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SLIDING MODE SPEED CONTROL FOR DC DRIVE SYSTEMS

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Abstract-In this study, the Sliding Mode Control theory of the Variable Structure System has been applied to the speed control of a dc motor. The dynamic performance of the sliding mode speed control system has been studied against system parameter variations and external load disturbance and the simulation results are given. The application of the sliding mode control theory to controller design for DC drive control system shows a robust system performance.

Keywords: Variable structure, sliding mode, DC motor

1. INTRODUCTION

The control of electrical drives plays an important role in the field of mechatronics, which merges mechanics, computer science, and electronics and control theory. From the control point of view, due to their simplicity dc motors have been the traditional choice where high dynamic performance is required. Hence high performance speed and position control drives predominantly used the dc motor. The control performance of the dc drive systems can be influenced by the plant uncertainties due to parameter variations, external disturbance, and unmodelled dynamics. Traditional linear controller such as Proportional-Integral (PI) controller, are highly sensitive to parameters variation. Also, the gains of the PI controller has to be selected carefully in order to obtain the desired response.

The Variable Structure Control (VSC) approach using the sliding mode has been the focus of many researches for the control of drive systems [1-3]. The method rests on the concept of changing the structure of the controller (Variable Structure System) in response to the changing state of the system in order to obtain the desired or optimal response. The sliding mode control theory of the VSC system provides a method to design a system in such a way that the controlled system is to be insensitive to parameter variations and external load disturbances [4, 5]. The approach is realized by the use of a high speed switching control law which forces the trajectory of the system to move to a predetermined path in the state variable space (called Sliding or Switching Surface and it is a line in case of two dimensions) and to stay in that surface thereafter. Before the system reaches the switching surface, there is a control directed towards the switching surface which is called reaching mode. The regime of a control system in the sliding surface is called Sliding Mode. In sliding mode a system's response remains insensitive to certain parameters variations and unknown disturbances. Figure 1 shows the modes of VSC systems for single input plants for which the switching surface is a line.

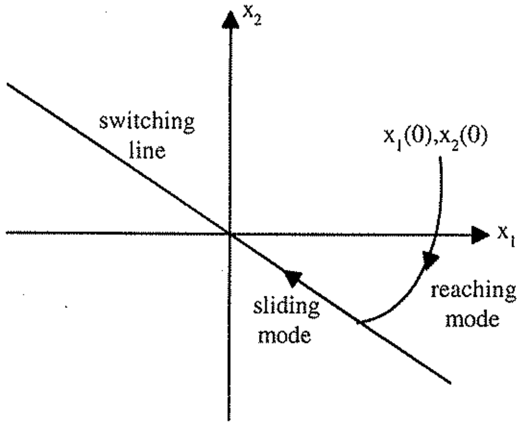

Figure 1. The Modes of Variable Structure Control Systems

In the Sliding mode the dynamics of the system is determined by the parameters of sliding surface and become insensitive to uncertainities and external disturbances or the change in system parameters These advantages of sliding mode control have been employed in the position and speed control of drive systems.

2. SLIDING MODE CONTROL OVERVIEW

The Variable Structure System (VSS) theory has been applied to nonlinear systems. One of the main features of this method is that one only needs to drive the error to a switching surface, after which the system is in sliding mode and robust against modelling uncertainties and disturbances. A Sliding Mode Controller is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that map plant state to a control surface, and the switching among different functions is determined by plant state that is represented by a switching function.

