a single LED and two users.

The optical channel gain for user k is given by [35]:

$$h_k = \frac{(m+1) A_k T(\psi_k) g(\psi_k)}{2\pi d_k^2} \cos^m(\phi_k) \cos(\psi_k), \quad (2)$$

where $m = -1/\log_2(\cos(\Phi_{1/2}))$ is the order of Lambertian emission; A_k is the effective area of the photodetector (PD); $T(\psi_k)$ is the optical filter gain; and $g(\psi_k) = n^2/\sin^2(\Psi_k)$ is the gain of the optical concentrator with a refractive index n . Since we assume that the users directly face the ceiling of the room, the irradiance angle and the incidence angle are

equal. Therefore, the optical channel gain of the line-of-sight (LOS) link can be expressed as a function of r_k as:

$$h_k = \frac{\zeta_k}{\sqrt{(L^2 + r_k^2)^{m+3}}}, \quad (3)$$

where $\zeta_k = (m + 1)A_k T(\psi_k)g(\psi_k)L^{m+1}/2\pi$.

3. Inter-Group Interference Management

The performance of the NOMA systems can be improved by user pairing into groups based on their channel conditions [36]. Considering the implementation of NOMA, multiple users are divided into multiple groups as in [37]. Basically, an algorithm referred to as next-largest-difference user pairing (NLUPA) is implemented in which the users are sorted in ascending order from lower to higher channel gains as $h^{[1]} \leq \dots h^{[k]} \leq \dots \leq h^{[K]}$. After that, each user with lower channel gain is grouped with a strong user. It is worth mentioning that, the efficiency of the grouping algorithm is determined based on the channel gain difference among the users of the same group.

In this work, we assume that K users are divided into G groups, each is composed of $K_g = 2$, $g \in G$, users as in [37]. The aim of this section is addressing interference management among multiple groups. Orthogonal transmission schemes can be easily implemented where different time or frequency slots are assigned to different groups, and therefore, inter-group interference can be managed as noise. However, this way leads to limit the capacity of the VLC network. In [38], BIA was implemented for RF networks to manage the interference among multiple users maximizing the multiplexing gain. In [39,40], BIA is considered for VLC networks to minimize CSI at transmitters, which is difficult to provide in RF or VLC networks. It is shown that BIA provides high performance for VLC networks compared with zero forcing (ZF) or maximum ratio combining (MRC) schemes. In [41], BIA is adopted to manage the inter-group interference in a MISO RF scenario avoiding the limitations of orthogonal transmission schemes.

In the following, the methodology of BIA is defined first for a simple VLC scenario for the sake of easy understanding, and then, the VLC general case is considered.

3.1. Motivational Example

For illustrative purposes, we consider an example comprises $L = 2$ optical APs serving $K = 6$ users. The users are divided into $G = 3$ groups, each with $K_g = 2$ users. Notice that, the purpose of this example is managing the interference among multiple groups considering BIA, and therefore, the following notations and equations are derived regardless of NOMA, which is presented in Section 4 managing the intra-group interference. Basically, the transmission based on BIA occurs over a block named supersymbol, which comprises multiple symbol extensions, i.e., time slots. The supersymbol of BIA for this particular example is composed of 4 time slots as shown in Figure 2. That is, the transmitted signal can be expressed as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}[1] \\ \mathbf{x}[2] \\ \mathbf{x}[3] \\ \mathbf{x}[4] \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_2 \\ \mathbf{0}_2 \\ \mathbf{0}_2 \end{bmatrix}}_{\mathbf{W}^{[1]}} \mathbf{s}^{[1]} \sqrt{P_1} + \underbrace{\begin{bmatrix} \mathbf{I}_2 \\ \mathbf{0}_2 \\ \mathbf{I}_2 \\ \mathbf{0}_2 \end{bmatrix}}_{\mathbf{W}^{[2]}} \mathbf{s}^{[2]} \sqrt{P_2} + \underbrace{\begin{bmatrix} \mathbf{I}_2 \\ \mathbf{0}_2 \\ \mathbf{0}_2 \\ \mathbf{I}_2 \end{bmatrix}}_{\mathbf{W}^{[3]}} \mathbf{s}^{[3]} \sqrt{P_3}, \quad (4)$$

where $\mathbf{x}[n] \in \mathbb{R}_+^{L \times 1}$ is the signal transmitted over the time slot n , $\mathbf{I}$ and $\mathbf{0}$ are 2×2 identity and zero matrices, respectively. Moreover, $\mathbf{s}^{[g]} = (\mathbf{s}^{[1,g]} + \mathbf{s}^{[2,g]})$ is the symbol transmitted to $K_g = 2$ users belonging to group g , $g \in G$. Furthermore, P_g and $\mathbf{W}^{[g]}$ are the power allocated and the precoding matrix of group g , respectively. Notice that, over the supersymbol that comprised 4 time slots, a resource block composed of two time slots, i.e., each time slot corresponds to one transmitter, allocated to each group. Focussing on group 1, i.e., $g = 1$, the first and second time slots form the resource block selecting $\mathbf{h}(1)$ and $\mathbf{h}(2)$, respectively. Therefore, the received signal, for example, by user 1 is given by

$$\begin{aligned} \begin{bmatrix} y^{[1,1]}[1] \\ y^{[1,1]}[2] \\ y^{[1,1]}[3] \\ y^{[1,1]}[4] \end{bmatrix} &= \underbrace{\begin{bmatrix} \mathbf{h}^{[1,1]T}(1) \\ \mathbf{h}^{[1,1]T}(2) \\ \mathbf{0}_{2,1}^T \\ \mathbf{0}_{2,1}^T \end{bmatrix}}_{\text{rank}=2} (\mathbf{s}^{[1,1]} + \mathbf{s}^{[2,1]}) \sqrt{P_1} + \\ &\underbrace{\begin{bmatrix} \mathbf{h}^{[1,1]T}(1) \\ \mathbf{0}_{2,1}^T \\ \mathbf{h}^{[1,1]T}(1) \\ \mathbf{0}_{2,1}^T \end{bmatrix}}_{\text{rank}=2} (\mathbf{s}^{[1,2]} + \mathbf{s}^{[2,2]}) \sqrt{P_2} + \underbrace{\begin{bmatrix} \mathbf{h}^{[1,1]T}(1) \\ \mathbf{0}_{2,1}^T \\ \mathbf{0}_{2,1}^T \\ \mathbf{h}^{[1,1]T}(1) \end{bmatrix}}_{\text{rank}=2} \\ &(\mathbf{s}^{[1,3]} + \mathbf{s}^{[2,3]}) \sqrt{P_3} + \begin{bmatrix} z^{[1,1]}[1] \\ z^{[1,1]}[2] \\ z^{[1,1]}[3] \\ z^{[1,1]}[4] \end{bmatrix}. \end{aligned} \quad (5)$$

Notice that, the users of group 1 receive their information over the first and second time slots, and therefore, the interference between them must be managed based on NOMA. Moreover, they receive interference over the first time slot due to transmission to adjacent groups 2 and 3. Based on BIA, orthogonal transmission is carried out over second, third and fourth time slots as in (5). Therefore, the users of group 1 can dedicate the third and fourth time slots to measure the interference received over the first time slot. That is, the signal received by user 1 after interference subtraction is given by

$$\begin{aligned} \mathbf{y}^{[1,1]} &= \begin{bmatrix} \mathbf{h}^{[1,1]T}(1) \\ \mathbf{h}^{[1,1]T}(2) \end{bmatrix} (\mathbf{s}^{[1,1]} + \mathbf{s}^{[2,1]}) \sqrt{P_1} + \\ &\begin{bmatrix} z^{[1,1]}[1] - z^{[1,1]}[3] - z^{[1,1]}[4] \\ z^{[1,1]}[2] \end{bmatrix}. \end{aligned} \quad (6)$$

Notice that, user 2 of group 1 follows the same procedure. As a result, the symbols transmitted to the users of group 1 can be decoded where the interference among groups is aligned and subtracted. It is worth mentioning that, over a supersymbol comprised 4 time slots, 2 DoF can be achieved by group 1. The groups 2 and 3, following the same methodology above, can achieve 2 DoF each. Therefore, for the considered example, the sum DoF equals $\frac{12}{3}$ considering the number of users that form each group. However, applying NOMA, there is a certain level of interference among the users of the same group that must be taken into consideration, which is the aim of this work, in order to further enhance the performance of the VLC network.

