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ASPECTS OF THE GURSON MODEL AND ITS APPLICATIONS

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Abstract- Ductile fracture in metals can involve the generation of considerable porosity caused by nucleation, growth and coalescence of microvoids. This process takes place on micro-level and can not describe by traditional constitutive laws such as von Mises theory. Hence, A. L. Gurson developed a theory which takes account of void growth and showed the role of hydrostatic stress in plastic yield and void growth. In this model the void volume fraction f (the portion of void in the material) is the single damage parameter; its evolution is defined by the incompressibility of the matrix material. (For Lameitre's model the damage variable D is relevant.) To model the material damage by using the Gurson damage approach a series of single elements including different types of loading are used. In the single element cases the results of the Gurson model and von Mises are also compared. In calculations the MARC finite elements software is used to calculate stress, strains and f the void volume fraction.

Keywords: Ductile Damage, Gurson Model, Void Growth

1. INTRODUCTION

Ductile fracture is a common cause of failure in engineering structures. A damaged ductile material consists of two parts: matrix medium and damage, e.g. voids. Ductile frature in metals can involve the generation of considerable porosity caused by nucleation, growth, and coalescence of microvoids. This process takes place on micro-level and cannot be described by traditional constitutive laws such as von Mises theory. Hence, A. L. Gurson introduced a model for ductile fracture which includes the influence of hydrostatic stress on the evolution of plasticity condition and combines plasticity with damage by introducing porosity of a metal [1]. This model is called the Gurson model. It assumes cylindrical and spherical voids surrounded by homogenous, incompressible von Mises material (matrix). The Gurson model takes only void growth into account. This model has no ability to predict void nucleation and coalescence. Nucleation and coalescence of microvoids was incorporated later by A. Needleman and V. Tvergaard [2,3,4,5, 6].

The process of ductile fracture consists of three sequential stages (see Figue 1):

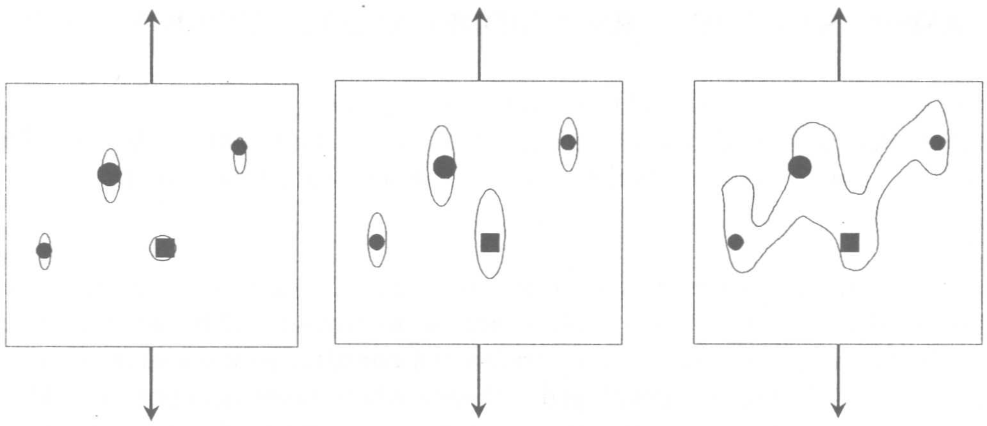

Figure 1. Mechanism of ductile failure by voids

2. GURSON MODEL

Gurson developed a theory which takes account of void growth, and showed the role of hydrostatic stress in plastic yield and void growth. He employed simplified physical models for ductile porous materials (aggregates of voids and ductile matrix), idealizing the matrix material as rigid-perfectly plastic and obeying the von Mises yield criterion. In order to get an analytical expression for the yield function, Gurson considered – as an abstraction of a matrix with voids – cylindrical and spherical domains, respectively. Each of them contains a void of the same shape in its centre (Figure 2 and Figure 3).

