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AN ALGORITHM FOR SEGMENT STABILITY

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Abstract- In this note an algorithm for testing on stability-unstability of polynomial segments with stable end-points is given. The algorithm is based on well-known segment lemma and on approximate positive real roots of suitable polynomial equation. In this paper an upper bound for absolute error of aproximate root that guarantees the segment stability or unstability is established. In the case of unstability the algorithm, differs from the existing algorithms, explains the segment behavior in the parameter space. Some illustrative examples also are given.

Keywords- Stable Polynomial, Segment Stability, Segment Lemma.

1. INTRODUCTION

In this paper we consider real, stable (Hurwitz) polynomials, i.e. polynomials, having all their roots in the open left half of complex plane.

Let $a(s)$ and $b(s)$ be two nth order stable polynomials:

$$a(s)=a_0+a_1s+a_2s^2+\cdots+a_ns^n, \tag{1}$$

$$b(s)=b_0+b_1s+b_2s^2+\cdots+b_ns^n. \tag{2}$$

Denote their convex combination by

$$a(s,\lambda)=(1-\lambda)a(s)+\lambda b(s), \quad \lambda\in[0,1]. \tag{3}$$

In the following the word segment means the family of polynomials $\{a(s,\lambda):\lambda\in[0,1]\}$.

We are interested in segment stability. Recall that, if for all $\lambda\in[0,1]$ the polynomial $a(s,\lambda)$ is stable then the segment is said to be stable. If there exists $\lambda\in(0,1)$ such that the polynomial $a(s,\lambda)$ is unstable then the segment is said to be unstable.

It is well known that the stability of polynomials $a(s)$ and $b(s)$ does not guarantee the stability of convex combinations.

Without loss of generality we assume that the coefficients of $a(s)$ and $b(s)$ are positive; If the coefficients of $a(s)$ and $b(s)$ have opposite signs then convex combination is necessarily unstable.

There are many results concerning the stability of the convex combination of stable polynomials (see, e.g., [1-7]). Bialas [2] shows that a segment is stable if and only if $\det|H(a)+\lambda H(b)|$ has no positive real roots, where $H(a)$ is the Hurwitz matrix associated with the polynomial $a(s)$. In [3], it was shown that if $a(s)$ and $b(s)$ have the same even (odd) parts then segment is stable. For a stable polynomials or matrix, Fu and Barmish [6] determines the maximal perturbation bounds under which stability is preserved. The algorithm given in [4] is based on the solving for the positive real roots of five polynomials. Rantzer [7] proved that if the polynomial $g(s) = a(s)\text{-}b(s)$ satisfies the growth condition then convex combination is stable.

In this paper we give an algorithm for testing segment stability and unstability. Our algorithm as the algorithm in [4], is based on the well known Segment Lemma in [5]. But the algorithm given in [4] is based on the solving for the positive roots of a five polynomials, whereas our algorithm is based on the solving for the positive approximate roots of a only one polynomial. If the segment is unstable, our algorithm differs from other algorithms also allows us to describe the segment's behavior during the motion starting from one end point to another.

Putting $s = j\omega$ with $\omega > 0$ we have

$$a(j\omega) = (a_0\text{-}a_2\omega^2\text{+}a_4\omega^4\text{-}\cdots)\text{+}j(a_1\omega\text{-}a_3\omega^3\text{+}a_5\omega^5\text{-}\cdots) = a^e(\omega) + ja^o(\omega)$$

$$b(j\omega) = (b_0\text{-}b_2\omega^2\text{+}b_4\omega^4\text{-}\cdots)\text{+}j(b_1\omega\text{-}b_3\omega^3\text{+}b_5\omega^5\text{-}\cdots) = b^e(\omega) + jb^o(\omega)$$

Lemma 1.1. (Segment Lemma). There exists $\lambda \in [0,1]$ such that the polynomial $a(s,\lambda)$ has a pure imaginary root $s = j\omega \Leftrightarrow$ the relations

$$a^e(\omega)b^o(\omega)\text{-}a^o(\omega)b^e(\omega) = 0 \tag{4}$$

$$a^e(\omega)b^e(\omega) \le 0 \tag{5}$$

$$a^o(\omega)b^o(\omega) \le 0 \tag{6}$$

The value of λ, mentioned in Lemma can be calculated as

$$\lambda(\omega) = \frac{a^e(\omega)}{a^e(\omega) - b^e(\omega)} \text{ or } \lambda(\omega) = \frac{a^o(\omega)}{a^o(\omega) - b^o(\omega)} \tag{7}$$