Time slot	Group 1	Group 2	Group 3
1	$\mathbf{h}^{[1]} (1)$	$\mathbf{h}^{[2]} (1)$	$\mathbf{h}^{[3]} (1)$
2	$\mathbf{h}^{[1]} (2)$	$\mathbf{h}^{[2]} (1)$	$\mathbf{h}^{[3]} (1)$
3	$\mathbf{h}^{[1]} (1)$	$\mathbf{h}^{[2]} (2)$	$\mathbf{h}^{[3]} (1)$
4	$\mathbf{h}^{[1]} (1)$	$\mathbf{h}^{[2]} (1)$	$\mathbf{h}^{[3]} (2)$

Figure 2. The supersymbol of BIA for a VLC network with $L = 2$ transmitters and $G = 3$ groups.

3.2. General Case

According to [38], the supersymbol of BIA is divided into two blocks referred to as Block 1 and Block 2. In the example above, Block 1 is given by the first time slot, while Block 2 is formed by the second, third and fourth time slots.

Let us consider a general case in which L optical APs serving K users divided into G groups. In contrast to BIA in [38], in this work, the supersymbol of BIA is given by the number of transmitters and groups as in [41]. In Block 1, all groups receive their information simultaneously generating severe inter-group interference. Therefore, Block 1 comprises $(L - 1)^G$ time slots. In Block 2, orthogonal transmission is carried out in order to give the users of each group enough dimensions to measure the interference received over Block 1. Therefore, Block 2 comprises $G(L - 1)^{G-1}$ time slots. As a result, $(L - 1)^{G-1}$ resource blocks are allocated for each group. Notice that, the first $L - 1$ time slots of each resource block allocated for each group are provided over Block 1, while the last time slot is provided over Block 2. To conclude, the length of the supersymbol of BIA implemented for inter-group interference mitigation is given by

$$\Gamma_{BIA} = (L - 1)^G + G(L - 1)^{G-1}. \quad (7)$$

The methodology of BIA is explained in more details in [38].

4. NOMA-Based VLC

As in [37], the users are divided into multiple groups, each with strong and weak users. In NOMA, two main processes must be considered. Firstly, superposition coding (SC) at transmitters where two different levels of power are assigned to the signals of the strong and weak users through an appropriate PA method. These signals are then superposed into a single signal representing the instantaneous electrical current that drives the transmitter.

Secondly, performing SIC at the users side giving each user the ability to cancel the interference received due to the transmission to users with lower channel gains. In SIC, the strong users decode the signals with high power and subtract them from the received signal until the desired signal is decoded. Both SC and SIC are conducted in the electrical domain. In Figure 3, the concept of NOMA in a VLC system is summarized.

The instantaneous electrical current that drives optical transmitters is given by:

$$\mathbf{x}_e = \sum_{g=1}^G \mathbf{w}^{[g]} \sum_{k_g=1}^{K_g} \sqrt{a_{k_g} P_g} \mathbf{s}^{[k_g, g]} + I_{DC}, \quad (8)$$

where a_{k_g} is the power allocation factor and I_{DC} is a DC current to ensure that the total current is unipolar. Here, $\mathbf{s}^{[k_g, g]}$ is assumed to be a vector that contains the symbols transmitted to the group over resource blocks allocated from BIA implemented to align the interference among multiple groups.

The optical transmitted signal is given by [26]:

$$\mathbf{X}_o = \eta \mathbf{X}_e, \quad (9)$$

where η is the efficiency of the transmitter to convert the electrical current to optical power (W/A). The received signal by user k_g belonging to group g after canceling the inter-group interference is given by :

$$\mathbf{y}^{[k_g, g]} = \mathbf{h}^{[k_g, g]} \mathbf{W}^{[g]} \sum_{k_g=1}^{K_g} \sqrt{a_{k_g} P_g} \mathbf{s}_\ell^{[k_g, g]} + \mathbf{z}^{[k_g, g]}, \quad (10)$$

where $\mathbf{s}_\ell^{[k_g, g]}$ contains the symbols transmitted to group g over the resource block ℓ . Notice that, $(L-1)^{G-1}$ resource blocks allocated to each group due to the implementation of BIA. Although, in (10) the inter-group interference is canceled, user k_g is subject to multi-user interference resulted from applying NOMA. In the following, this issue is addressed in more details aiming to enhance the performance of NOMA.

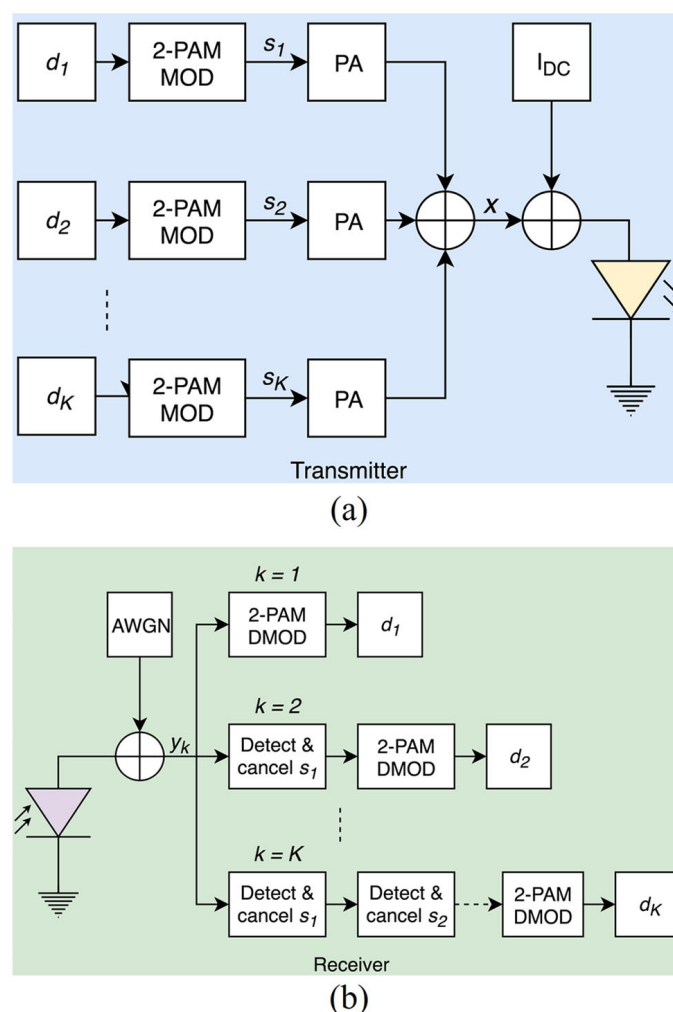


Figure 3. The concept of a NOMA-based VLC system: (a) Transmitter (b) Receiver.

Multi-User Interference in NOMA

From now on, without loss of generality, we focus on a generic group and eliminate the index of the group for the sake of simplicity. To explain the multi-user interference in NOMA, the received signal in (10) is rewritten as:

$$y_k = c_k \left(\underbrace{\sum_{i=1}^{k-1} \sqrt{a_i} s_i}_{\text{interference}} + \underbrace{\sqrt{a_k} s_k}_{\text{signal}} + \underbrace{\sum_{j=k+1}^K \sqrt{a_j} s_j}_{\text{noise}} \right) + z_k, \quad (11)$$

where $c_k = v h_k \eta \sqrt{P_T}$ and v is the PD responsivity. There are two types of user interference in NOMA systems in (11); the interference from users with lower decoding order, i.e., users with higher powers, and the interference from users with higher decoding order, i.e., users with lower powers. The first type should be successfully canceled during SIC process, while the second type can be considered as noise. The interference can be controlled through the PA process. To ensure the success of SIC, the power of the signal to be decoded should be higher than the power of the second-type interference plus noise. Otherwise, the interference could become severe and leads to unsuccessful SIC, which results in a potential performance degradation. On the other hand, the powers of the signals with higher decoding orders should be high enough to be distinguished from the noise. Consequently, there is a trade-off between reducing the interference to decode the first-user signal and reducing the effect of the noise to decode the second-user signal. In other words, the higher the power allocated to the first user the less interference from the second user. However, the effect of AWGN on the second-user signal will be increased.

5. SER Analysis

Typically, VLC can be realized as an intensity modulation/direct detection (IM/DD) system [42], which requires that the modulated signal has to be both real-valued and non-negative (unipolar). Hence, more efficient modulation schemes, such as QAM cannot be directly implemented for IM/DD because they are complex-valued and bipolar in nature. Optical orthogonal frequency division multiplexing (O-OFDM) systems are the most commonly utilized solution [43]. In O-OFDM, a real-valued time-domain signal is obtained at the expense of a 50% reduction of the spectral efficiency due to Hermitian symmetry constraint. A unipolar time-domain signal can then be obtained by adding a positive DC to the OFDM signal, such as in the DC-biased O-OFDM (DCO-OFDM) [44], or using techniques that obtain a unipolar time-domain signal such as in asymmetrically clipped O-OFDM (ACO-OFDM) with a further 50% reduction of the data rate [44].

