

Supplementary Material: Suppression of Nonlinear Optical Effects in DWDM-PON by Frequency Modulation Non-Coherent Detection

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S1. Theoretical Model for the DFB laser

The DFB laser is simulated by a traveling wave model (TWM) [16]:

$$\frac{dN(z,t)}{dt} = I(t)/(eV) - N(z,t)/\tau_c - v_g P_s(z,t)g(z,t)/[1 + \varepsilon P_s(z,t)], \quad (S1a)$$

$$\left(\frac{1}{v_g} \frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right) F(z,t) \quad (S1b)$$

$$= \left\{ -j\delta + \frac{1}{2} \left[\frac{\Gamma g(z,t)}{1 + \varepsilon P_s(z,t)} - \alpha \right] \right\} F(z,t) + j\kappa R(z,t) + \tilde{s}^f(z,t),$$

$$\left(\frac{1}{v_g} \frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right) R(z,t) \quad (S1c)$$

$$= \left\{ -j\delta + \frac{1}{2} \left[\frac{\Gamma g(z,t)}{1 + \varepsilon P_s(z,t)} - \alpha \right] \right\} R(z,t) + j\kappa F(z,t) + \tilde{s}^r(z,t),$$

where $N(z,t)$ is the carrier density, $I(t)$ the injected current, e the electron charge, V the active region volume, τ_c the carrier lifetime, $v_g = c/n_g$ the group velocity, c the speed of light, n_g the group index, $P_s(z,t) = n_{eff}/(2hv_0) \sqrt{\varepsilon_0/\mu_0} \Gamma/(dwv_g) \cdot [|F(z,t)|^2 + |R(z,t)|^2]$ the photon density distribution, $n_{eff} = n_{eff}^0 - \lambda_0/(4\pi) \alpha_{LEF} \cdot \Gamma g(z,t)$ effective index, n_{eff}^0 effective index without injection, λ_0 the peak gain wavelength, α_{LEF} linewidth enhancement factor, Γ the optical confinement factor, $g(z,t) = a \ln[N(z,t)/N_0]$ the material optical gain, a the material gain coefficient, N_0 the transparent carrier density, h Planck's constant, v_0 the optical frequency corresponding to λ_0 , ε_0 the permittivity of a vacuum, μ_0 the permeability of a vacuum, d thickness of active region, w width of active region, $F(z,t)$ the slowly varying envelopes of the forward propagating fields, $R(z,t)$ the slowly varying envelopes of the backward propagating fields, ε nonlinear gain suppression coefficient, j the imaginary unit, $\delta = [2\pi n_{eff}^0/\lambda_0 - 1/2 \alpha_{LEF} \Gamma g(z,t) - \pi/\Lambda]$ the phase detuning factor from the Bragg wavelength, Λ Bragg grating period, α the optical modal loss, and κ grating coupling coefficient.

The magnitude of the spontaneous emission noise fields $\tilde{s}^f(z,t,\lambda_i)$ and $\tilde{s}^r(z,t,\lambda_i)$ are approximated as Gaussian random processes with a zero mean and satisfy following autocorrelation function [1]:

$$\langle |\tilde{s}^{f,r}(z,t)| |\tilde{s}^{f,r}(z',t')| \rangle = 2 \sqrt{\frac{\mu_0 \Gamma g_{sp} h v_0}{\varepsilon_0 n_{eff}}} \delta(z - z') \delta(t - t'), \quad (S2)$$

where γ indicates the spontaneous coupling factor, g_{sp} the spontaneous emission gain, and $\delta(\cdot)$ Dirac's delta function. Again, the phase of the spontaneous emission noise fields is assumed to be uniformly distributed between $0-2\pi$.

The finite bandwidth of the gain profile is modeled by an infinite impulse response (IIR) filter approach [32,33]:

$$|H(\omega)|^2 = \{(1 - \eta)^2 / [1 + \eta^2 - 2\eta \cos(\omega \Delta t)]\}, \quad (S3)$$

where η indicates the filter coefficient that controls the filter bandwidth and Δt the time marching step in simulation.