If we can find the positive roots $\omega_1^*, \omega_2^*, \ldots, \omega_k^*$ of the polynomial equation (4), then by checking (5) and (6), one may conclude about segment stability or unstability. Unfortunately it is almost impossible to find out exact roots. Therefore, we develope an algorithm based on approximate roots of (4).

2. THE SEGMENT STABILITY ALGORITHM

The following proposition is a direct consequence of the Segment Lemma.

Proposition 2.1. Let $a(s)$ and $b(s)$ be two nth order stable polynomials. Then the segment is unstable $\Leftrightarrow$ there exists $\omega > 0$ such that either the relations (4) and

$$a^e(\omega)b^e(\omega) < 0$$

or the relation (4) and

$$a^o(\omega)b^o(\omega) < 0$$

holds.

If the Eqn.(4) has no positive solution then the segment is stable. Therefore, in the following we assume that the Eqn.(4) has at least one positive solution.

Assume that $\omega_1, \omega_2, \ldots, \omega_k$ are distinct approximate positive roots of (4) and ε is their absolute error. Thus for all $i = 1, 2, \ldots, k$ we have

$$\omega_i - \varepsilon \le \omega_i^* \le \omega_i + \varepsilon \tag{8}$$

(recall that ω_i^* is exact positive solution of (4)). Note that for ε sufficiently small one has

$$\omega_i - \varepsilon > 0. \tag{9}$$

Inequalities (9) will be in force throughout the paper. Define

$$
\begin{array}{rcl}
\underline{a}^e(\omega_i) & = & a_0 - a_2(\omega_i+\varepsilon)^2 + a_4(\omega_i-\varepsilon)^4 - a_6(\omega_i+\varepsilon)^6 + \dots \\
\overline{a}^e(\omega_i) & = & a_0 - a_2(\omega_i-\varepsilon)^2 + a_4(\omega_i+\varepsilon)^4 - a_6(\omega_i-\varepsilon)^6 + \dots \\
\underline{b}^e(\omega_i) & = & b_0 - b_2(\omega_i+\varepsilon)^2 + b_4(\omega_i-\varepsilon)^4 - b_6(\omega_i+\varepsilon)^6 + \dots \\
\overline{b}^e(\omega_i) & = & b_0 - b_2(\omega_i-\varepsilon)^2 + b_4(\omega_i+\varepsilon)^4 - b_6(\omega_i-\varepsilon)^6 + \dots \\
\underline{a}^o(\omega_i) & = & a_1(\omega_i-\varepsilon) - a_3(\omega_i+\varepsilon)^3 + a_5(\omega_i-\varepsilon)^5 - a_7(\omega+\varepsilon)^7 + \dots \\
\overline{a}^o(\omega_i) & = & a_1(\omega_i+\varepsilon) - a_3(\omega_i-\varepsilon)^3 + a_5(\omega_i+\varepsilon)^5 - a_7(\omega-\varepsilon)^7 + \dots \\
\underline{b}^o(\omega_i) & = & b_1(\omega_i-\varepsilon) - b_3(\omega_i+\varepsilon)^3 + b_5(\omega_i-\varepsilon)^5 - b_7(\omega+\varepsilon)^7 + \dots \\
\overline{b}^o(\omega_i) & = & b_1(\omega_i+\varepsilon) - b_3(\omega_i-\varepsilon)^3 + b_5(\omega_i+\varepsilon)^5 - b_7(\omega-\varepsilon)^7 + \dots
\end{array}
\tag{10}
$$