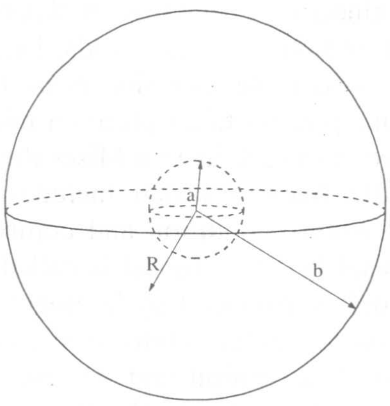

Figure 2. Spherical void in a matrix

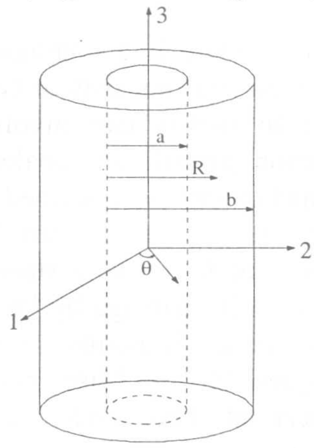

Figure 3. Long circular cylindrical void in a matrix

Gurson approximated a solid with a volume fraction f of voids by a homogenous body with a cylindrical or spherical cavitiy. An approximate rigid-plastic limit analysis of this situation was used to develop the yield condition. The Gurson model characterizes the porosity by a single-scale internal variable f the void volume fraction. This dilatant plastic yield criterion is known under the name of "*Gurson's criterion*". For cylindrical voids with parallel axis it holds:

$$\Phi = \left(\frac{\sigma_e}{\overline{\sigma}}\right)^2 + 2f\cosh\left(\frac{\sqrt{3}}{2}\frac{\sigma_h}{\overline{\sigma}}\right) - 1 - f^2 = 0 \tag{1}$$

and for spherical voids it was derived:

$$\Phi = \left(\frac{\sigma_e}{\overline{\sigma}}\right)^2 + 2f\cosh\left(\frac{1}{2}\frac{\sigma_h}{\overline{\sigma}}\right) - 1 - f^2 = 0 \tag{2}$$

where f is void volume fraction (void volume fraction serves here as a *damage* parameter.), $\overline{\sigma}$ is the actual yield stress of the matrix material, σ_e von Mises equivalent stress of the homogenized material and σ_h the trace of the Cauchy stress tensor (σ_{kk}). These are defined as follows:

$$\sigma_e = \sqrt{\frac{3}{2}S_{ij}S_{ij}} \tag{3}$$

where S_{ij}

$$S_{ij} = \sigma_{ij} - \sigma_h\delta_{ij} \tag{4}$$

The yield stress $\overline{\sigma}$ of the fully dense matrix material is a fuction of the equivalent plastic strain in the matrix $\overline{\varepsilon}$,

$$\overline{\sigma} = \overline{\sigma}(\overline{\varepsilon}) \tag{5}$$

3. CONSTITUTIVE EQUATIONS

Plastic part of the rate of the deformation $\dot{\varepsilon}^p_{ij}$ is derived from yield potential; the presence of pressure in the yield condition results in nondeviatoric plastic strain. Plastic flow is assumed to be normal to the yield surface with $\dot{\Lambda}$ as the non-negative plastic flow multiplier [7,8, 9, and 10]. The plastic part of the rate of deformation tensor, $\dot{\varepsilon}^p_{ij}$, is given by the flow rule

$$\dot{\varepsilon}^p_{ij} = \dot{\Lambda}\frac{\partial\Phi}{\partial\sigma_{ij}} \tag{6}$$

Plastic multiplier $\dot{\Lambda}\,(\geq 0)$ is a plastic flow proportionally factor and calculated using the consistency condition $\dot{\Phi} = 0$ in the case of plasticity. Evolution equations need to be specified for the internal variables f and $\overline{\sigma}$. The portion of voids in the material is expressed by void volume fraction. In general, the evolution of void volume fraction results from both nucleation of new voids and from growth of existing voids. The growth rate of existing voids is determined by requiring the matrix material to be plastically incompressible. The growth rate is assumed proportional to hydrostatic part the stress tensor:

$$\dot{f} = (1-f)\dot{\varepsilon}^p_{kk} \tag{7}$$

rearranging the above equation yields

$$\dot{f} = (1-f)\frac{\partial\Phi}{\partial\sigma_{kk}}\dot{\Lambda} = (1-f)\frac{\partial\Phi}{\partial\sigma_h}\dot{\Lambda} \tag{8}$$

