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A MATHEMATICAL MODEL FOR THE SEPARATION OF VISCOUS LIQUID FLOW BY MAGNETIC FIELD

Hikmet Gafaroglu Hassanov

Azerbaijan State Oil Research and Project Institute (AzNSETLI),
Aga - Neymatullah str.39, 370033, Baku, Azerbaijan
hhassanov@yahoo.com

Abstract - Inverse problem of the two - phase liquid flow separation on components under magnetic field action is firstly resolved at given law of the moving boundary alteration. Automodel solution is established, which may be approximated till various areas of dimensionless variable θ. It is found the relationship between the phase radius r_0 and magnetic properties of the liquid involved. The dependence of velocity function on rheological parameters of the liquid is estimated. The changes of the liquid flow separation due to effect of the alternating magnetic field are shown. It is established, what liquids should be combined in total flow in order to acquire the best effect of separation.
Keywords - Magnetic field effect, two – phase liquid, moving boundary , magnetic susceptibility

1. INTRODUCTION

Separation of multi-component liquids (mixtures, solutions, emulsions) by means of physical field is a problem of great interest from both practical and theoretical points of view. Opportunity to separate the moving liquids for their phases allows us to facilitate the most of technological processes (see, for example [1]) and decrease expenses in oil and chemical industry, medicine, etc. Recently, universal external agent able to separate any multiphase liquid is absent. Each of them has some advantages and lacks. For example, magnetic field is sufficiently effective if the separated phases of the liquid have essential magnetic properties (susceptibility and / or permeability), as it was shown in [2]. High - frequency electro - magnetic fields [3,4] and direct electric field [5,6] may also be considered as the physical agents able to separate two - component liquids. Nevertheless, the analysis of existing experimental works gives an opportunity to conclude that the magnetic field (m.f.) is one of the most appropriate for practical purposes.

Despite of sufficient number of the experimental research in this area, the question about theoretical investigation of two - component liquids separation on their components is open so far. Moreover, review of present literature showed an absence of theoretical papers devoted to this problem. The main reason is that from mathematical point of view the problem belongs to the class of the problem with moving boundary. For this class the integration of the equations conjugates with serious mathematical difficulties. In present paper ones make an attempt to give the theoretical solution of the problem of two - component liquid flow separation on its components under the m.f. action.

2. MATHEMATICAL STATEMENT OF THE PROBLEM

As a basic model for the process of separation one takes the following. Certainly, in moving the visco - plastic liquid within cylindrical pipe there is a separation of this liquid for two phases: the first - plastic phase with the radius r_0 and the second - visco-plastic one expanding throughout $r_0 \le r \le R$ (R - radius of considered cylindrical pipe). The complete mathematical description of this process is given in [7], where in terms of the inverse problem for the boundary of plastic area it is developed the automodel solution of non - stationary process of the liquid separation for two phases. The boundary change law is accepted to be as the most appropriate to observed experimental data

$$r_0 = \alpha\sqrt{t} \ ,$$

hereafter the coefficient α is a certain constant parameter that describes the separation rate of the visco - plastic liquid.

Let's propose that in switching on the permanent m.f. the separation of a two - phase liquid realizes by the same scheme like considered in [7]. Such an approach allows getting the automodel solution of the problem of the two - phase liquid separation by means of the permanent m.f. In similar approximation the value r_0 describes the boundary of the one phase after separating.

Let's consider the linear non - stationary motion of two - phase liquid in cylindrical pipe. It is taken that at $t \le 0$ there is the steady flow with velocity distribution by the law $-\frac{B_1}{\eta}r$, here B_1 is constant magnitude characterizing applied m.f. (detailed consideration of the question of m.f. action on the viscous liquid with account of magnetic and rheological properties of the last is investigated in [8]). Pressure drop ΔP at $t > 0$ is accepted to be equal to zero. The axis z is directed along the axis of the cylindrical pipe.

From linearity of motion of the two - phase liquid it follows that $v_r = v_\varphi = 0$. Taking into consideration cylindrical symmetry of problem involved, one has $\frac{\partial v_z}{\partial \varphi} = 0$ and ultimately from the continuity equation $\frac{\partial v_z}{\partial z} = 0$. Then, the equation of motion will be represented by the following form

$$\rho\frac{\partial v_z}{\partial t} = \eta\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right) - B_1 r \qquad (1)$$

under the condition that the value B_1 is determined by the parameters of magnet used and rheological specifications of the liquid [8]

$$B_1 = \pm 2\frac{M}{\mu\mu_0}B^{(2)}$$

herein M is the magnetic moment of unit volume of the liquid, $B^{(2)}$ - permanent factor describing magnet and naturally changing in dependence on construction and type of the magnet, the poles forms, distance between them etc, upper and lower signs " + " and " – " correspond to the liquids with positive and negative magnetic moments, respectively.

