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ON THE CONSTRUCTION OF DIFFERENTIAL INCLUSION WITH PRESCRIBED INTEGRAL FUNNEL

Kh.G.Guseinov O. Özer and S.A.Düzce
Anadolu University, Faculty of Science,
Department of Mathematics, 26470, Eskisehir, Turkey
kguseynov@anadolu.edu.tr, oozer@anadolu.edu.tr, saduzce@anadolu.edu.tr

Abstract- In this article, the inverse problem of the differential inclusion theory is considered. For a given $\varepsilon > 0$ and a given special type set valued map $t \to V(t)$, $t \in [t_*, t^*]$, it is required to define differential inclusion such that the Hausdorff distance between the reachable sets of the differential inclusion with initial set $(t_*, V(t_*))$ and $V(t)$ would be less than ε for every $t \in [t_*, t^*]$.

Keywords- Differential Inclusion, Integral Funnel, Set Valued Map.

1. INTRODUCTION

In this article the inverse problem of the differential inclusion theory (DI) is considered. For a given $\varepsilon > 0$ and a given special type set valued map $t \to V(t)$, $t \in [t_*, t^*]$, it is required to define DI such that the Hausdorff distance between the reachable sets of the DI with initial set $(t_*, V(t_*))$ and $V(t)$ would be less than ε for every $t \in [t_*, t^*]$. Note that the notions strong and weak invariant sets with respect to DI are of great importance in construction of such DI (see, e.g. [1 - 5]).

In [1-9], direct problems were considered i.e. the various properties of integral funnels and reachable sets of the DI were studied. The inverse problem was investigated in works [10 - 12]. In the offered article, the searching DI are defined so that the right hand sides of the DI satisfy the conditions, which guarantee the existence and extendability of the solutions.

Let $t \to V(t)$, $t \in [t_*, t^*]$, be a set valued map,

$$V = gr V(\cdot) = \{(t,x) \in [t_*, t^*] \times R^n : x \in V(t)\}$$

be a closed set. For $(t,x) \in [t_*, t^*] \times R^n$ we denote

$$D_*^+ V(t,x) = \{d \in R^n : \exists x(\tau) \in V(\tau), \tau > t, \lim_{\tau \to t+0} (x(\tau) - x)/(\tau - t) = d\},$$

$$D_*^- V(t,x) = \{d \in R^n : \exists x(\tau) \in V(\tau), \tau < t, \lim_{\tau \to t-0} (x(\tau) - x)/(\tau - t) = d\}.$$

The sets $D_*^+ V(t,x)$ and $D_*^- V(t,x)$ are said respectively to be lower right hand side and lower left hand side derivative sets of the set valued map $t \to V(t)$ calculated at the point (t,x). These sets are closed and they have nearly connection with lower Bouligand contingent cone, used in many problems of the set valued and nonsmooth analysis (see, for example [1 - 5]).

Let $B = \{x \in R^n : \|x\| \le 1\}$. We denote by symbol $\alpha(A,C)$ the Hausdorff distance between the sets $A \subset R^n$ and $C \subset R^n$. It is defined as

$$h(A,C) = \max\left\{ \underset{a \in A}{Sup}\, dist(a,C), \underset{c \in C}{Sup}\, dist(c,A) \right\}$$

where $dist(a,C) = \inf_{c \in C} \|a - c\|$, $\|\cdot\|$ means the Euclidean norm. We denote the interior of $E \subset R^n$ by $\operatorname{int} E$ and the boundary of $E \subset R^n$ by ∂E.

From now on we will assume that $V_* \subset R^n$, $V^* \subset R^n$ are convex, compact sets, $\operatorname{int} V_* \ne \varnothing$, $\operatorname{int} V^* \ne \varnothing$ and the set valued map $t \to V(t)$, $t \in [t_*, t^*]$ is defined as

$$V(t) = \left(1 - \frac{t - t_*}{t^* - t_*}\right) V_* + \frac{t - t_*}{t^* - t_*} V^*, \tag{1.1}$$

for every $t \in [t_*, t^*]$. It is obvious that $V = gr V(\cdot) \subset [t_*, t^*] \times R^n$ and $V(t) \subset R^n$ are convex, compact sets for every $t \in [t_*, t^*]$, the set valued map $t \to V(t)$, $t \in [t_*, t^*]$, is continuous.