Then in view of positivity of coefficients $a_0, a_1, \dots a_n, b_0, b_1, \dots b_n$, inequalities (8), (9) we have got the following inequalities: For all $i = 1, 2, \dots, k$

$$\underline{a}^e(\omega_i) \le a^e(\omega_i^*) \le \overline{a}^e(\omega_i) \tag{11}$$

$$\underline{b}^e(\omega_i) \le b^e(\omega_i^*) \le \overline{b}^e(\omega_i) \tag{12}$$

$$\underline{a}^o(\omega_i) \le a^o(\omega_i^*) \le \overline{a}^o(\omega_i) \tag{13}$$

$$\underline{b}^o(\omega_i) \le b^o(\omega_i^*) \le \overline{b}^o(\omega_i) \tag{14}$$

Also note that

$$
\begin{array}{rcccl}
\lim\limits_{\varepsilon\to 0} \underline{a}^e(\omega_i) & = & \lim\limits_{\varepsilon\to 0} \overline{a}^e(\omega_i) & = & a^e(\omega_i^*) \\
\lim\limits_{\varepsilon\to 0} \underline{b}^e(\omega_i) & = & \lim\limits_{\varepsilon\to 0} \overline{b}^e(\omega_i) & = & b^e(\omega_i^*) \\
\lim\limits_{\varepsilon\to 0} \underline{a}^o(\omega_i) & = & \lim\limits_{\varepsilon\to 0} \overline{a}^o(\omega_i) & = & a^o(\omega_i^*) \\
\lim\limits_{\varepsilon\to 0} \underline{b}^o(\omega_i) & = & \lim\limits_{\varepsilon\to 0} \overline{b}^o(\omega_i) & = & b^o(\omega_i^*)
\end{array}
\tag{15}
$$

Consider the following Table 1 and Table 2, where the symbols '+' and '–' indicate the signs of the corresponding expressions.

Table 1: Stability Table

$\underline{a}^e(\omega_i)$	+			
$\overline{a}^e(\omega_i)$		–		
$\underline{b}^e(\omega_i)$	+			
$\overline{b}^e(\omega_i)$		–		
$\underline{a}^o(\omega_i)$			+	
$\overline{a}^o(\omega_i)$				–
$\underline{b}^o(\omega_i)$			+	
$\overline{b}^o(\omega_i)$				–

Table 2: Unstability Table

$\underline{a}^e(\omega_i)$	+			
$\overline{a}^e(\omega_i)$		–		
$\underline{b}^e(\omega_i)$		+		
$\overline{b}^e(\omega_i)$	–			
$\underline{a}^o(\omega_i)$			+	
$\overline{a}^o(\omega_i)$				–
$\underline{b}^o(\omega_i)$				+
$\overline{b}^o(\omega_i)$			–	

Now we give upper bound for ε which guaranties that for each approximate root ω_i satisfying (8) at least one column of the Table 1 or Table 2 occurs (actually at most two column may occur).

First of all we define the cutoff frequency $\omega_c > 0$ such that for all $\omega > \omega_c$ the vectors $a(\omega)=(a^e(\omega),a^o(\omega))$ and $b(\omega)=(b^e(\omega),b^o(\omega))$ are not colinear. Since the colinearity is given by Eqn.(4) one can take ω_c to be the number, greater than largest positive root of (4). In the sequel we assume that

$$0 \le \omega \le \omega_c. \tag{16}$$

Let i be fixed. Denote

$$\begin{array}{lllllll} D_1 & = & a^e(\omega_i^*)-\underline{a}^e(\omega_i) & , & D_2 & = & \overline{a^e}(\omega_i)-a^e(\omega_i^*) \\ D_3 & = & a^o(\omega_i^*)-\underline{a}^o(\omega_i) & , & D_4 & = & \overline{a^o}(\omega_i)-a^o(\omega_i^*) \\ D_5 & = & b^e(\omega_i^*)-\underline{b}^e(\omega_i) & , & D_6 & = & \overline{b^e}(\omega_i)-b^e(\omega_i^*) \\ D_7 & = & b^o(\omega_i^*)-\underline{b}^o(\omega_i) & , & D_8 & = & \overline{b^o}(\omega_i)-b^o(\omega_i^*) \end{array}$$

(recall that, ω_i^* and ω_i is exact and approximate roots respectively satisfying (8)). After direct calculations we get the following