The initial and boundary conditions are given in the next form which is the most corresponding to matter of the process considered

$$v_z(r,t=0) = -\frac{B_1}{\eta} r\,, \tag{2}$$

$$\frac{\partial v_z}{\partial r}[r_0(t),t] = 0\,, \tag{3}$$

$$\frac{\partial v_z}{\partial t}[r_0(t),t] = -\frac{B_1}{\rho} r\,, \tag{4}$$

$$r_0(t=0) = 0 \tag{5}$$

How it was marked above, let's take that the boundary of one separated phase of the liquid r_0 changes by the law

$$r_0(t) = \alpha\sqrt{t} \tag{6}$$

In this case it is obvious

$$v_z[r_0(t),t] = -\frac{2}{3}\frac{\alpha B_1}{\rho} t^{3/2} \tag{7}$$

3. SOLVING THE PROBLEM

Solution of the formulated problem is automodel and takes the following form

$$v_z(r,t) = B_2 t^{3/2} f(\theta) \tag{8}$$

where

$$B_2 = \frac{2}{3}\frac{\alpha B_1}{\rho} \quad , \quad \theta = r\sqrt{\frac{\rho}{\eta t}}$$

and θ - the dimensionless variable (analog of this variable has been successfully used in [9] as " Boltzmann transformation "). So, our problem is reduced to determining the function $f(\theta)$. Inserting the relationship (8) into the equation (1), after non - complex mathematical calculations one has

$$\theta\frac{d^2 f(\theta)}{d\theta^2} + \left(1+\frac{\theta^2}{2}\right)\frac{d\,f(\theta)}{d\theta} - \frac{3}{2}\theta\, f(\theta) = B_3\theta^2 \tag{9}$$

where the value B_3 is defined as

$$B_3 = \frac{3}{2\alpha}\sqrt{\frac{\eta}{\rho}}$$

For the function $f(\theta)$ the boundary conditions are altered and may be represented like

$$f(\theta_0)=-1 \quad , \tag{10}$$

$$\frac{\partial f(\theta_0)}{\partial \theta}=0 \quad , \quad \theta_0=\alpha\sqrt{\frac{\rho}{\eta}} \tag{11}$$

The equation (9) belongs to the class of non - homogeneous hypergeometric equations, solutions of which is a difficult task. So, by means of the following substitution

$$f(\theta)=g(\theta)+\frac{B_3}{9}\theta^3 \tag{12}$$

let's transform the equation (9) into appropriate homogeneous equation

$$\theta\frac{d^2 f(\theta)}{d\theta^2}+\left(1+\frac{\theta^2}{2}\right)\frac{d f(\theta)}{d\theta}-\frac{3}{2}\theta f(\theta)=0 \tag{13}$$

The substitution (12) may be established due to the next scheme. One takes $f(\theta)=g(\theta)+k\theta^3$, where k - the coefficient that should be defined. Then, it is obvious. obtained

$$\frac{d f(\theta)}{d\theta}=\frac{d g(\theta)}{d\theta}+3k\theta^2 ,$$

$$\frac{d^2 f(\theta)}{d\theta^2}=\frac{d^2 g(\theta)}{d\theta^2}+6k\theta$$

and after inserting the last formulae into the equation (9) one yields, $\kappa={B_3}/{9}$. The equation (13) can be resolved by various methods, for example, contour integrating [10]. However, such a solution has not obviousness. So, it is worthwhile to look for the solution as an infinite series. One accepts that the function $g(\theta)$ is expressed by means of the series

$$g(\theta)=\sum_{i=0}^{\infty} a_i\theta^i \tag{14}$$

and finds all the required values of the coefficients a_i and indices i . For this it is necessary to find all the derivatives of the relationship (14)

$$\frac{d g(\theta)}{d\theta}=\sum_{i=0}^{\infty} i a_i\theta^{i-1} ,$$

$$\frac{d^2 g(\theta)}{d\theta^2}=\sum_{i=0}^{\infty} i(i-1)a_i\theta^{i-2}$$

and introduce them into the left side of the equation (13) instead of the function $g(\theta)$ and its derivatives. Having selected the summands with the same degree of variable θ , one yields the following series

$$\sum_{i=0}^{\infty} \alpha_i \theta^{i-1} \left\{ i(i-1) + \left(1+\frac{\theta^2}{2}\right) i - \frac{3}{2}\theta^2 \right\} = 0$$

Since the equation (13) is homogeneous, it is obvious that every coefficient at any degree of θ should be equal to zero. This condition gives the infinite system

$$-\frac{3}{2}\alpha_0\theta + \left(1-\theta^2\right)\alpha_1 + \left(4-\frac{1}{2}\theta^2\right)\alpha_2\theta + 9\alpha_3\theta^4 + \cdots = 0$$