In this work, PAM with DC-biasing is adopted as a modulation scheme to avoid the reduction in spectral efficiency imposed by the requirements of using higher-order modulation schemes in VLC systems. In this section, we derive SER expressions for the users in NOMA-based VLC systems. First, we derive the SER expressions when the modulation scheme for both users is 2-PAM. Then, we generalize the derivation for M-PAM.

5.1. Derivation of First User SER

In NOMA systems, the signal constellation corresponding to the signal of user k can be expressed by the vector $\mathbf{s}_k = (s_{k1}, \dots, s_{kM})$, where s_{km} is the symbol m for user k and M is the total number of symbols. Figure 4 shows the constellations of both users and the superposed signal in a 2-user NOMA system with 2-PAM signaling. $\mathbf{s}_1 = (-\sqrt{a_1 E_s}, \sqrt{a_1 E_s})$ and $\mathbf{s}_2 = (-\sqrt{a_2 E_s}, \sqrt{a_2 E_s})$, where a_k and E_s are the power allocation factor and the maximum energy per symbol before the PA, respectively. The signal constellation corresponding to the superposed transmitted signal (x) consists of four constellation points, i.e., $\mathbf{s}_x = \{s_{x1}, s_{x2}, s_{x3}, s_{x4}\}$, where $s_{x1} = -\sqrt{a_1 E_s} - \sqrt{a_2 E_s}$, $s_{x2} = -\sqrt{a_1 E_s} + \sqrt{a_2 E_s}$, $s_{x3} = \sqrt{a_1 E_s} - \sqrt{a_2 E_s}$, and $s_{x4} = \sqrt{a_1 E_s} + \sqrt{a_2 E_s}$. The symbols s_{x1} , s_{x2} , s_{x3} , and s_{x4} are assumed to be equally probable, i.e., $Pr\{s_{x1}\} = Pr\{s_{x2}\} = Pr\{s_{x3}\} = Pr\{s_{x4}\} = \frac{1}{4}$.

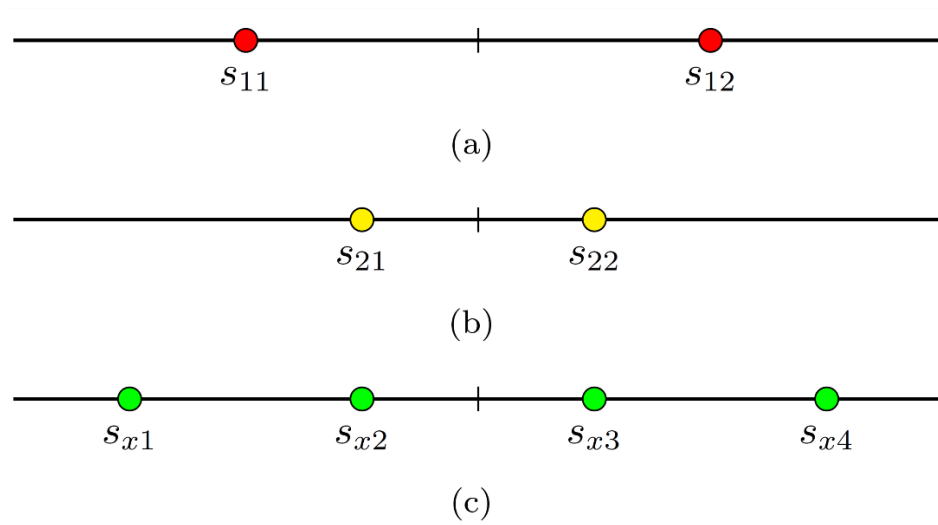


Figure 4. The signal constellation of: (a) weak user (u_1), (b) strong user (u_2), and (c) superposed transmitted signal (x).

The received symbol can be expressed as:

$$\tilde{y}_k = g_k s_x + \tilde{n}_k, \quad (12)$$

where $g_k = v\eta h_k$ is the total channel gain and the signal $s_x \in \{s_{x1}, s_{x2}, s_{x3}, s_{x4}\}$. Since the noise n is assumed to be zero-mean AWGN with N_0 variance, i.e., $\tilde{n}_k \sim \{0, N_0\}$, the probability density function (PDF) of $\tilde{n}_k$ is given by:

$$Pr(\tilde{n}_k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\tilde{n}_k - \mu)^2}{2\sigma^2}\right), \quad (13)$$

where $\mu = 0$ is the mean and $\sigma^2 = N_0$ is the variance. We assume that g_k is previously known at the receiver. Hence, equalization is performed by dividing the received symbol $\tilde{y}_k$ by g_k , then:

$$y_k = s_x + n_k, \quad (14)$$

where $n_k = \tilde{n}_k/g_k$ is the additive Gaussian noise scaled by the total channel gain. The conditional PDF of the equalized received signal y_k given s_{x3} was sent is given by:

$$Pr(y_k|s_{x3}) = \frac{1}{\sqrt{2\pi N_k}} \exp\left(-\frac{(y_k - s_{x3})^2}{2N_k}\right), \quad (15)$$

where $N_k = N_0/g_k^2$. Similarly,

$$Pr(y_k|s_{x4}) = \frac{1}{\sqrt{2\pi N_k}} \exp\left(-\frac{(y_k - s_{x4})^2}{2N_k}\right). \quad (16)$$

The optimal decision threshold for the maximum likelihood (ML) receiver of the first user is $\lambda_{th} = 0$. Therefore, the ML receiver should detect the symbol $s_{11} = -\sqrt{a_1 E_s}$ if either s_{x1} or s_{x2} was sent. Similarly, it should detect the symbol $s_{12} = +\sqrt{a_1 E_s}$ if either s_{x3} or s_{x4} was sent.

Figure 5 shows the conditional PDF of the received signal y_k given s_{x3} was sent and the conditional PDF of the received signal y_k given s_{x4} was sent. The dark region represents the probability of error.

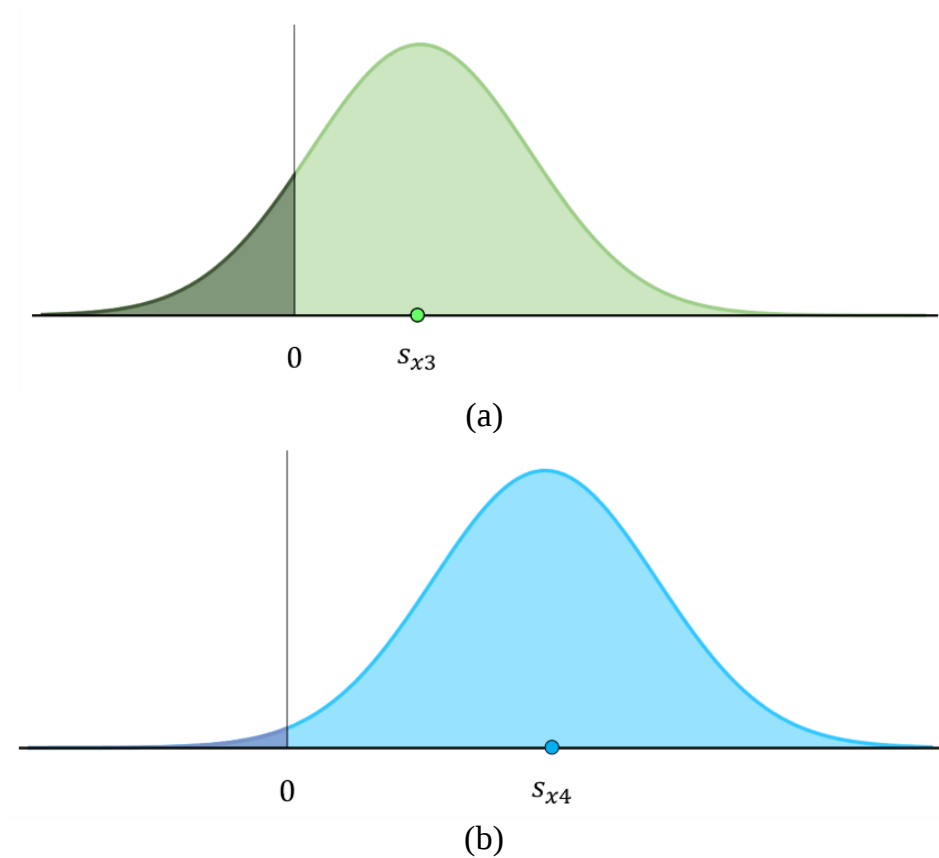


Figure 5. The conditional PDF (a) $Pr(y_k|s_{x3})$. (b) $Pr(y_k|s_{x4})$. The dark region represents the probability of error.