Proposition 1.1. [13] There exist $\alpha_* > 0$ and a set valued map $t \to W_*(t)$, $t \in [t_* - \alpha_*, t^* + \alpha_*]$, such that $W_*(t) = V(t)$ for every $t \in [t_*, t^*]$, $W_*(t_* - \alpha_*) \ne \varnothing$, $W_*(t^* + \alpha_*) \ne \varnothing$ and

$$W_* = gr W_*(\cdot) = \{(t,x) \in [t_* - \alpha_*, t^* + \alpha_*] \times R^n : x \in W_*(t)\}$$

is convex, compact set.

2. PRELIMINARY STUDIES

Consider the DI

$$\dot{x} \in F(t,x) \tag{2.1}$$

where $x \in R^n$ - is the phase state vector, $t \in [t_0, \vartheta]$ is the time. Absolutely continuous function $x(\cdot) : [t_0, \vartheta] \to R^n$ satisfying the inclusion $\dot{x}(t) \in F(t, x(t))$ for almost all $t \in [t_0, \vartheta]$ is said to be a solution of the DI (2.1) (see, e.g. [6]). By symbol $X(t_1, X_1)$ we denote the totality of solutions of the DI (2.1) satisfying the condition $x(t_1) \in X_1$, where $t_1 \in [t_0, \vartheta]$, $X_1 \subset R^n$. We set

$$X(t; t_1, X_1) = \{x(t) \in R^n : x(\cdot) \in X(t_1, X_1)\},$$
$$H(t_1, X_1) = \{(t,x) \in [t_0, \vartheta] \times R^n : x \in X(t; t_1, X_1)\}.$$

The set $X(t; t_1, X_1)$ is called the reachable set of the DI (2.1) at the time moment t. The set $H(t_1, X_1)$ is called the integral funnel of the DI (2.1) with initial set (t_1, X_1).

Let $\varepsilon > 0$ be some fixed number. We will study the following problem. It is required to define a DI so that the inequality $\alpha(X(t; t_*, V_*), V(t)) \le \varepsilon$ holds for every $t \in [t_*, t^*]$ where the sets $V(t)$, $t \in [t_*, t^*]$, are defined by (1.1).

For $(t,x) \in [t_*, t^*] \times R^n$, $\nu \in (0, \alpha_*)$ we set

$$F_{\nu}^{*}(t,x)=[W_{*}(t+\nu)-x]/\nu, \tag{2.2}$$

$$\Phi_{\nu}^{*}(t,x)=[W_{*}(t-\nu)-x]/(-\nu) \tag{2.3}$$

where $\alpha_{*}>0$ and the set valued map $t\to W_{*}(t)$, $t\in\left[t_{*}-\alpha_{*},t^{*}+\alpha_{*}\right]$ are defined in proposition 1.1. It is obvious that $F_{\nu}^{*}(t,x)=[V(t+\nu)-x]/\nu$ if $t+\nu\leq t^{*}$ and $\Phi_{\nu}^{*}(t,x)=[V(t-\nu)-x]/(-\nu)$ if $t-\nu\geq t_{*}$. Consider some properties of the set valued maps $(t,x)\to F_{\nu}^{*}(t,x)$ and $(t,x)\to\Phi_{\nu}^{*}(t,x)$ defined on $[t_{*},t^{*}]\times R^{n}$. Denote

$$a=\max\left\{\|x\|:(t,x)\in W_{*}\right\}.$$

(2.4)

Proposition 2.1. The sets $F_{\nu}^{*}(t,x)\subset R^{n}$ and $\Phi_{\nu}^{*}(t,x)\subset R^{n}$ are convex, compact for any $(t,x)\in[t_{*},t^{*}]\times R^{n}$. The set valued maps $(t,x)\to F_{\nu}^{*}(t,x)$ and $(t,x)\to\Phi_{\nu}^{*}(t,x)$ are continuous with respect to (t,x) in $[t_{*},t^{*}]\times R^{n}$ and are Lipschitz with respect to x with constant $\frac{1}{\nu}$. The inequalities

$$\max\left\{\|f\|:f\in F_{\nu}^{*}(t,x)\right\}\leq\frac{1}{\nu}(a+\|x\|),\qquad \max\left\{\|\varphi\|:\varphi\in\Phi_{\nu}^{*}(t,x)\right\}\leq\frac{1}{\nu}(a+\|x\|)$$

are true for any $(t,x)\in[t_{*},t^{*}]\times R^{n}$ where $a\geq 0$ is defined by relation (2.4).