The plastic work rate for the porous solid is set equal to the matrix plastic work rate. Accordingly,

$$\sigma_{ij}\dot{\varepsilon}^{p}_{ij} = (1-f)\bar{\sigma}\dot{\bar{\varepsilon}} \tag{9}$$

From Eq. 9, the plastic flow proportionality factor, $\dot{\Lambda}$, in the flow rule Eq. 6 is determined to be,

$$\dot{\Lambda} = \frac{(1-f)\bar{\sigma}\dot{\bar{\varepsilon}}}{\sigma_{kl}\dfrac{\partial \Phi}{\partial \sigma_{kl}}} \tag{10}$$

Here, $\dot{\bar{\varepsilon}}$ is the matrix von Mises equivalent strain rate, which is determined from the matrix strain hardening relation. For an elastic-plastic solid, we write

$$\varepsilon_{ij} = \varepsilon^{e}_{ij} + \varepsilon^{p}_{ij} \tag{11}$$

In circumstances where the elastic strains remain small, although the plastic strains may be large, it is convenient to use a hypoelastic approximation for ε^{e}_{ij},

$$\hat{\sigma}_{ij} = \frac{2E}{2(1+\nu)}\varepsilon^{e}_{ij} + \frac{E\nu}{(1+\nu)(1-2\nu)}\varepsilon^{e}_{kk} = \frac{2G}{(1-2\nu)}\left[(1-2\nu)\varepsilon^{e}_{ij} + \nu\varepsilon^{e}_{kk}\delta_{ij}\right] = D^{e}_{ijkl}\varepsilon^{e}_{kl} \tag{12}$$

where E is Young's modulus, ν is Poisson's ratio, δ_{ij} is Kronecker's delta. For plastic loading,

$$\hat{\sigma}_{ij} = D^{ep}_{ijkl}\varepsilon_{kl} \tag{13}$$

4. SINGLE ELEMENT CASES

The material damage has been assessed by Gurson's criterion by using a series of single elements. In calculations the MARC finite element software is used to calculate stress, strains and f the void volume fraction. A material element or "cell" of size LxL is taken out of the body. This material element is analyzed in plane strain. Three cases, illustrated in Figure 4-5-6, are considered here: (i) uniaxial tension, (ii) biaxial tension (volumetric expansion) (iii) constrained axial tension.

The uniaxial strain stress response of the elastic-plastic material that surrounds the void is described by

$$\frac{\bar{\sigma}}{\sigma_o} = \left(\frac{\bar{\sigma}}{\sigma_o} + \frac{3G}{\sigma_o}\bar{\varepsilon}\right)^{N}, \quad \bar{\sigma} > \sigma_o \tag{14}$$

with $E/\sigma_o = 300$, $\nu = 0.3$ and $N = 0.1$ and G is shear modulus. The Gurson model is invoked using the strain-controlled nucleation model. Void volume fraction is described by $f_N = 0.1$, $\varepsilon_N = 0.3$ and $S = 0.04$. (f_N, the void volume fraction of void forming particles; ε_N, the normal distribution around the mean value and S the standard deviation). For the biaxial tension example, the voids were assumed to have already nucleated at the beginning of the deformation with initial void volume fraction, $f_o = 0.04$. In the other examples, the initial porosity was set to zero.