Therefore, the probability of error given s_{x3} was sent is given by:

$$\begin{aligned}
 Pr\{e|s_{x3}\} &= \frac{1}{\sqrt{2\pi N_k}} \int_{-\infty}^0 \exp\left(-\frac{(y_k - s_{x3})^2}{2N_k}\right) dy_k \\
 &= \frac{1}{\sqrt{2\pi}} \int_{\frac{s_{x3}}{\sqrt{N_k}}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz \\
 &= Q\left(\frac{s_{x3}}{\sqrt{N_k}}\right) \\
 &= Q(\sqrt{\gamma_k} \Gamma_1),
 \end{aligned} \tag{17}$$

where $\gamma_k = \frac{g_k^2 E_s}{N_0}$, $\Gamma_1 = \sqrt{a_1} - \sqrt{a_2}$, and $Q(\cdot)$ is the Q-function defined by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-\frac{u^2}{2}) du$. Similarly, the probability of error given s_{x4} was sent is given by:

$$Pr\{e|s_{x4}\} = Q\left(\frac{s_{x4}}{\sqrt{N_k}}\right) = Q(\sqrt{\gamma_k} \Gamma_2), \tag{18}$$

where $\Gamma_2 = \sqrt{a_1} + \sqrt{a_2}$.

The SER for the first user can be expressed as:

$$\begin{aligned}
 SER_1 &= Pr\{s_{x1}\}Pr\{e|s_{x1}\} + Pr\{s_{x2}\}Pr\{e|s_{x2}\} \\
 &\quad + Pr\{s_{x3}\}Pr\{e|s_{x3}\} + Pr\{s_{x4}\}Pr\{e|s_{x4}\}.
 \end{aligned} \tag{19}$$

Since the constellation of x is symmetric, then $Pr\{e|s_{x1}\} = Pr\{e|s_{x4}\}$, and $Pr\{e|s_{x2}\} = Pr\{e|s_{x3}\}$. Therefore, (19) can be reduced to:

$$SER_1 = \frac{1}{2}(Pr\{e|s_{x3}\} + Pr\{e|s_{x4}\}) . \quad (20)$$

By substituting (17) and (18) in (20), we obtain:

$$SER_1 = \frac{1}{2}Q(\sqrt{\gamma_1} \Gamma_1) + \frac{1}{2}Q(\sqrt{\gamma_1} \Gamma_2). \quad (21)$$

5.2. Derivation of Second User SER

For the sake of the second user, since it is required to cancel the interference coming from the signal of the first user, the SER of the second user depends on whether the first user symbol was successfully detected or not. Therefore, the SER for the second user can be calculated as:

$$SER_2 = (1 - SER_{2 \rightarrow 1})SER_{2 \rightarrow 2} + SER_{2 \rightarrow 1} , \quad (22)$$

where $SER_{2 \rightarrow 1}$ is the SER for the second user to detect the signal of the first user and $SER_{2 \rightarrow 2}$ is the SER for the second user to detect its own signal after detecting the signal of the first user successfully. In (22), we consider the worst-case scenario in which the second user cannot decode its signal successfully whenever the signal of the first user was unsuccessfully detected. $SER_{2 \rightarrow 1}$ can be calculated in a similar way to (21). Therefore:

$$SER_{2 \rightarrow 1} = \frac{1}{2}Q(\sqrt{\gamma_2} \Gamma_1) + \frac{1}{2}Q(\sqrt{\gamma_2} \Gamma_2). \quad (23)$$

After canceling the interference from the signal of the first user successfully, the second user will detect its signal as if there is no interference. Therefore, $SER_{2 \rightarrow 2}$ can be calculated easily as:

$$SER_{2 \rightarrow 2} = Q(\sqrt{a_2 \gamma_2}). \quad (24)$$

Combining (22) and (24), the SER for the second user can be calculated as:

$$SER_2 = Q(\sqrt{a_2 \gamma_2}) + SER_{2 \rightarrow 1}(1 - Q(\sqrt{a_2 \gamma_2})). \quad (25)$$

Hence, the SER of the second user can be calculated using (23) and (25).

5.3. Derivation of General SER Expression

The general case for high order PAM schemes can be derived following the same fashion proposed in Sections 5.1 and 5.2. Let M_1 and M_2 be the modulation orders for user 1 and user 2, respectively. The symbols alphabet for user 1 could be represented as

$$A_{1m_1} = 2m_1 - 1 - M_1, \quad m_1 = 1, 2, \dots, M_1 \quad (26)$$

Similarly, The symbols alphabet for user 2 could be represented as

$$A_{2m_2} = 2m_2 - 1 - M_2, \quad m_2 = 1, 2, \dots, M_2 \quad (27)$$

Therefore, user 1 and user 2 symbols can be expressed as

$$s_{km_k} = A_{km_k} \zeta_k \sqrt{a_k E_s}, \quad k = 1, 2 \quad (28)$$

where $\zeta_k = \sqrt{\frac{3}{M_k^2 - 1}}$ is a normalization factor. The signal constellation corresponding to the superposed signal (x) consists of $M_1 M_2$ constellation points. The superimposed symbol is expressed as follows:

$$s_x = \sqrt{E_s}(A_{1m_1} \zeta_1 \sqrt{a_1} + A_{2m_2} \zeta_2 \sqrt{a_2}). \quad (29)$$

Figure 6 shows the constellations of both users and the superposed signal when both users have 4-PAM signaling. The vertical lines represent the boundaries of the decision regions for user 1.

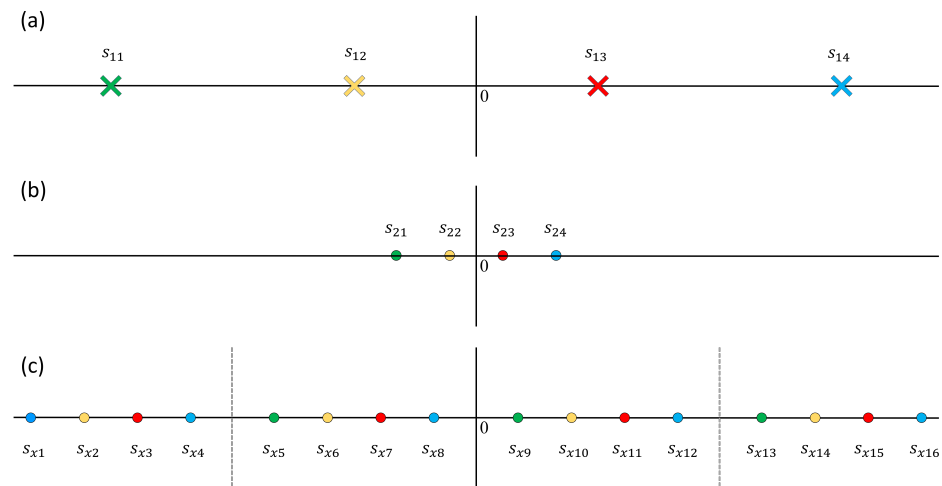


Figure 6. Signal constellation when both users have 4-PAM. (a) user 1, (b) user 2, and (c) superposed signal (x).

To calculate the SER for user 1, we follow the same procedures in Section 5.1 to find $Pr\{e|s_{x_k}\}$ and exploit the constellation symmetry around the y-axis to reduce the calculations. Thus, we need only to calculate the error probability for the symbols on the right-hand side or left-hand side. To further reduce the calculations, the symmetry of the error probability of some symbols in the decision region can be exploited. For example, in 4-PAM case $Pr\{e|s_{x_9}\} = Pr\{e|s_{x_{12}}\}$ and $Pr\{e|s_{x_{10}}\} = Pr\{e|s_{x_{11}}\}$. Therefore, the general expression for SER for user 1 can be expressed as:

$$SER_1 = \frac{2(M_1 - 1)}{M_1 M_2} \sum_{k=1}^{M_2/2} (Q(\sqrt{\gamma_1} \Gamma_1) + Q(\sqrt{\gamma_1} \Gamma_2)), \quad (30)$$

where $\Gamma_1 = \xi_1 \sqrt{a_1} - (2k - 1)\xi_2 \sqrt{a_2}$ and $\Gamma_2 = \xi_1 \sqrt{a_1} + (2k - 1)\xi_2 \sqrt{a_2}$.

The SER for user 2 in the general case can be calculated following same procedures in Section 5.2. Equation (22) is still applicable for determining SER_2 in the general case, but the terms $SER_{2 \rightarrow 1}$ and $SER_{2 \rightarrow 2}$ need to be updated. $SER_{2 \rightarrow 1}$ can be calculated in a similar way to (30). Therefore:

$$SER_{2 \rightarrow 1} = \frac{2(M_1 - 1)}{M_1 M_2} \sum_{k=1}^{M_2/2} (Q(\sqrt{\gamma_2} \Gamma_1) + Q(\sqrt{\gamma_2} \Gamma_2)). \quad (31)$$

After canceling the interference from user 1 signal, the second user will decode its signal as if there is no interference. Therefore, $SER_{2 \rightarrow 2}$ can be calculated easily as:

$$SER_{2 \rightarrow 2} = 2 \left(1 - \frac{1}{M_2}\right) Q(\sqrt{a_2 \gamma_2}). \quad (32)$$

Combining (22) and (32), the SER for the second user can be calculated as:

$$SER_2 = 2 \left(1 - \frac{1}{M_2}\right) Q(\sqrt{a_2 \gamma_2}) + SER_{2 \rightarrow 1} \left(\left(\frac{2}{M_2} - 1\right) Q(\sqrt{a_2 \gamma_2}) \right). \quad (33)$$

Hence, the SER of the second user can be calculated using (31) and (33).