For $(t,x)\in[t_{*},t^{*}]\times R^{n}$ we set

$$r_{\nu}^{*}(t,x)=\rho(F_{\nu}^{*}(t,x),\Phi_{\nu}^{*}(t,x))$$

(2.5)

where $\rho(A,C)=\inf\left\{\|a-c\|:a\in A,c\in C\right\}$ for the sets $A\subset R^{n}$, $C\subset R^{n}$.

Proposition 2.2. Suppose that $t^{*}-t_{*}\geq 2\nu$. Then, for any $(t,x)\in V$ such that $t\in[t_{*}+\nu,t^{*}-\nu]$, the equality $r_{\nu}^{*}(t,x)=0$ holds.

The following proposition characterizes some properties of the function $r_{\nu}^{*}(\cdot):[t_{*},t^{*}]\times R^{n}\to[0,\infty)$.

Proposition 2.3. The function $r_{\nu}^{*}(\cdot):[t_{*},t^{*}]\times R^{n}\to[0,\infty)$ is continuous with respect to (t,x), is Lipschitz with respect to x with Lipschitz constant $2\frac{1}{\nu}$, $\left|r_{\nu}^{*}(t,x)\right|\leq 2\frac{1}{\nu}(a+\|x\|)$ for any $(t,x)\in[t_{*},t^{*}]\times R^{n}$, where $a\geq 0$ is defined by relation (2.4). Moreover, the function $r_{\nu}^{*}(\cdot):[t_{*},t^{*}]\times R^{n}\to[0,\infty)$ is right continuous as $t=t_{*}$ and left continuous as $t=t^{*}$.

Now we formulate proposition characterizing relationship of the sets $F_{\nu}^{*}(t,x)$ and $\Phi_{\nu}^{*}(t,x)$ with the derivative sets of the set valued map $t\to V(t)$, $t\in[t_{*},t^{*}]$, where the sets $V(t)\subset R^{n}$ are defined by (1.1).

Proposition 2.4. The inclusions

$$F_{\nu}^{*}(t,x)\subset D_{*}^{+}V(t,x)\ \text{for any}\ (t,x)\in V,\ t\in[t_{*},t^{*}),$$

$$\Phi_\nu^*(t,x) \subset D_*^- V(t,x) \text{ for any } (t,x)\in V,\ t\in (t_*,t^*]$$

are satisfied.

3. APPROXIMATION

Consider DI

$$\dot{x}\in F_\nu^*(t,x),\ (t,x)\in [t_*,t^*]\times R^n \tag{3.1}$$

$$\dot{x}\in F_\nu^*(t,x)+r_\nu^*(t,x)B,\ (t,x)\in [t_*,t^*]\times R^n \tag{3.2}$$

where the set valued map $(t,x)\to F_\nu^*(t,x)$, $(t,x)\in [t_*,t^*]\times R^n$, and the function $r_\nu^*(\cdot):[t_*,t^*]\times R^n\to [0,\infty)$ are defined by (2.1) and (2.5), respectively.

We denote by symbols $X_\nu^1(t_1,X_1)$ and $X_\nu^2(t_1,X_1)$ the totality of solutions of the DI (3.1) and (3.2), respectively, satisfying the condition $x(t_1)\in X_1$ where $X_1\subset R^n$, $t_1\in [t_*,t^*]$. Further, we set

$$X_\nu^1(t;t_1,X_1)=\{x(t)\in R^n : x(\cdot)\in X_\nu^1(t_1,X_1)\}$$
$$X_\nu^2(t;t_1,X_1)=\{x(t)\in R^n : x(\cdot)\in X_\nu^2(t_1,X_1)\}$$
$$H_\nu^1(t_1,X_1)=\{(t,x)\in [t_*,t^*]\times R^n : x\in X_\nu^1(t;t_1,X_1)\}$$
$$H_\nu^2(t_1,X_1)=\{(t,x)\in [t_*,t^*]\times R^n : x\in X_\nu^2(t;t_1,X_1)\}$$

Formulate the theorem establishing a connection between integral funnels of the DI (3.1), (3.2) and the set $V = gr V(\cdot)\subset [t_*,t^*]\times R^n$.

Theorem 3.1. For every $\nu\in (0,\alpha_*)$ where $\alpha_*>0$ is defined in proposition 1.1, the inclusions

$$H_\nu^1(t_*,V(t_*))\subset V\subset H_\nu^2(t_*,V(t_*))$$

and consequently

$$X_\nu^1(t;t_*,V(t_*))\subset V(t)\subset X_\nu^2(t;t_*,V(t_*))$$

are fulfilled for every $t\in [t_*,t^*]$.