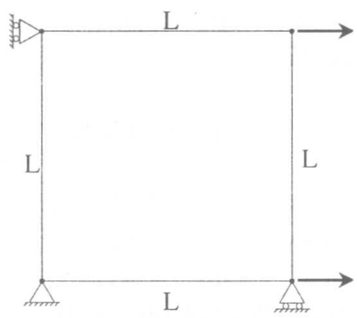

Figure 4 Uniaxial tension

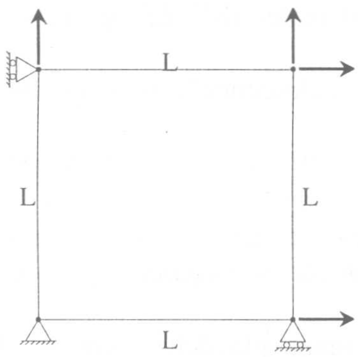

Figure 5 Biaxial tension

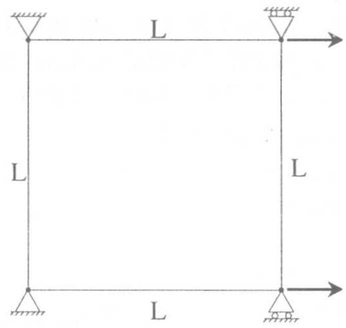

Figure 6 Constrained tension

5. CONCLUSION

Figure 7 plots σ_e/σ_o (equivalent stress/yield stress) versus ε^p_{eq} (equivalent plastic strain). The results of the Gurson damage model and von Mises approach are compared. Uniaxial tension results in Figure 8 shows an initial increase in porosity through void nucleation centered around a strain of 0.3.

Figure 9 plots hydrostatic stress as a function of equivalent plastic strain for biaxial tension case. Constituve softening behavior is observed as the hydrostatic stress drops off sharply with increased porosity. The effect of void volume fraction on equivalent plastic strain is showed in Figure 10.

Figure 11 plots hydrostatic stress as a function of equivalent plastic strain for constrained axial extension. As can be seen in Figure 11, the stress rises sharply during the initial stages of deformation for which the porosity is very low and drops suddenly as the strains to cause void nucleation are reached. The initial material response is very close to that of a von Mises material since f is small which allows the hydrostatic stress and triaxiality ($\sigma_h/\bar{\sigma}$) to become large. As plastic strain begins to accumulate, void nucleation becomes significant at uniaxial strains of about 0.022. At this point, the high triaxility leads to a burst of void growth and a rapid contraction of the yield surface as indicated by the sudden drop in stress. The effect of void volume fraction on equivalent plastic strain is showed in Figure 12.

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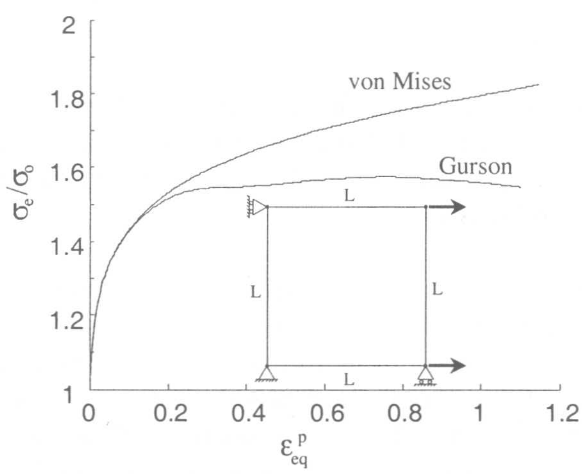
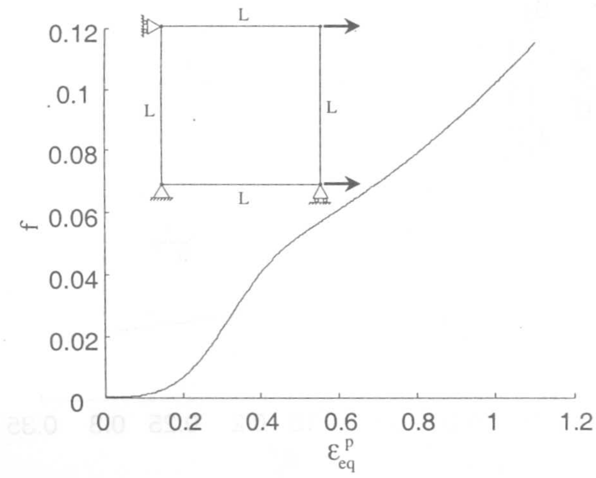


Figure 7 Single element uniaxial tension case (plane strain)