Without loss of generality, consider the design of a sliding mode controller for the following second order system:

$$\ddot{x} = f(x, \dot{x}, t) + bu(t) \qquad (1)$$

Here it is assumed that $b>0$. $u(t)$ is the input to the system. The following is a possible choice of the structure of a sliding mode controller [6]

$$u = -k\,\mathrm{sgn}(S) + u_{eq} \qquad (2)$$

where u_{eq} is called equivalent control which is used when the system state is in the sliding mode [7]. In other words the control that makes the derivative of the switching function equal to zero is called equivalent control. k is a constant, representing the maximum controller output. S is called switching function because the control action switches depending on its sign on the two sides of the switching surface $S=0$. In VSC the goal is to keep the system motion on the manifold S which is defined as [6]

$$S = \dot{e} + \lambda e \qquad (3)$$

where $e=x-x_d$ and x_d is the desired state. λ is a constant. The definition of e here requires that k in (2) be positive. sgn(S) is a sign function, which is defined as

$$\operatorname{sgn}(S)=\begin{cases}-1 & if \quad S<0\\ 1 & if \quad S>0\end{cases} \tag{4}$$

Since the aim is to force the system states to the sliding surface, the control strategy adopted here will guarantee the system trajectory move toward and stay on the sliding surface $S=0$ from any initial condition if the following condition meets

$$S\dot{S}\le -\eta|S| \tag{5}$$

where η is a positive constant that guarantees the system trajectories hit the sliding surface in finite time [6].

Using a sign function often causes chattering in practice. One solution is to introduce a boundary layer around the switching surface [7]

$$u=-ksat(\frac{S}{\phi})+u_{eq} \tag{6}$$

where constant factor ϕ defines the thickness of the boundary layer. $sat(\frac{S}{\phi})$ is a saturation function that is defined as

$$sat(\frac{S}{\phi})=\begin{cases}\dfrac{S}{\phi} & if \quad \left|\dfrac{S}{\phi}\right|\le 1\\ \operatorname{sgn}(\dfrac{S}{\phi}) & if \quad \left|\dfrac{S}{\phi}\right|>1\end{cases} \tag{7}$$

This controller is actually a continuous approximation of the ideal relay control [6]. The consequence of this control scheme is that invariance property of sliding mode control is lost. The system robustness is a function of the width of the boundary layer.

A variation of the above controller structure is to use a hyperbolic tangent function instead of saturation function [8, 9]

$$u=k\tanh(\frac{S}{\phi})+u_{eq} \tag{8}$$

It is proven that if k is large enough, the sliding mode controllers given by equations (2), (6), and (8) are guaranteed to be asymptotically stable [7, 8]. For a 2-dimensional system, the controller structures are illustrated in Figure 2.

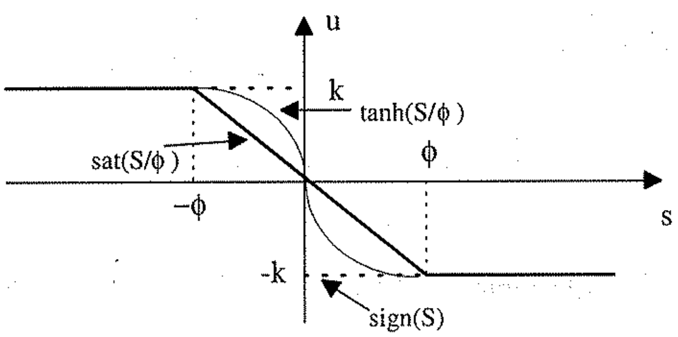

Figure 2. The Controller Structures for 2D Systems

2.1 The Controller Design

The model used in this paper assumes a constant motor field as shown in Figure 3 and the electrical time constant is much smaller than the mechanical time constant i.e. the armature current dynamics is much faster than the mechanical dynamics.

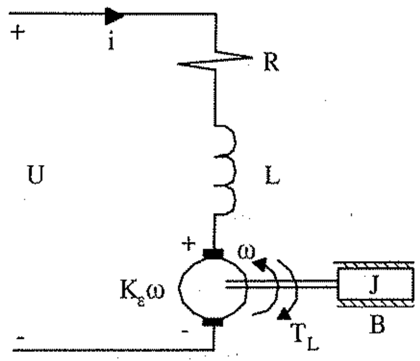

Figure 3. The DC Motor

From Figure 3, the equations describing the motor dynamics are

$$L\dot{i} = U - Ri - K_e\omega \tag{9}$$

$$J\dot{\omega} = -B\omega + K_t i - T_L \tag{10}$$

where

i= armature current	J= total inertia
ω=shaft speed	B= viscous friction
R= armature resistance	K_t= torque constant
L= armature inductance	K_e= back emf constant
U= terminal voltage	T_L= load torque

The block diagram of the speed control of dc motor is shown in Figure 4.