Proof. According to the proposition 2.4, $F_\nu^*(t,x)\subset D_*^+V(t,x)$ for any $(t,x)\in V$, $t\in [t_*,t^*)$. Then, it follows from proposition 2.1 and theorem 1 [1] that the set V is positively strongly invariant with respect to the DI (3.1). It means that for any $(t_1,x_1)\in V$, $x(\cdot)\in X_\nu^1(t_1,X_1)$ the inclusion $x(t)\in V(t)$ is verified for any $t\in [t_1,t^*]$. It follows from here and from the theorem 1.1 [9] that

$$H_\nu^1(t_*,V(t_*))\subset V$$

According to the proposition 2.4, we have $\Phi_\nu^*(t,x)\subset D_*^- V(t,x)$ for any $(t,x)\in V$, $t\in (t_*,t^*]$ where the set $\Phi_\nu^*(t,x)\subset R^n$, $(t,x)\in [t_*,t^*]\times R^n$, is defined by (2.3). Then, it follows from definition of the function $(t,x)\to r_\nu^*(t,x)$ that

$$\Phi_{\nu}^{*}(t,x)\cap[F_{\nu}^{*}(t,x)+r_{\nu}^{*}(t,x)B]\neq\varnothing \text{ for any } (t,x)\in V,\ t\in(t_{*},t^{*}].$$

We obtain that

$$[F_{\nu}^{*}(t,x)+r_{\nu}^{*}(t,x)B]\cap D_{*}^{-}V(t,x)\neq\varnothing \text{ for any } (t,x)\in V, \tag{3.3}$$

where $t\in(t_{*},t^{*}]$. Then it follows from propositions 2.1, 2.3, relation (3.3) and theorem 2 [1] that the set V is negatively weakly invariant with respect to the differential inclusion (3.2). This means that for every fixed $(t_1,x_1)\in V$, there exists $x(\cdot)\in X_{\nu}^{2}(t_1,x_1)$ such that the inclusion $x(t)\in V(t)$ is verified for every $t\in[t_{*},t_1]$. We have from here and from theorem 1.1 [9] that $V\subset H_{\nu}^{2}(t_{*},V(t_{*}))$. Theorem was proved.

Consider DI

$$\dot{x}\in F_{\nu}^{*}(t,x)+rB,\ (t,x)\in[t_{*},t^{*}]\times R^{n} \tag{3.4}$$

By symbol $Y_{\nu}^{r}(t_1,Y_1)$ we denote the totality of solutions of the DI (3.4) satisfying the condition $x(t_1)\in Y_1$, where $Y_1\subset R^{n}$, $t_1\in[t_{*},t^{*}]$ and we set

$$Y_{\nu}^{r}(t;t_1,Y_1)=\{x(t)\in R^{n}: x(\cdot)\in Y_{\nu}^{r}(t_1,Y_1)\}$$

Proposition 3.1. Let $X_{*}\subset R^{n}$, $Y_{*}\subset R^{n}$ be compact sets. Then

$$h\left(X_{\nu}^{1}(t;t_{*},X_{*}),Y_{\nu}^{r}(t;t_{*},Y_{*})\right)\leq h(X_{*},Y_{*})\exp\left[-\frac{1}{\nu}(t-t_{*})\right]+r\nu\left[1-\exp\left(-\frac{1}{\nu}(t-t_{*})\right)\right]$$

holds for any $t\in[t_{*},t^{*}]$.

Proof. Let $t_1\in[t_{*},t^{*}]$, $x_1\in X_{\nu}^{1}(t_1;t_{*},X_{*})$. Then there exists $x(\cdot)\in X_{\nu}^{1}(t_{*},X_{*})$ such that $x(t_1)=x_1$. Consequently, there exist $x_{*}\in X_{*}$ and measurable function $p_{*}(\cdot):[t_{*},t^{*}]\to R^{n}$ such that $p_{*}(t)\in W_{*}(t+\nu)$ a.e. in $[t_{*},t^{*}]$ and

$$x(t)=x_{*}-\frac{1}{\nu}\int_{t_{*}}^{t}x(\tau)\,d\tau+\frac{1}{\nu}\int_{t_{*}}^{t}p_{*}(\tau)\,d\tau$$

(3.5)