6. Power Allocation in NOMA

In NOMA systems, the users are multiplexed in the power domain, hence, the power allocated to each user should be carefully determined. For PA in NOMA-based VLC systems, two main power constraints should be satisfied: the first constraint is requires that the total power should be kept constant since the light sources in VLC systems are used for both illumination and data transmission, i.e., $\sum_{i=1}^K p_i = P_T$. The second constraint is that the powers allocated to the users with lower decoding order, i.e., lower channel gains, should be greater than the powers assigned to the users with higher decoding order, i.e., higher channel gains. In other words, NOMA users are allocated with power levels inversely proportional to their channel gain, i.e., $p_1 > p_2 > \dots > p_K$. This constraint is to ensure successful decoding during SIC [45].

Since the indoor VLC attocells have small coverage area and due to the fact that the users may have the same horizontal distance, the users in an attocell typically have close or similar channel gains [46]. Although the performance of NOMA is degraded when the channel gains are close or similar, it is shown that NOMA can always increase the sum rate of VLC systems even if the users have similar channel gains. However, using NOMA for pairing users with close or similar channel gains may increase the effect of multi-user interference on decoding the signals of the users.

To solve the previous issue, we propose a low-complexity PA method that guarantees different powers allocated to the users in all cases even if the users have similar channel gains. The proposed PA method is a decoding-order-based PA (DOPA) in which the powers are allocated to the users based on their indices in the decoding order and their channel gains. In DOPA, the power allocated to user k is given by:

$$p_k = \frac{(K - k + 1)^\alpha h_{K-k+1}}{\sum_{i=1}^K (K - i + 1)^\alpha h_{K-i+1}} P_T, \quad (34)$$

where α is an optimization parameter that should be predetermined to maximize the target performance metric such as the system sum rate, the fairness among the users or the energy efficiency. The powers allocated to the users with DOPA satisfy the following constraints:

- $\sum_{i=1}^K p_i = P_T$
- $p_k > p_{k+1}, \forall k \in \{1, 2, \dots, K - 1\}$ and $\alpha > 0$

The first point represents the first PA constraint in NOMA-based VLC. The sum of all powers allocated to the users using DOPA can be calculated as:

$$\sum_{k=1}^K p_k = \sum_{k=1}^K \frac{(K - k + 1)^\alpha h_{K-k+1}}{\sum_{i=1}^K (K - i + 1)^\alpha h_{K-i+1}} P_T. \quad (35)$$

Let $u = K - i + 1$ and $v = K - k + 1$, then:

$$\begin{aligned} \sum_{k=1}^K p_k &= \sum_{v=1}^K \frac{v^\alpha h_v}{\sum_{u=1}^K u^\alpha h_u} P_T \\ &= \frac{\sum_{v=1}^K v^\alpha h_v}{\sum_{u=1}^K u^\alpha h_u} P_T = P_T. \end{aligned} \quad (36)$$

Therefore, the total power of DOPA is always constant. The second point states that the powers allocated to the users by DOPA are distinctive and follow the optimal decoding order, i.e., $p_1 > p_2 > \dots > p_K$. Using (34), p_{k+1} can be calculated by:

$$p_{k+1} = \frac{(K - k)^\alpha h_{K-k}}{\sum_{i=1}^K (K - i + 1)^\alpha h_{K-i+1}} P_T. \quad (37)$$

Let $m = K - k$ and substitute in (34) and (37), then:

$$p_k = \frac{(m+1)^\alpha h_{m+1}}{\sum_{i=1}^K (K-i+1)^\alpha h_{K-i+1}} P_T. \quad (38)$$

and

$$p_{k+1} = \frac{(m)^\alpha h_m}{\sum_{i=1}^K (K-i+1)^\alpha h_{K-i+1}} P_T. \quad (39)$$

From (38) and (39), we obtain

$$p_k = \left(\frac{m+1}{m} \right)^\alpha \left(\frac{h_{m+1}}{h_m} \right) p_{k+1}. \quad (40)$$

Since $h_m \leq h_{m+1}$ is given and $m+1 > m$, it is clear from (40) that p_k is always greater than p_{k+1} for all possible values of h_m and h_{m+1} as long as $\alpha > 0$. Therefore, the proposed DOPA satisfies the PA constraints of NOMA-based VLC.

Figure 7 shows the PA coefficients of the first user (u_1) and the second user (u_2) using DOPA with different values of the optimization parameter (α). Whether the users have similar or different horizontal distances, i.e., similar or different channel gains, the power allocated to u_1 is always higher than the power allocated to u_2 except when both users have similar horizontal distances and $\alpha = 0$. As α increases, the powers allocated to the users become more distinct. However, for high values of α , the power of u_2 becomes very low. Consequently, u_2 becomes more susceptible to the AWGN. Therefore, the value of α should be determined carefully.

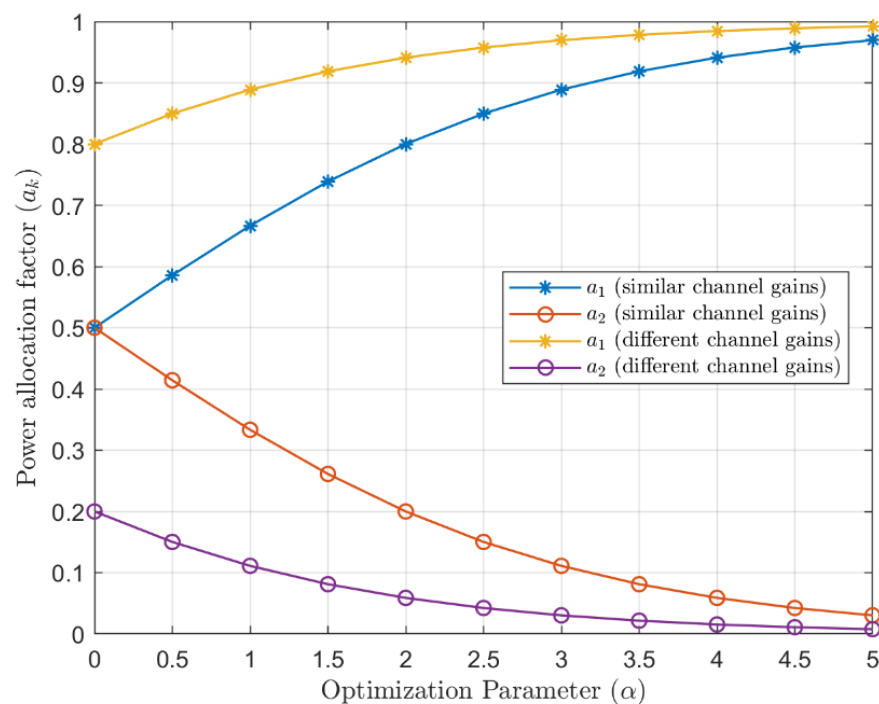


Figure 7. PA coefficients using DOPA with the optimization parameter.

The optimal value of α that minimizes SER_1 is infinity since SER_1 reduces as the power allocated for the first user is increased. On the other hand, to get the optimal value of the optimization parameter α that minimizes the SER of the second user in a 2-user NOMA, the previous SER analysis is used. The problem can be formulated as follows:

$$\hat{\alpha} = \arg \min_{\alpha} (\text{SER}_2), \quad (41)$$

where SER_2 is given by (25). Using $K = 2$ in (34), we obtain:

$$\begin{aligned} a_1 &= \frac{2^\alpha h_2}{h_1 + 2^\alpha h_2} \\ a_2 &= \frac{h_1}{h_1 + 2^\alpha h_2} \end{aligned} \quad (42)$$

Substituting (25) in (41), the problem becomes:

$$\hat{\alpha} = \arg \min_{\alpha} (Q(\sqrt{\gamma_2 a_2}) + (\frac{1}{2}Q(\sqrt{\gamma_2} \Gamma_1) + \frac{1}{2}Q(\sqrt{\gamma_2} \Gamma_2))\Theta), \quad (43)$$

where $\Theta = 1 - Q(\sqrt{a_2 \gamma_2})$.