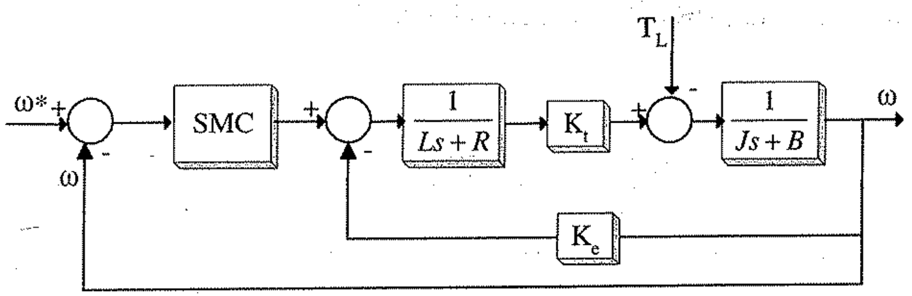

Figure 4. The Block Diagram of Sliding Mode Control of Dc Motor

If the error is chosen as the difference between the desired and the actual speeds as

$$x_1 = e = \omega^* - \omega \tag{11}$$

$$\dot{x}_1 = x_2 = \dot{e} = \dot{\omega}^* - \dot{\omega} \qquad \text{and} \tag{12}$$

$$\dot{x}_2 = \ddot{\omega}^* - \ddot{x}_1 \tag{13}$$

then the equation of motion with respect to this error becomes

$$\dot{x}_1 = x_2 \tag{14}$$

$$\dot{x}_2 = -a_1 x_1 - a_2 x_2 + f(t) - bU \tag{15}$$

where

$$a_1 = \frac{RJ + BL}{JL} \quad ; \quad a_2 = \frac{RB + K_e K_t}{JL} \quad ; \quad b = \frac{K_t}{JL}$$

and $f(t)$ is the time function depending on ω^*, T_L and their time derivatives and is given by

$$f(t) = \ddot{\omega}^* + a_1 \dot{\omega}^* + a_2 \omega^* - c\dot{T}_L - dT_L \tag{16}$$

where

$$c = \frac{1}{J} \quad ; \qquad d = \frac{R}{JL}$$

for discontinuous control the switching function and the associated controller are designed as

$$S = \lambda e + \dot{e} = \lambda x_1 + \dot{x}_1 \tag{17}$$

$$U = k\,\mathrm{sgn}(S) + U_{eq} \tag{18}$$

where λ and k are constants and λ determines the rate of convergence. The first term in equation (18) which is discontinuous, is used to reject disturbances and to suppress parametric uncertainties and the second term which is continuous, is used to control the overall behaviour of the system. According to these equations, the speed tracking error x_1 decays exponentially after sliding mode occurs in the manifold $S=0$. The system motion in sliding mode is independent of parameters a_1, a_2, b and disturbances in $d(t)$. To enforce the sliding mode, the control gain k should be selected such that

$$S\dot{S} < 0 \tag{19}$$

Imposing the condition in Equation (19) we get

$$\dot{S} = -a_1 x_1 - (a_2 - \lambda) x_2 + d(t) - bk\,\mathrm{sgn}(S) \tag{20}$$

and sliding mode exists if the inequality

$$k > \frac{1}{b} \left| -a_1 x_1 - (a_2 - \lambda) x_2 + d(t) \right| \tag{21}$$

holds.