Proposition 2.2. Let

$$\max\{a_0,a_1,\ldots,a_n,b_0,b_1,\ldots,b_n\}=M$$

$$\max\left\{\frac{2\omega_c-(n+2)\omega_c^{n+1}+n\omega_c^{n+3}}{(1-\omega_c^2)^2},\frac{1+\omega_c^2-(n+1)\omega_c^n+(n-1)\omega_c^{n+2}}{(1-\omega_c^2)^2}\right\}=K_1$$

$$\max\left\{\frac{2\omega_c-(n+1)\omega_c^n+(n-1)\omega_c^{n+2}}{(1-\omega_c^2)^2},\frac{1+\omega_c^2-(n+2)\omega_c^{n+1}+n\omega_c^{n+3}}{(1-\omega_c^2)^2}\right\}=K_2$$

Then for all $p=1,2,\ldots,8$ and for all i

$$0\le D_p\le 2MK_1\varepsilon \quad (\text{if } n \text{ is even}) \ , \quad 0\le D_p\le 2MK_2\varepsilon \quad (\text{if } n \text{ is odd})$$

Now, for the stable polynomial $a(s)$ (and $b(s)$) we define lower bound for the distance of the curve $a(j\omega)$ $(0\le\omega\le\omega_c)$ from origin (it is well-known that as ω goes from 0 to $+\infty$ the curve $a(j\omega)$ will cut counterclockwisely the real and imaginary axes alternately a total of n times).

Let t be a natural number such that

$$\frac{\pi}{4}(t-1)\le \arg a(j\omega_c)\le\frac{\pi}{4}t,$$

and $\omega^0=0,\omega^1,\omega^2,\ldots,\omega^t=\omega_c$ be real numbers satisfying

$$\omega^1<\omega^2<\ldots<\omega^{t-1}<\omega_c \ , \ (q-1)\frac{\pi}{4}<\arg a(j\omega^q)<q\frac{\pi}{4} \quad (q=1,2,\ldots,t).$$

Denote by L_q the straight line passing from $a(j\omega^{q-1})$ and $a(j\omega^q)$ and by d_a^q the distance from origin to L_q.

Let $d_a=\min\limits_q d_a^q$. Then, by arc convexity theorem [1, p.316]

$$|a(j\omega)|\ge d_a \quad (0\le\omega\le\omega_c).$$

Similarly, one may choose d_b such that

$$|b(j\omega)|\ge d_b \quad (0\le\omega\le\omega_c).$$

Proposition 2.3. Let two-dimensional vectors $a=(a_1,a_2)$, $b=(b_1,b_2)$ are colinear and

$$\|a\| \geq d \quad , \quad \|b\| \geq d$$

then at least one of the following statements is valid:

1. $|a_1| \geq \frac{d}{\sqrt{2}}$ and $|b_1| \geq \frac{d}{\sqrt{2}}$ 2. $|a_2| \geq \frac{d}{\sqrt{2}}$ and $|b_2| \geq \frac{d}{\sqrt{2}}$

The proof is obvious.

Proposition 2.4. Let $d = \min\{d_a, d_b\}$ and ε is chosen such that

$$2MK_1\varepsilon < \frac{d}{\sqrt{2}} \quad \text{(if } n \text{ is even)}$$

or (17)

$$2MK_2\varepsilon < \frac{d}{\sqrt{2}} \quad \text{(if } n \text{ is odd)}$$

Then for each ω_i at least one column of the Table 1 or Table 2 occurs.

If ε satisfies (17) then

$$0 \leq D_p < \frac{d}{\sqrt{2}} \quad (p = 1,2,\ldots,8). \tag{18}$$

Proposition 2.5. Let ε be arbitrary positive number, satisfying (17). Then

1. Segment is stable $\Leftrightarrow$ for all $i = 1,2,\ldots,k$ at least one column of the Table 1 occurs.
2. Segment is unstable $\Leftrightarrow$ there exist i such that at least one column of the Table 2 occurs.