Şekil 8. Void volume fraction versus equivalent plastic

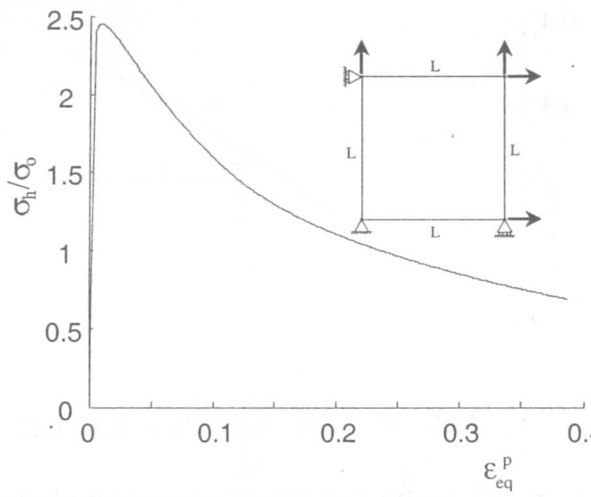


Figure 9 Hydrostatic stress versus equivalent plastic strain ($f_o = 0.04$)

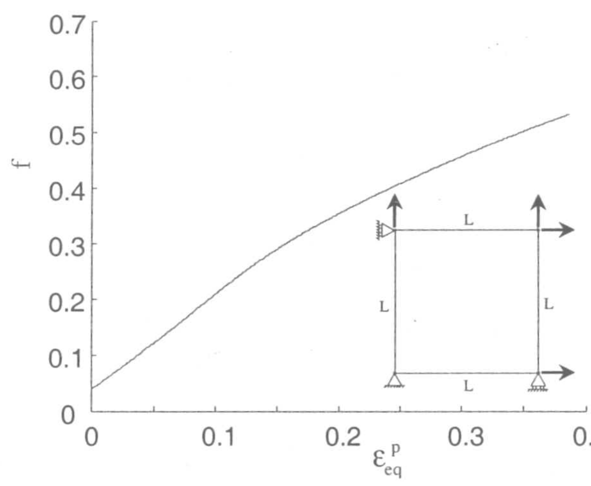
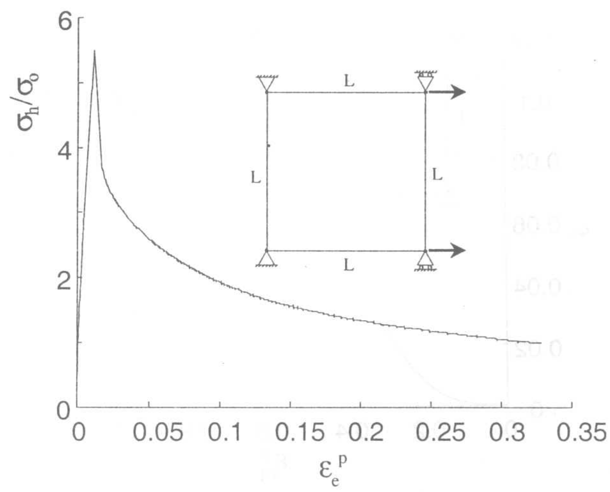


Figure 10 Void volume fraction versus equivalent plastic strain ($f_o = 0.04$)



Şekil 11 Hydrostatic stress versus equivalent plastic strain

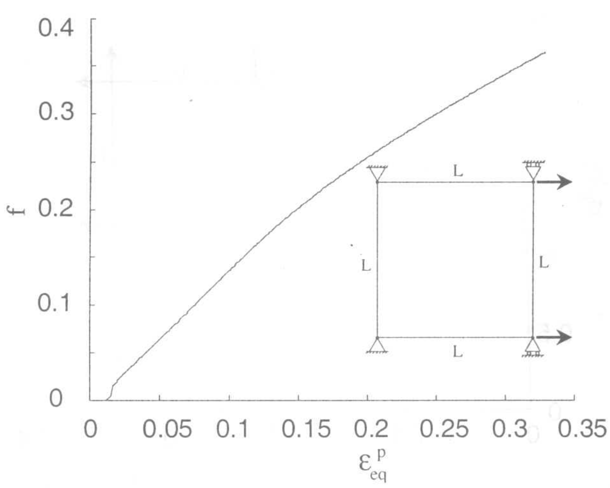


Figure 12 void volume fraction versus equivalent plastic strain