S2. Theoretical Model for the SOA

The numerical model that we have adopted to describe the SOA is given as [34–36]:

$$\frac{dN(z, t)}{dt} = \frac{I(t)}{eV} - \frac{N(z, t)}{\tau_c} - v_g \sum_{i=-M}^M P_s(z, t, \lambda_i) g(z, t, \lambda_i) / [1 + \varepsilon P_{tot}(z, t)], \quad (S4a)$$

$$\left(\frac{1}{v_g} \frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right) F(z, t, \lambda_i) \quad (S4b)$$

$$= \left\{ j \left[\frac{1}{2} \alpha_{LEF} \Gamma g(z, t, \lambda_i) \right] + \frac{1}{2} \left[\frac{\Gamma g(z, t, \lambda_i)}{1 + \varepsilon P_{tot}(z, t)} - \alpha \right] \right\} F(z, t, \lambda_i) + \tilde{s}^f(z, t, \lambda_i),$$

$$\left(\frac{1}{v_g} \frac{\partial}{\partial t} - \frac{\partial}{\partial z} \right) R(z, t, \lambda_i) \quad (S4c)$$

$$= \left\{ j \left[\frac{1}{2} \alpha_{LEF} \Gamma g(z, t, \lambda_i) \right] + \frac{1}{2} \left[\frac{\Gamma g(z, t, \lambda_i)}{1 + \varepsilon P_{tot}(z, t)} - \alpha \right] \right\} R(z, t, \lambda_i) + \tilde{s}^r(z, t, \lambda_i),$$

where $N(z, t)$ is the carrier density, $I(t)$ the injected current, e the electron charge, V the active region volume, τ_c the carrier lifetime, $v_g = c/n_g$ the group velocity, c the speed of light, n_g the group index, $P_s(z, t, \lambda_i) = n_{eff} / (2h\nu_i) \sqrt{\varepsilon_0 / \mu_0} \Gamma / (dwv_g) \cdot [|F(z, t, \lambda_i)|^2 + |R(z, t, \lambda_i)|^2]$ the photon density distribution of the i^{th} wavelength channel, λ_i is the wavelength of the i^{th} ($i = 0, \mp 1, \mp 2, \dots, \mp M$) channel in the sliced spectrum, $n_{eff} = n_{eff}^0 - \lambda_0 / (4\pi) \alpha_{LEF} \Gamma g(z, t, \lambda_i)$ effective index, n_{eff}^0 effective index without injection, λ_0 the peak gain wavelength, α_{LEF} linewidth enhancement factor, Γ the optical confinement factor, $g(z, t, \lambda_i) = a \ln[N(z, t) / N_0] [1 - 0.5(\lambda_i - \lambda_0 / \Delta\lambda_g)^2]$ the material optical gain, a the material gain coefficient, N_0 the transparent carrier density, $\Delta\lambda_g$ the gain profile width, h Planck's constant, ν_i the optical frequency corresponding to λ_i , ε_0 the permittivity of a vacuum, μ_0 the permeability of a vacuum, d thickness of active region, w width of active region, $F(z, t, \lambda_i)$ the slowly varying envelopes of the forward propagating fields, $R(z, t, \lambda_i)$ the slowly varying envelopes of the backward propagating fields, ε nonlinear gain suppression coefficient, $P_{tot}(z, t) = \sum_{i=-M}^M P_s(z, t, \lambda_i)$ the total photon density distribution in all wavelength channel, j the imaginary unit, and α the optical modal loss.

The magnitude of the spontaneous emission noise fields $\tilde{s}^f(z, t)$ and $\tilde{s}^r(z, t)$ are treated in a similar way as equation (S2) in section 1:

$$\langle |\tilde{s}^{f,r}(z, t)| |\tilde{s}^{f,r}(z', t')| \rangle = 2 \sqrt{\frac{\mu_0 \Gamma \gamma g_{sp} h \nu_0}{\varepsilon_0 n_{eff}}} \delta(z - z') \delta(t - t'), \quad (S5)$$

where γ the spontaneous coupling factor, $R_{sp}(z, t, \lambda_i)$ the spontaneous emission rate, d_z the length of a subsection introduced by the spatial discretization of the active region along the wave propagation direction, and $\delta(\cdot)$ Dirac's delta function. The phase of the spontaneous emission noise fields is assumed to be uniformly distributed between $0-2\pi$.