Although the problem (43) cannot be solved easily by setting the first derivative (with respect to α) of (43) equal to zero and finding the solution of the resulting equation, it is convex and can be solved numerically or using optimization software. Figure 8, presents the optimal values of α that minimize SER_2 . The Golden Section Search and Parabolic Interpolation algorithm [47] are used to find $\hat{\alpha}$ that satisfies (43). As can be seen, the optimal value of α for high γ_2 is almost fixed. For example, when $h_2 = 2h_1$, $\hat{\alpha} = 1$, and when $h_2 = 3h_1$, $\hat{\alpha} \approx 0.4$.

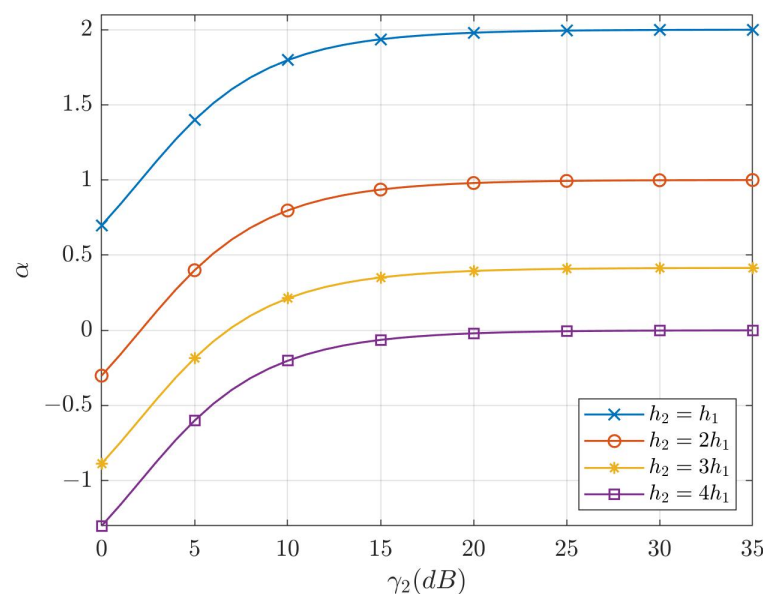


Figure 8. Optimal values of the optimization parameter α that minimize SER_2 .

It is worth noting that both GRPA, NGDPA, and proposed DOPA depend on the channel gains of the users, which can be obtained through channel estimation. In the proposed DOPA, the decoding order of the users is included to guarantee different powers allocated to the users. Moreover, the proposed DOPA is more flexible than GRPA and NGDPA since it has the optimization parameter α which could be predetermined for minimizing the symbol error rate or maximizing the target performance metric, such as the system sum rate, the fairness, or the energy efficiency.

In terms of complexity, both GRPA, NGDPA, and proposed DOPA with fixed α have nearly the same complexity. For the optimal α , the complexity of the proposed DOPA increases depending on the applied optimization algorithm to find optimal α . Nonetheless, the complexity of the proposed DOPA, in this case, can be significantly reduced by pre-determining the $\hat{\alpha}$ values for different SNR values and channel gain differences. Then, the predetermined values can be stored in lookup tables instead of optimizing during runtime.

7. Performance Evaluation

In this section, analytical and simulation results are presented for the SER performance of a NOMA-based VLC system. DOPA is compared with GRPA and NGDPA. Table 1 displays the simulation parameters.

The simulation for SERs computation is performed as follows. First, a frame of 1000 uniformly-distributed random symbols is generated for each user. Then, the generated symbols are modulated using M-PAM, depending on the modulation order of each user. After that, the modulated signals are normalized and the PA is applied to the normalized signals. Then, the signals are superimposed into one signal and the channel effect is added to the superimposed signal depending on the signal-to-noise ratio (SNR). User 1 signal is demodulated directly from the received signal. In the case of User 2, SIC is performed on the received signal by decoding User 1 signal and then subtracting it from the received signal. Then, User 2 signal is demodulated from the resulting signal. The demodulated signals are compared to the corresponding generated signals and the errors are computed. Finally, the above steps are repeated for 1000 iterations and the cumulative errors are used to find the SER for each user.

Table 1. Simulation Parameters.

Parameter	Notation	Value
Vertical distance	L	2 m
LED input power	P_T	1 W
LED semi-angle	$\Phi_{1/2}$	60°
LED efficiency	η	1 W/A
PD FOV	Ψ	60°
PD area	A	10^{-4} m^2
PD responsivity	v	0.6 A/W
Optical filter gain	T	1
Refractive index	n	1.5
Noise power spectral density	N_0	$10^{-15} \text{ A}^2/\text{Hz}$

7.1. SER Evaluation

In Figure 9, theoretical and simulated SER of two users, u_1 and u_2 , in a NOMA-based VLC system are plotted in two cases. In the first case, the power allocation coefficients of u_1 and u_2 are $a_1 = 0.85$ and $a_2 = 0.15$, respectively, while in the second case $a_1 = 0.9$ and $a_1 = 0.1$. From Figure 6, it is obvious that simulation outcomes match well with theoretical analysis, which validates and confirms the derived SER expressions. Moreover, it can be observed that the SER of u_2 is higher than the SER of u_1 at the same SNR value because the power allocated to u_2 is lower and u_2 needs to decode u_1 signal first before decoding its own signal.

Figures 10 and 11 show the comparison of DOPA, GRPA, and NGDPA in terms of SER_1 and SER_2 , respectively. When both users have similar channel gains, GRPA allocates equal power level to both users. Hence, the interference generated by the second user on the first user is very high and cannot be neglected. Similarly, the second user cannot successfully cancel the interference inserted by the first user. This explains the bad performance of GRPA in this case for both users. On the other hand, NGDPA allocates zero power to the second user when both users have similar channel gains. In this case, there is no interference at the first user which explains its good performance. However, the second signal is lost. Unlike

GRPA and NGDPA, DOPA offers better performance for both users because it allocates different non-zero power levels to the users.

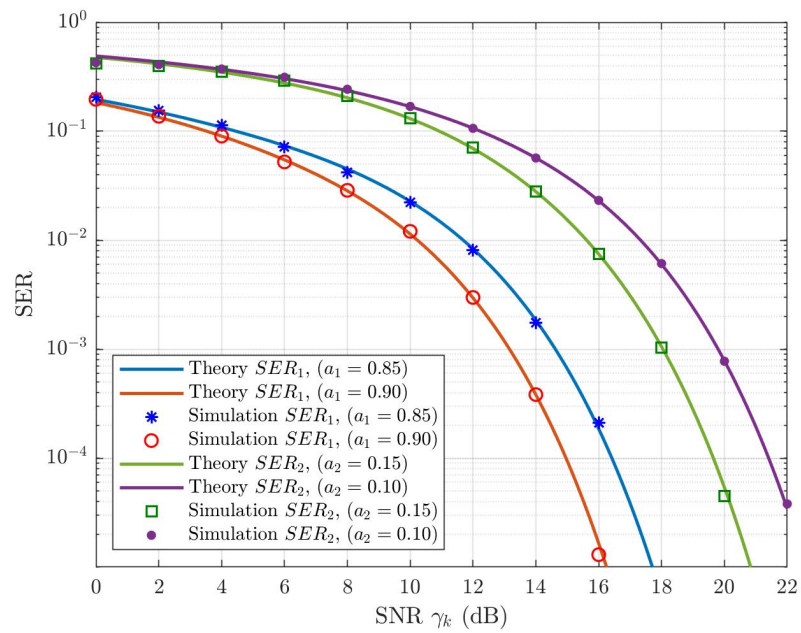


Figure 9. Theoretical and simulated SER of two users in a NOMA-based VLC system.

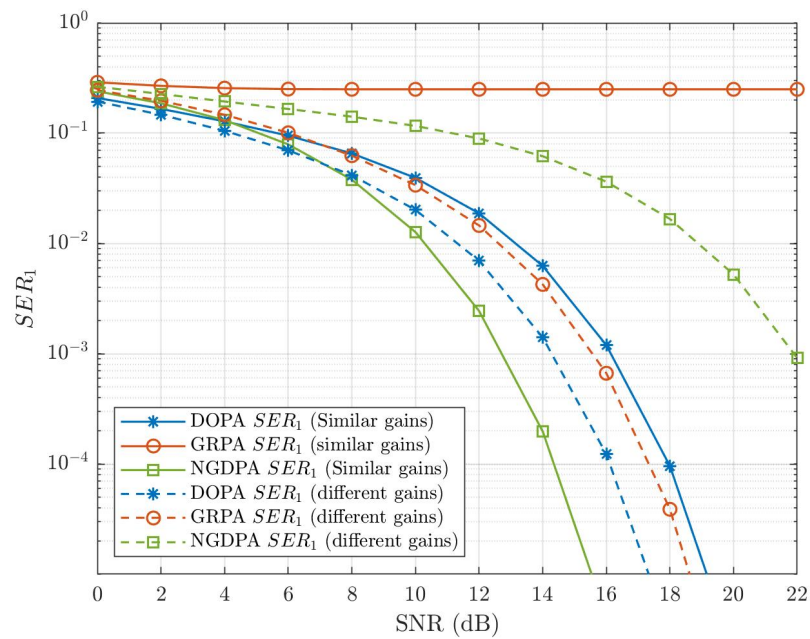


Figure 10. SER_1 using DOPA, GRPA and NGDPA. $\alpha = 2$ for similar channel gains and $\alpha = 1$ for different channel gains.