After a finite time interval, the system state will reach the sliding line $S=0$. Thereafter, the system response depends only on the design parameter λ.
Since the reaching condition (equation (19)) is satisfied with the parameter k which is found in equation (21), the task is to find the ideal continuous control that yields the system response on the sliding surface. i.e. U_{eq}, which is the average of the switching

control that satisfies the reaching condition. The equivalent control should satisfy the condition

$$\dot{S} = 0 \tag{22}$$

That is, the state is on the sliding line and does not leave the line. Using equation (17) and assuming ω^* and T_L are constant, the time derivatives are all zero hence $\dot{S}$ can be found as

$$\dot{S} = \ddot{e} + \lambda \dot{e} \tag{23}$$

using equations (11-13)

$$\dot{S} = (\lambda - a_1)\dot{e} + a_2(\omega^* - e) - bU - dT_L \tag{24}$$

and

$$U_{eq} = U\big|_{\dot{S}=0} = \frac{(\lambda - a_1)\dot{e} + a_2(\omega^* - e) - dT_L}{b} \tag{25}$$

3. SIMULATION RESULTS

In this section the performance of the sliding mode controlled drive system given in Figure 4 is illustrated. In the simulations, the mechanical inertia and the damping constants are significantly varied to allow the transfer function of the mechanical side of the dc motor to be changed from that of the nominal values $J=5xJ_n$ and $B=5xB_n$. J_n and B_n represent the nominal parameter values. A 100 rad/s speed command is applied to the system at time $t=0$. The speed and the torque response of the sliding mode controlled dc motor with nominal system parameters and with $J=5xJ_n$ and $B=5xB_n$ and with external load $T_L=2Nm$ applied at $t=0.25s$, are shown in Figure 5 (a)-(b). It is evident from Figure 6 that the controlled speed can track the speed command quickly both with and without parameter variations. Hence it has a good performance in speed regulation and the robustness to disturbance is obtained using the sliding mode control system.

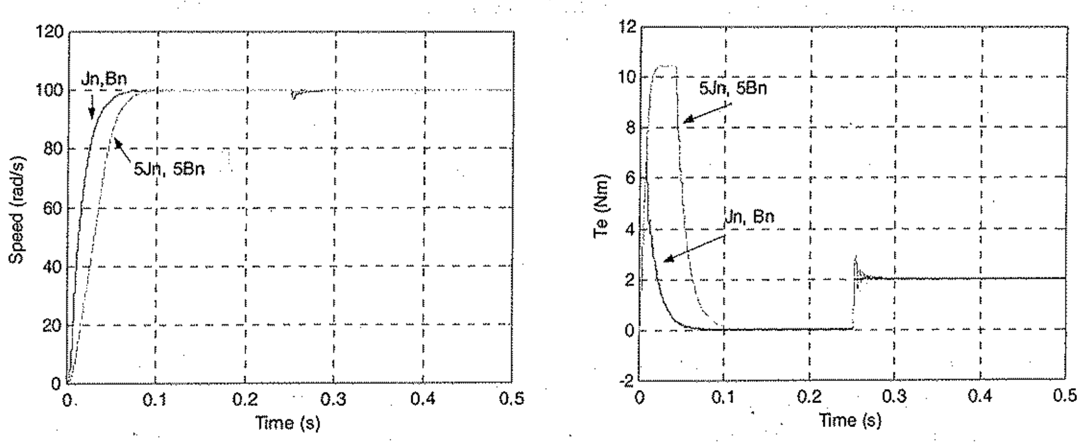

Figure 5. The speed and torque response with $T_L=2\ Nm$ applied at $t=0.25s$

The phase plane when *2Nm* load torque applied at *t=0.25s* with nominal system parameters are shown in Figure 6 (a). Since it is impossible for the control input to be switched at infinite rate, chattering occurs in sliding mode controlled systems. Figure 6-(b) shows the phase plane of the same system as in Figure 6 (a) but with an imperfect control switching to enlarge this chattering. The speed response and the control effort when speed reversal is used at time t=0.15s and 0.35s are shown in Figure 7.