Proof. 1. $\Rightarrow$). Assume that segment is stable. For all $i = 1,2,\ldots,k$ the vectors $a(\omega_i^*) = (a^e(\omega_i^*), a^o(\omega_i^*))$, $b(\omega_i^*) = (b^e(\omega_i^*), b^o(\omega_i^*))$ are colinear. Let, for example, statement 1) from Proposition 2.3 is valid:

$$|a^e(\omega_i^*)| \geq \frac{d}{\sqrt{2}} \quad , \quad |b^e(\omega_i^*)| \geq \frac{d}{\sqrt{2}}. \tag{19}$$

Since segment is stable, then $a^e(\omega_i^*)$ and $b^e(\omega_i^*)$ have the same sign and assertion follows from (18), (19).

$\Leftarrow$). By (11)-(14) we obtain that for all $i = 1,2,\ldots,k$ either the inequality

$$a^e(\omega_i^*)b^e(\omega_i^*) > 0 \tag{20}$$

or the inequality

$$a^o(\omega_i^*)b^o(\omega_i^*) > 0 \tag{21}$$

holds. If (20) holds then by (4) $a^o(\omega_i^*)b^o(\omega_i^*) \geq 0$. If (21) holds then also by (4) $a^e(\omega_i^*)b^e(\omega_i^*) \geq 0$. In each cases by Proposition 2.1 segment is not unstable and thus it is stable.

2. $\Rightarrow$). Assume that segment is unstable. By Proposition 2.1 and Proposition 2.3 there exist i such that

$$a^e(\omega_i^*)b^e(\omega_i^*) < 0 \text{ and } |a^e(\omega_i^*)| \geq \frac{d}{\sqrt{2}} \quad , \quad |b^e(\omega_i^*)| \geq \frac{d}{\sqrt{2}}$$

or

$$a^o(\omega_i^*)b^o(\omega_i^*)<0 \text{ and } \left|a^o(\omega_i^*)\right| \ge \frac{d}{\sqrt{2}}, \quad \left|b^o(\omega_i^*)\right| \ge \frac{d}{\sqrt{2}}$$

holds. Then assertion follows from (18).

The implication $\Leftarrow$) follows from Proposition 2.1 and inequalities (11)-(14).

Suppose that the segment is unstable. Then there exists i such that at least one column of the Table 2 occurs (Proposition 2.5). This i corresponds the value (see (7))

$$\lambda(\omega_i^*) = \frac{a^e(\omega_i^*)}{a^e(\omega_i^*) - b^e(\omega_i^*)} \quad \text{or} \quad \lambda(\omega_i^*) = \frac{a^o(\omega_i^*)}{a^o(\omega_i^*) - b^o(\omega_i^*)}$$

for which the polynomial $a(s,\lambda(\omega_i^*))$ has pure imaginary root $s = j\omega_i^*$ (recall that ω_i^* is exact root of (4) satisfying (5), (6)). In view of (11)-(14) we have the following

Proposition 2.6. Let i be an index for which at least one of the columns of Table 2 occurs. Then,

If 1. column of the Table 2 occurs then $\lambda(\omega_i^*) \in \left[\dfrac{\underline{a}^e(\omega_i)}{\underline{a}^e(\omega_i) - \underline{b}^e(\omega_i)}, \dfrac{\overline{a}^e(\omega_i)}{\overline{a}^e(\omega_i) - \overline{b}^e(\omega_i)} \right]$

If 2. column occurs then $\lambda(\omega_i^*) \in \left[\dfrac{\overline{a}^e(\omega_i)}{\overline{a}^e(\omega_i) - \overline{b}^e(\omega_i)}, \dfrac{\underline{a}^e(\omega_i)}{\underline{a}^e(\omega_i) - \underline{b}^e(\omega_i)} \right]$

If 3. column occurs then $\lambda(\omega_i^*) \in \left[\dfrac{\underline{a}^o(\omega_i)}{\underline{a}^o(\omega_i) - \underline{b}^o(\omega_i)}, \dfrac{\overline{a}^o(\omega_i)}{\overline{a}^o(\omega_i) - \overline{b}^o(\omega_i)} \right]$

If 4. column occurs then $\lambda(\omega_i^*) \in \left[\dfrac{\overline{a}^o(\omega_i)}{\overline{a}^o(\omega_i) - \overline{b}^o(\omega_i)}, \dfrac{\underline{a}^o(\omega_i)}{\underline{a}^o(\omega_i) - \underline{b}^o(\omega_i)} \right]$

The intervals above depend on ε and their lengths are sufficiently small. Therefore these intervals give approximate values of $\lambda(\omega_i^*)$ and its error.