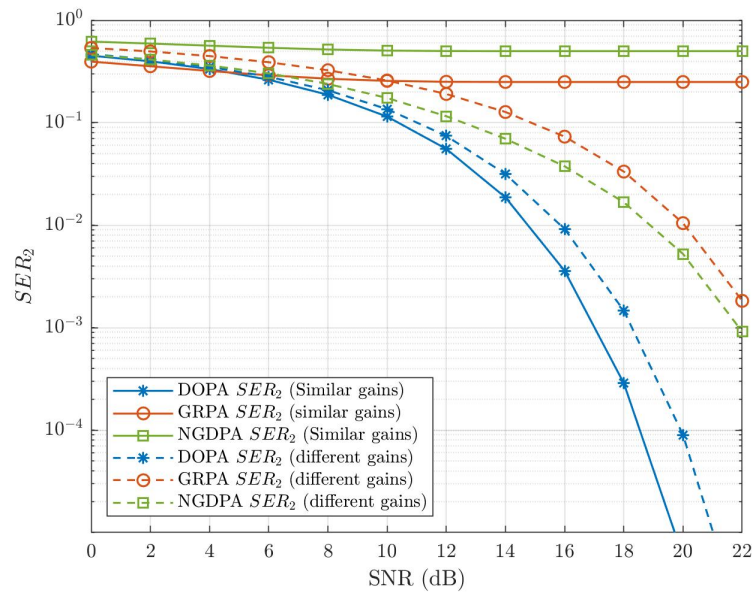


Figure 11. SER_2 using DOPA, GRPA and NGDPA. $\alpha = 2$ for similar channel gains and $\alpha = 1$ for different channel gains.

Figure 12 shows how the SER of u_1 varies with α at different SNR values. Without loss of generality, the horizontal distances of the users are assumed to be $r_1 = r_2 = 1.5$ m (i.e., both users are on the circumference of a circle with a radius r of 1.5). From Figure 12, as α increases, SER_1 is decreased. This is because of that the higher the value of α , the higher the power allocated to u_1 and the lower the power allocated to u_2 , which means the multi-user interference is decreased. Therefore, SER_1 is also decreased. Moreover, SER_1 decreases as the SNR increases and the value of α that gives a specific SER_1 is also decreased. For example, when the SNR is 15 dB, the value of α is about 3.2 to achieve SER_1 of 10^{-4} , whereas the value of α is about 2 when the SNR is 18 dB to achieve the same SER_1 .

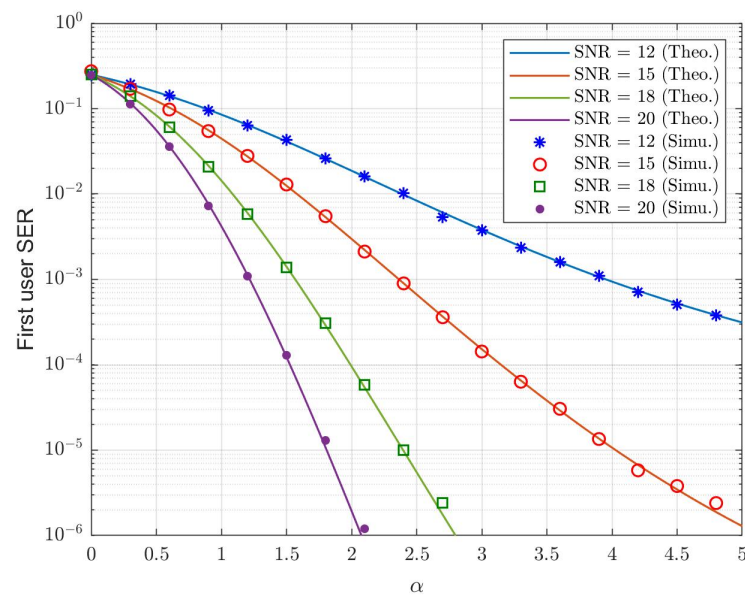


Figure 12. SER_1 with α at different SNR values.

Figure 13, shows how the SER of u_2 varies with α at different SNR values. As the value of α increases, the SER_2 starts to decrease until it reaches a minimum value and then

increases again. When the SNR value increases, the SER_2 decreases faster and reaches a lower minimum value and increases faster again. The SER_2 initially decreases because the multi-user interference is reduced when α increases. Hence, u_2 can more accurately cancel the interference from u_1 during the SIC process. However, as α increases, the power allocated to u_2 decreases. Thus, the AWGN added to the signal of u_2 increases. Therefore, the SER_2 finally increases again. From Figure 13, we can also notice that the optimal value of α that gives the minimum value of the SER_2 is roughly the same regardless of the SNR value.

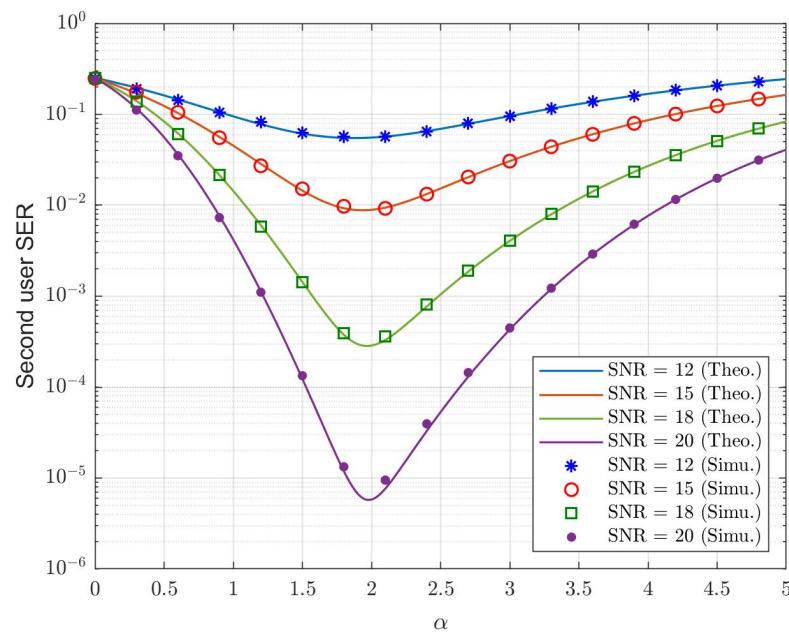


Figure 13. SER_2 with α at different SNR values.

7.2. Achievable Rate

The sum rate is an important metric to show the performance of the VLC network. In Figure 14, the achievable sum rate of the proposed DOPA against different values of the SNR ranging from 10 dB to 20 dB is depicted. The aim of this figure is showing the effectiveness of formulating the optimization problem considering the SER of the second user in comparison with benchmarking schemes, i.e., GRPA and NGDPA. It is shown that the DOPA scheme provides a greater sum rate, which is increased considerably as the value of the SNR increases, due to allocating the power among the users of each group, i.e., the first and second users, considering the minimization of the interference between them. It is worth mentioning that the interference among groups is avoided by implementing BIA. On the other hand, the GRPA scheme achieves a poor sum rate compared with the DOPA, due to the fact that the power is allocated equally between the first and second users, and therefore, the users of each group generate severe interference between them resulting in the degradation of the sum rate. The worst scenario is shown by the NGDPA scheme, it is easy to see that the sum rate is slightly increased as the SNR increases. Where for the highest value of the SNR, which equals to 20 dB, a low sum-rate around 2.5 [bits/s/Hz] is achieved. This is because, the NGDPA allocates a zero power level for the second user belonging to each group contradicting the concept of NOMA, i.e., the second users in the network achieve zero rates. As a result, allocating the power based on the objective function of the proposed DOPA scheme makes NOMA more suitable for the VLC network achieving high rates.

The achievable user rate of the proposed DOPA scheme is derived after considering the approach used for inter-group interference management in Section 3 power allocation approach in Section 6. Note that, after the formation of multiple groups, the bandwidth

is divided according to the transmission block of BIA aiming to dedicate resources for each group, while inter-group interference is aligned into less dimensions than the useful information (please see the motivational example and general case in Section 3). It is worth mentioning that inter-group interference in the proposed scheme is aligned at the cost of increasing the noise that results from interference cancellation. In other words, a given user in a certain group must cancel the signals intended to other groups. Following this methodology, the overall noise in our work is given by inter-group interference cancellation noise, background noise and successive interference cancellation (SIC) noise. Subsequently, the useful signal intended to each user belonging to a certain group is determined after performing power allocation according to our approach reported in detail in Section 6. Finally, Shannon capacity is used to calculate the data rate received by each user. Note that the user rates of all the counterparts schemes considered in the paper are calculated following the same fashion, while they differ in power allocation between the weak and strong users.