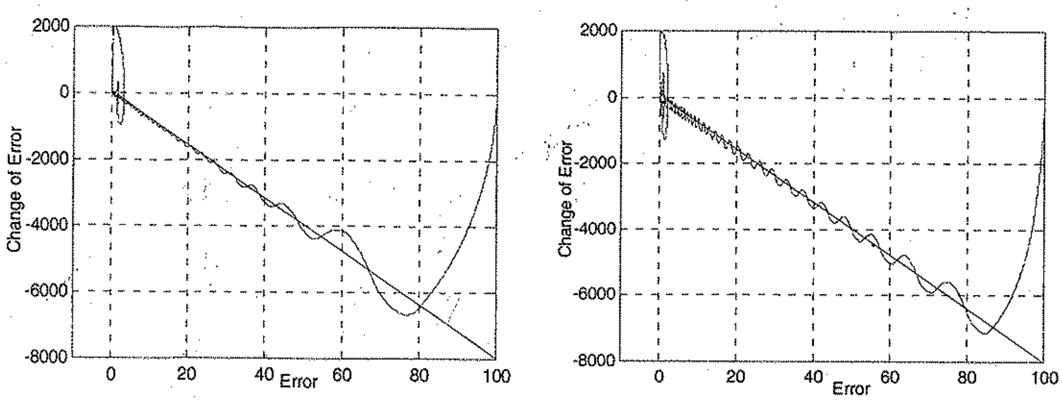

Figure 6. (a)The phase plane with *2Nm* load torque applied at *t=2.5s* (b) imperfect control switching, with nominal system parameters

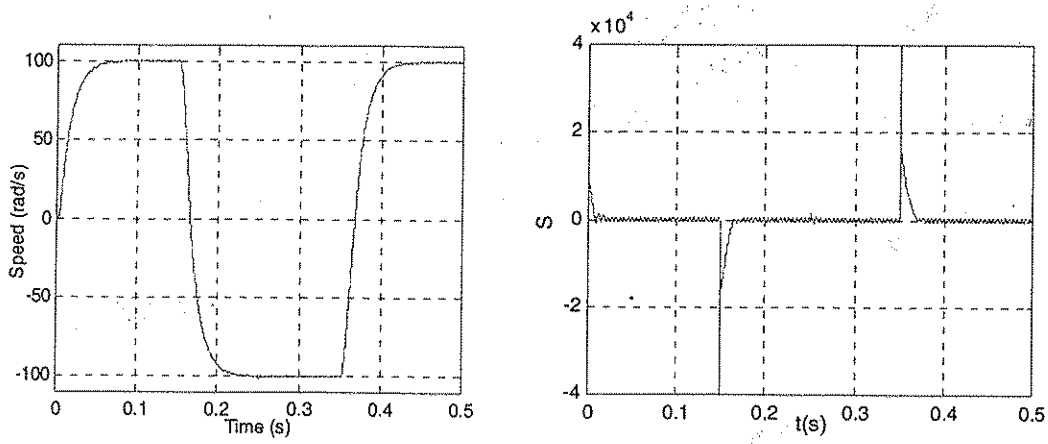

Figure 7. The speed response and the control effort with speed reversal

4. CONCLUSION

In this paper, the speed control of dc motor using sliding mode controller has been simulated. The speed control system is simulated using both nominal system parameters and five times increased system parameters. The Simulation results are compared to see the effect of the system parameter variations and external load disturbances on the speed control system. Simulation results from testing the speed control of dc motor drive show that the sliding mode controller provide high performance dynamic characteristics and are robust with regard to plant parameter variations and external load disturbance. The robust control performance of the speed control system in both command tracking and load regulation are obvious.

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Appendix

The dc motor parameters used in this study are as follows

$R=0.5\ \Omega$	$L=0.001\ H$
$J=0.001\ kg\ m^2$	$B=0.0001\ Nm\ s/rad$
$Ke=0.001V\ s/rad$	$Kt=4.2\ Nm/A$