A vector $(p_0, p_1, \ldots, p_n) \in R^{n+1}$ with positive coordinates is said to belong to Hurwitz region of R^{n+1} if the polynomial

$$p(s) = p_0 + p_1 s + \ldots + p_n s^n \tag{22}$$

is stable.

In the case of unstability Proposition 2.6 allows us to calculate approximate values of all λ and its errors for which the polynomial $a(s,\lambda)$ has pure imaginary roots. This, in turn, enables us to determine all values of λ for which the segment acrosses the boundary of Hurwitz region.

Proposition 2.7. If the vector $(p_0, p_1, \ldots, p_n)$ belongs to the boundary of Hurwitz region then the polynomial (22) has pure imaginary root $s = j\omega$.

Proof is obvious.

Let ε be arbitrary number, satisfying (17). Denote by $\{i_1, i_2, \ldots, i_m\}$ the set of all i for which at least one of the column of the Table 2 occurs. Then by Proposition 2.6 each $i \in \{i_1, i_2, \ldots, i_m\}$ corresponds at least one interval $[\underline{\lambda}_i, \overline{\lambda}_i]$ which contains the exact value $\lambda(\omega_i^*)$. If for some i more than one column of Table 2 occurs then such i corresponds more than one interval. In this case it must be taken an intersection of these intervals (this intersection is nonempty, because it contains $\lambda(\omega_i^*)$). Therefore, for a given ε, each $i \in \{i_1, i_2, \ldots, i_m\}$ corresponds a unique interval $[\underline{\lambda}_i, \overline{\lambda}_i]$.

Suppose that these intervals do not intersect each other (this corresponds to the case when the polynomial $a(s,\lambda)$ has no two different pure imaginary roots $s = j\omega_1$, $s = j\omega_2$, $\omega_1 > 0$, $\omega_2 > 0$) and are written in the increasing order. Choose arbitrary

$$\lambda_1 \in (\overline{\lambda}_1, \underline{\lambda}_2), \lambda_2 \in (\overline{\lambda}_2, \underline{\lambda}_3), \ldots, \lambda_{m-1} \in (\overline{\lambda}_{m-1}, \underline{\lambda}_m),$$

and check for stability or unstability of polynomials $a(s,\lambda_1), a(s,\lambda_2), \ldots, a(s,\lambda_{m-1})$. Then, conclude about segment's behavior. For example, suppose $m = 3$ and $a(s,\lambda_1)$ is stable, $a(s,\lambda_2)$ is unstable. Then:

1. The segment starting from $a(s) = a(s,0)$ is an inner tangent of the boundary of the Hurwitz region for some value $\lambda \in [\underline{\lambda}_1, \overline{\lambda}_1]$,
2. For some $\lambda \in [\underline{\lambda}_2, \overline{\lambda}_2]$ the segment leaves the Hurwitz region,
3. For some $\lambda \in [\underline{\lambda}_3, \overline{\lambda}_3]$ the segment re-enters the Hurwitz region.

The algorithm

1. For given stable polynomials $a(s)$ and $b(s)$ write the Eqn.(4).
2. If the Eqn.(4) has no positive root then the segment is stable. Suppose that the Eqn.(4) has at least one positive root. Choose an appropriate ε (see (17)). Find all approximate, positive distinct roots $\omega_1, \omega_2, \ldots, \omega_k$ of the equation (4) with absolute error ε.
3. Calculate the numbers (10) for all $i = 1, 2, \ldots, k$. If for all ω_i at least one column of the Table 1 occurs then the segment is stable. If there exists ω_i such that at least one column of the Table 2 occurs then the segment is unstable.
4. Suppose the segment is unstable and let $\{i_1, i_2, \ldots, i_m\}$ denote the set of all i for which at least one of column of the Table 2 occurs (remember that ε is chosen such that for each ω_i at least one column of the Table 1 or Table 2 occurs). For each $i \in \{i_1, i_2, \ldots, i_m\}$ calculate the corresponding interval $[\underline{\lambda}_i, \overline{\lambda}_i]$ which contains $\lambda(\omega_i^*), (i = 1, 2, \ldots, m)$. Suppose that these intervals are disjoint and are written in the increasing order.
5. Choose arbitrary $\lambda_1 \in (\overline{\lambda}_1, \underline{\lambda}_2), \lambda_2 \in (\overline{\lambda}_2, \underline{\lambda}_3), \ldots, \lambda_{m-1} \in (\overline{\lambda}_{m-1}, \underline{\lambda}_m)$, and check for stability or unstability of polynomials $a(s,\lambda_1), a(s,\lambda_2), \ldots, a(s,\lambda_{m-1})$. Then conclude about segment behavior.