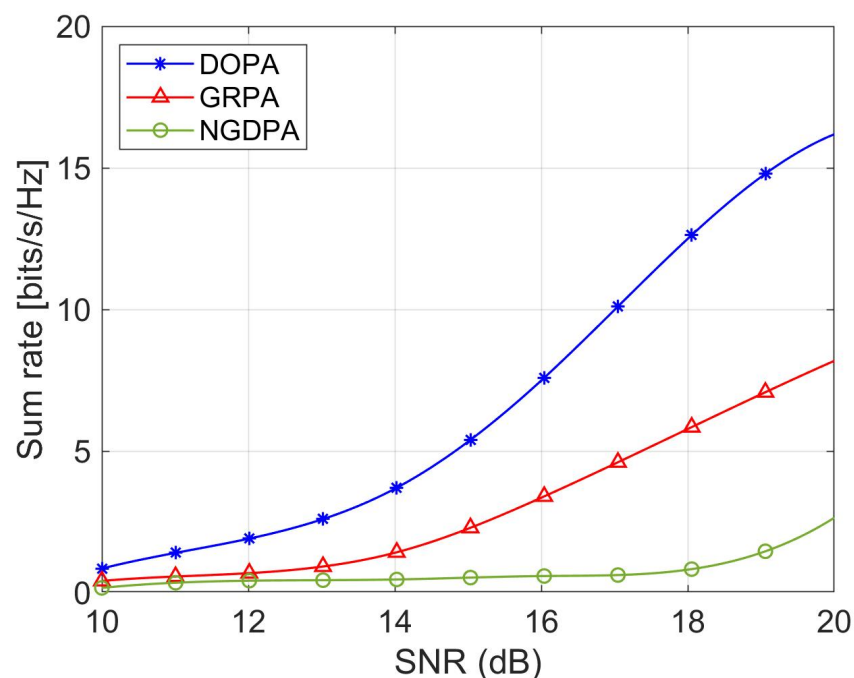


Figure 14. The sum rate of the proposed DOPA against the SNR for $\alpha = 1$ and $K = 10$ users, in comparison with the benchmark GRPA and NGDPA schemes.

7.3. Transmitting an Image

To further show the impact of the multi-user interference on the system performance, we assume that an image is sent for each user simultaneously. These images have the same size and there is no data or channel encryption. Furthermore, it is assumed that the users have similar channel gains and have SNR = 16 dB. The impact of the multi-user interference on the received images is shown in Figure 15. The top row images are for u_1 and the second-row images are for u_2 . The transmitted images for u_1 and u_2 are shown in Figure 15a. The received images are shown in different cases. In Figure 15b, the received images are shown when GRPA is used. In this case, the multi-user interference can be clearly observed in both images, which means that the interference from u_2 is too high to be ignored by u_1 and the interference from u_1 can also be observed in u_2 image because of the unsuccessful SIC. The high multi-user interference, in this case, is due to the equal powers allocated to the users by GRPA.

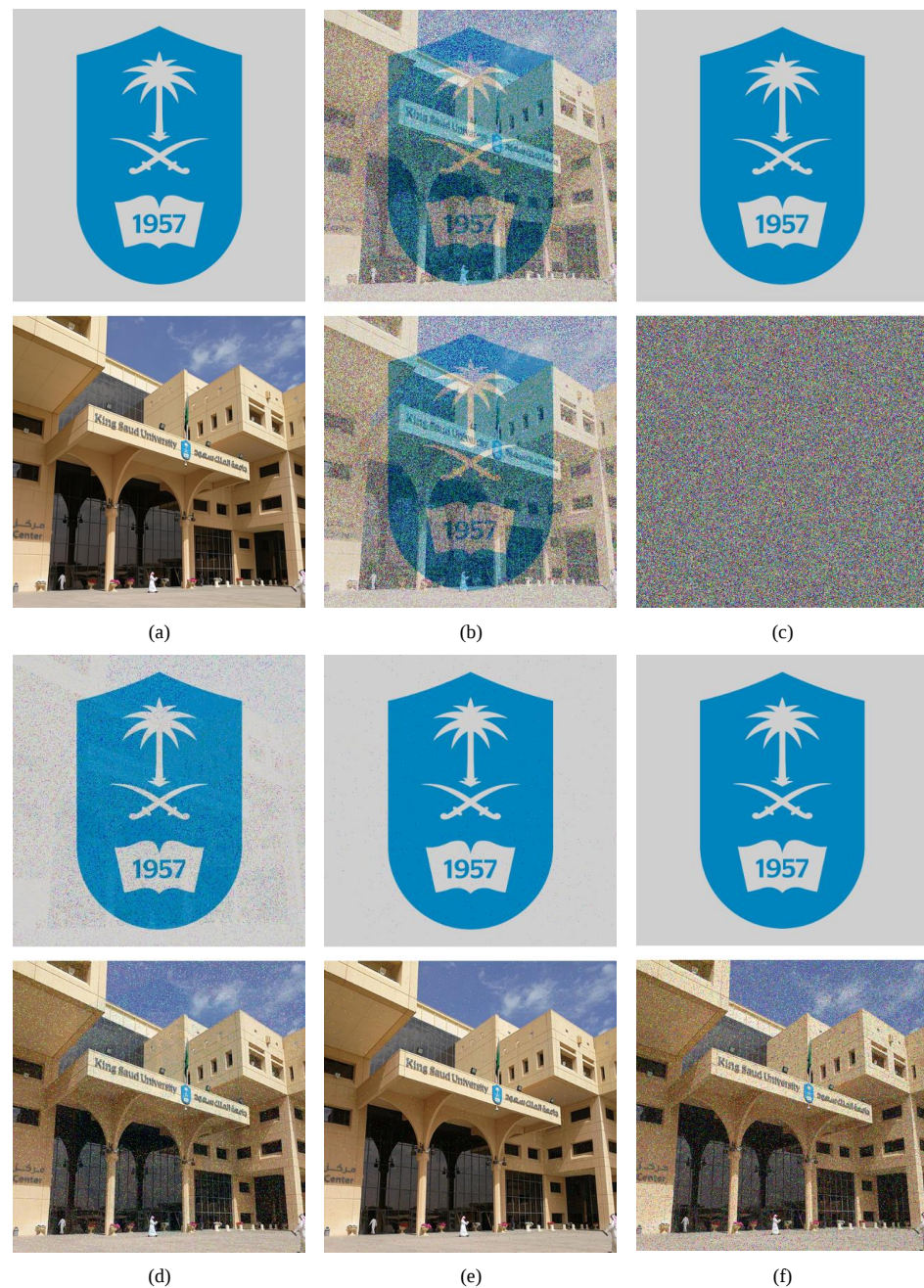


Figure 15. The impact of multi-user interference on the received images. The top row images are for u_1 and the second-row images are for u_2 . (a) Transmitted images. (b–f) Received images with: (b) GRPA, (c) NGDPA, (d) DOPA ($\alpha = 1$), (e) DOPA ($\alpha = 2$), and (f) DOPA ($\alpha = 4$).

In Figure 15c, the received images are shown when NGDPA is used. In this case, the multi-user interference has been totally eliminated from u_1 image; however, u_2 image has become just noise since NGDPA allocates zero power to the second user when the users' channel gains are equal. Figure 15d–f show the received images when the proposed DOPA is used with $\alpha = 1$, $\alpha = 2$, and $\alpha = 4$, respectively. When $\alpha = 1$, the multi-user interference has been reduced compared with GRPA case because DOPA allocates different powers to the users even if they have similar channel gains. When $\alpha = 2$, which is the optimal value in this case as shown in Figure 13, the multi-user interference has been significantly reduced in both images. When $\alpha = 4$, the interference in u_1 image has been reduced further, however, the effect of the AWGN on u_2 image has been increased, which is expected because the power allocated to u_2 decreases as α is increased.

8. Conclusions

In this paper, the effect of multi-user interference on the performance of a downlink NOMA-based VLC system has been presented. A closed-form expression for the user symbol error rate has been derived and verified by computer simulation. A low-complexity power allocation (PA) scheme has been proposed to reduce the inevitable multi-user interference. The proposed PA method is a decoding-order-based PA (DOPA) scheme in which the powers are allocated to the users based on their indices in the decoding order and their channel gains. It has been demonstrated that DOPA is able to reduce the interference in NOMA-based VLC systems, and achieve better sum rate compared to GRPA and NGDPA schemes.

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