Now, we choose $y_{*}\in Y_{*}$ such that $\|x_{*}-y_{*}\|\leq h(X_{*},Y_{*})$ and let

$$y(t)=y_{*}-\frac{1}{\nu}\int_{t_{*}}^{t}y(\tau)\,d\tau+\frac{1}{\nu}\int_{t_{*}}^{t}p_{*}(\tau)\,d\tau \tag{3.6}$$

Then, $y(\cdot)\in Y_{\nu}^{r}(t_{*},Y_{*})$ and $y_1=y(t_1)\in Y_{\nu}^{r}(t_1;t_{*},Y_{*})$. It follows from (3.5) and (3.6) that

$$\|x(t)-y(t)\|\leq h(X_{*},Y_{*})\exp\left[-\frac{1}{\nu}(t-t_{*})\right]$$

for any $t\in[t_{*},t^{*}]$. Since $x_1\in X_{\nu}^{1}(t_1;t_{*},X_{*})$ is arbitrarily chosen, $t_1\in[t_{*},t^{*}]$, we obtain that

$$X_{\nu}^{1}(t_1;t_{*},X_{*})\subset Y_{\nu}^{r}(t_1;t_{*},Y_{*})+h(X_{*},Y_{*})\exp\left[-\frac{1}{\nu}(t_1-t_{*})\right]\cdot B \tag{3.7}$$

Now, we choose an arbitrary $y_1 \in Y_\nu^r(t_1;t_*,Y_*)$. Then, there exists $y(\cdot)\in Y_\nu^r(t_*,Y_*)$ such that $y(t_1)=y_1$. Consequently, there exist $y_* \in Y_*$ and measurable function $p_*(\cdot):[t_*,t^*]\to R^n$ such that $p_*(t)\in W_*(t+\nu)+r\nu B$ a.e. in $[t_*,t^*]$ and

$$y(t)=y_* - \frac{1}{\nu}\int_{t_*}^{t} y(\tau)\,d\tau + \frac{1}{\nu}\int_{t_*}^{t} p_*(\tau)\,d\tau . \qquad (3.7)$$

Now we choose $x_* \in X_*$ such that $\|y_* - x_*\| \le h(X_*,Y_*)$ and let

$$p(t)=\{p\in W_*(t+\nu):\|p_*(t)-p\| = dist\,(p_*(t),W_*(t+\nu))\} \qquad (3.8)$$

where $t\in[t_*,t^*]$. Since the set valued map $t\to W_*(t+\nu)$, $t\in[t_*,t^*]$, is continuous, the sets $W_*(t+\nu)$ are convex and compact for every $t\in[t_*,t^*]$, the function $p_*(\cdot):[t_*,t^*]\to R^n$ is measurable then according to [4], the function $p(\cdot):[t_*,t^*]\to R^n$ defined by (3.8) is single valued and measurable. Since $p_*(t)\in W_*(t+\nu)+r\nu B$ a.e. in $[t_*,t^*]$ then

$\|p_*(t)-p(t)\|\le r\nu$ for almost all $t\in[t_*,t^*]$. $\qquad$ (3.9)

Let

$$x(t)=x_* - \frac{1}{\nu}\int_{t_*}^{t} x(\tau)\,d\tau + \frac{1}{\nu}\int_{t_*}^{t} p(\tau)\,d\tau$$

(3.10)

where $t\in[t_*,t^*]$. Then $x(\cdot)\in X_\nu^1(t_*,X_*)$ and $x_1=x(t_1)\in X_\nu^1(t_1;t_*,X_*)$. It follows from (3.7), (3.9) and (3.10) that

$$\|y(t)-x(t)\|\le h(X_*,Y_*)\exp\left[-\frac{1}{\nu}(t-t_*)\right]+r\nu\left(1-\exp\left[-\frac{1}{\nu}(t-t_*)\right]\right)$$

for any $t\in[t_*,t^*]$. Since $y_1\in Y_\nu^r(t_1;t_*,Y_*)$ is arbitrarily chosen, $t_1\in[t_*,t^*]$ we obtain from here that

$$Y_\nu^r(t_1;t_*,Y_*)\subset X_\nu^1(t_1;t_*,X_*)+\left\{h(X_*,Y_*)\exp\left[-\frac{1}{\nu}(t_1-t_*)\right]+r\nu\left[1-\exp\left(-\frac{1}{\nu}(t_1-t_*)\right)\right]\right\}B$$

It follows from here and (3.7) the validity of the proposition.