3. EXAMPLES

Example 3.1. Consider two stable polynomials $a(s)$ and $b(s)$:

$$a(s) = 2 + 8s + 13s^2 + 6s^3 + 5s^4 + s^5,\ b(s) = 1 + 4s + 5s^2 + 5s^3 + 3s^4 + s^5.$$

The Eqn.(4) has the form $2\omega^9 - 15\omega^7 + 32\omega^5 - 16\omega^3 = 0$ and has three positive roots;

$$\omega_1^* = 2,\quad \omega_2^* = 0.848070512\ldots,\quad \omega_3^* = 1.667566013\ldots$$

and acceptable cutoff frequency is $\omega_c = 2$.

Simple computations yield $M = 13$, $K_2 = 93$, $d = \frac{1}{\sqrt{2}}$, $\varepsilon < 0.0002$.

Therefore $\varepsilon = 0.0001$ is acceptable. For $\varepsilon = 0.0001$ the approximate roots are

$$\omega_1 = 2,\ \omega_2 = 0.8480,\ \omega_3 = 1.6675$$

By calculating the expressions (10) we conclude that for each ω_1, ω_2 and ω_3 at least one column of the Table 1 occurs, and therefore the segment is stable.

Example 3.2.[4] Consider the following stable polynomials

$$a(s) = 3 + s + 5s^2 + s^3 + s^4,\ b(s) = 1 + 2s + 3s^2 + 5s^3 + s^4.$$

The Eqn.(4) has the following form

$$4\omega^7 - 23\omega^5 + 21\omega^3 - 5\omega = 0$$

and has three positive roots;

$$\omega_1^* = 0.6367324701...,\ \omega_2^* = 0.8110794934...,\ \omega_3^* = 2.164883788...$$

Therefore the acceptable cutoff frequency is $\omega_c = 2.5$. Computation gives $M = 5$, $K_1 = 67.5$, and $d_a = 0.03270685678$, $d_b = 0.2461069279$.

With $d = \min\{d_a, d_b\} = 0.03270685678$ (17) gives $\varepsilon < 0.0000342...$, therefore $\varepsilon = 0.00001$ is acceptable. For this error, approximate roots are

$$\omega_1 = 0.63673,\ \omega_2 = 0.81107,\ \omega_3 = 2.16488$$

After calculating the expressions (10) we get that for ω_3 at least one column of the Table 1 occurs and for ω_1 and ω_2 at least one column of the Table 2 occurs. Using Proposition 2.6 we obtain

$$[\underline{\lambda}_1, \overline{\lambda}_1] = [0.209661605, 0.209941126],\ [\underline{\lambda}_2, \overline{\lambda}_2] = [0.9563087719, 0.956392263]$$

For $\lambda = 0.5 \in (\overline{\lambda}_1, \underline{\lambda}_2)$ convex combination $a(s,\lambda) = (1-\lambda)a(s) + \lambda b(s)$ is unstable and we conclude that segment is unstable and starting from $a(s,0) = a(s)$ leaves the Hurwitz region for some $\lambda \in [\underline{\lambda}_1, \overline{\lambda}_1]$ and re-enters for some $\lambda \in [\underline{\lambda}_2, \overline{\lambda}_2]$.

4. CONCLUSION

In this paper, we presented an algorithm for testing on stability-unstability of polynomial segments. The algorithm is based on the solving for the positive approximate roots of a polynomial equation.

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