For the odd and even values of index i two system of equations are obtained, respectively

$$\begin{cases} -\frac{3}{2}\alpha_0\theta + 4\alpha_2\theta = 0, \\ -\frac{1}{2}\alpha_2\theta^3 + 16\alpha_4\theta^3 = 0, \\ \dots\dots\dots\dots \end{cases} \qquad \begin{cases} \alpha_1 = 0, \\ -\frac{1}{2}\alpha_1\theta^2 + 9\alpha_3\theta^2 = 0, \\ \dots\dots\dots\dots \end{cases}$$

From these systems one gets the next result: all the coefficients with odd values of i are zero, but from the magnitudes a_i with the even i only a_0, a_2 and a_4 are different from zero, so that for the function $g(\theta)$ there is the following dependence

$$g(\theta) = a_0 + a_2\theta^2 + a_4\theta^4 \tag{15}$$

under the law between the constant coefficients a_i ($i = 0, 2, 4$) given below as

$$a_2 = \frac{3}{8}a_0 \quad , \quad a_4 = \frac{1}{32}a_2 = \frac{3}{256}a_0$$

It is clear that the coefficients a_2 and a_4 are expressed through a_0. For finding the coefficient a_0 one should use the boundary condition (10). Firstly, let's return to the original function $f(\theta)$. Taking into account the substitution (12), we have

$$f(\theta) = \frac{B_3}{9}\theta^3 + a_0 + a_2\theta^2 + a_4\theta^4$$

Now it is used the condition (10). The calculations give

$$a_0 = -\frac{1+\frac{B_3}{9}\theta_0^3}{1+\frac{3}{8}\theta_0^2 + \frac{3}{256}\theta_0^4}$$

and ultimately for the function $f(\theta)$ of the equation (9)

$$f(\theta)=\frac{B_3}{9}\theta^3-\left(1+\frac{B_3}{9}\theta_0^3\right)\frac{1+\frac{3}{8}\theta^2+\frac{3}{256}\theta^4}{1+\frac{3}{8}\theta_0^2+\frac{3}{256}\theta_0^4} \tag{16}$$

At infinitesimal values of the parameter θ which physically corresponds to stationary regime of the flow $t \to \infty$, from the relationship (16) one can acquire for the velocity function

$$f(\theta\to 0)=-\frac{1+\frac{B_3}{9}\theta_0^3}{1+\frac{3}{8}\theta_0^2+\frac{3}{256}\theta_0^4}$$

Having analyzed the formula (16) we are able to estimate the dependence of the function $f(\theta)$ on various rheological parameters of the liquid involved. For instance, the more the rate of the boundary separation α, the less the function $f(\theta)$ because the inequality $\frac{\partial f(\theta)}{\partial \alpha} \prec 0$ is valid. Physical explanation of the last aspect is obvious. The phase separation is followed by energy dissipation. It is clear that under weak rate of the flow separation the dissipation of energy is insufficient, naturally, the velocity function $f(\theta)$ is more than for case of the flow separation with greater rate.

4. CONCLUSION

So, the algorithm developed here allows to analytically resolve the inverse problem of the two - phase liquid separation by m.f. Obtained automodel solutions are valid under the law $r_0=\alpha\sqrt{t}$ for the boundary change of one phase. Certainly, the value α depends on the physical parameters of the phase composing the liquid flow. In [8] it has been revealed that the change of hydraulic characteristics of liquid under m.f. action is connected to magnetic susceptibility of the liquid. If two various by sign magnetic susceptibilities of phases in the liquid are existed, then under m.f. effect their separation takes place due to different "rheological" reaction of these phases on applied m.f. Accordingly to the results of [8], for diamagnetic liquids decrease of their viscosity with parameter $B^{(2)}$ under magnetic field should be observed, while for para - and ferromagnetic liquids ones have to indicate the increasing their viscosity. Naturally, the more difference between the magnetic permeabilities of considered phases μ_1 and μ_2, the greater separation process as a whole. In principle, in this case ones find two values of the liquid velocity: $v_z^{(+)}(r,t)$ for the liquids with $M>0$, and $v_z^{(-)}(r,t)$ - for the liquids with $M<0$. As a result one gets

$$v_z^{(+)}(r,t) = B_2 t^{3/2} f(\theta),$$
$$v_z^{(-)}(r,t) = -B_2 t^{3/2} f(\theta)$$

for the permanent m.f. effect. By other words, there is the different character of velocity change at the same function $f(\theta)$. The variant of various by sign magnetic susceptibilities of phases composing the initially unified flow considered here is the most optimal for the separation effect. Of course, the separation under m.f. will be realized for two phases with the univalent sign of their magnetic moments M as well, but for similar combination of the liquid phases magnetic properties the effect will not be so essential.

Hence, the more the value $\Delta\mu$, the greater the constant α (because there is the dependence $\alpha \sim \Delta\mu$), and the boundary of the phases separation r_0, respectively.

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