Theorem 3.2. Suppose $\nu_*\in(0,\alpha_*)$ where $\alpha_*>0$ is defined in proposition 1.1. Then, there exists $r_*\ge 0$ such that for every $\nu\in(0,\nu_*]$ the inequality

$$h\left(X_\nu^1(t;t_*,V(t_*)),V(t)\right)\le \nu r_*\cdot\left[1-\exp\left(-\frac{1}{\nu}(t-t_*)\right)\right]$$

is satisfied for any $t\in[t_*,t^*]$.

Proof. Since the function $r_\nu^*(\cdot):[t_*,t^*]\times R^n\to[0,\infty)$ is continuous with respect to (t,x), $V=grV(\cdot)\subset[t_*,t^*]\times R^n$ is compact set, then there exists $r_*\ge 0$ such that

$r^*_{\nu_*}(t,x) \le r_*$ for any $(t,x) \in V$. Let us choose an arbitrary $\nu \in (0,\nu_*]$. According to the theorem 3.1, we have

$$X^1_\nu(t;t_*,V(t_*)) \subset V(t) \text{ for every } t \in [t_*,t^*]. \tag{3.11}$$

Since $\Phi^*_\nu(t,x) \subset D^-_* V(t,x)$ for any $(t,x) \in V$, $t \in (t_*,t^*]$, it follows from the definition of the function $(t,x) \to r^*_\nu(t,x)$ that

$$\left[F^*_\nu(t,x) + r^*_\nu(t,x)B\right] \cap D^-_* V(t,x) \ne \varnothing, \tag{3.12}$$

for any $(t,x) \in V$, $t \in (t_*,t^*]$. It is possible to prove that $r^*_\nu(t,x) \le r^*_{\nu_*}(t,x)$ for every $(t,x) \in V$. Consequently, we obtain from (3.12) that

$$D^-_* V(t,x) \cap [F^*_\nu(t,x) + r_* B] \ne \varnothing \text{ for every } (t,x) \in \partial V,\ t \in (t_*,t^*]. \tag{3.13}$$

Consider DI

$$\dot{x} \in F^*_\nu(t,x) + r_* B,\ (t,x) \in [t_*,t^*] \times R^n \tag{3.14}$$

By symbol $Y^{r_*}_\nu(t_1,Y_1)$, we denote the totality of solutions of the DI (3.14) satisfying the condition $x(t_1) \in Y_1$, where $Y_1 \subset R^n$, $t_1 \in [t_*,t^*]$, and we set

$$Y^{r_*}_\nu(t;t_1,Y_1) = \{x(t) \in R^n : x(\cdot) \in Y^{r_*}_\nu(t_1,Y_1)\}$$

It follows from relation (3.13) and from theorem 2 [1] that the set V is negatively weakly invariant with respect to the DI (3.14). This means that for every $(t_1,x_1) \in V$ there exists $x(\cdot) \in Y^{r_*}_{\nu_*}(t_1,Y_1)$ so that the inclusion $x(t) \in V(t)$ is verified for any $t \in [t_*,t_1]$. We obtain by theorem 1.1 [9] that

$$V(t) \subset Y^{r_*}_\nu(t;t_*,V(t_*)) \text{ for any } t \in [t_*,t^*] \tag{3.15}$$

By the proposition 3.1 we have

$$h\left(X^1_\nu(t;t_*,V(t_*)), Y^{r_*}_\nu(t;t_*,V(t_*))\right) \le \nu r_* \cdot \left[1 - \exp\left(-\frac{1}{\nu}(t-t_*)\right)\right] \tag{3.16}$$

for any $t \in [t_*,t^*]$.

By virtue of (3.11), (3.15) and (3.16), the validity of the theorem is shown.

Validity of the following theorem follows from theorem 3.1 and theorem 3.2.

Theorem 3.3. For each $\varepsilon > 0$ there exists $\nu(\varepsilon) \in (0,\alpha_*)$ that for every $\nu \in (0,\nu(\varepsilon))$ the inequality $h\left(X^1_\nu(t;t_*,V(t_*)), V(t)\right) \le \varepsilon$ holds for any $t \in [t_*,t^*]$ where $\alpha_* > 0$ is defined in proposition 1.1.

If there exists $\nu_* \in (0,\alpha_*)$ such that $r^*_{\nu_*}(t,x) = 0$ for any $(t,x) \in \partial V$, then $H^1_{\nu_*}(t_*,V(t_*)) = V$.

4. CONCLUSION

The obtained results can be applied in mathematics modeling where it is required to specify the dynamic of the system through measurement of the phase state of the system.

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