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ON THE BUCKLING ANALYSIS OF PLATES REINFORCED WITH STIFF RIBS

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Abstract: This paper deals with analytical buckling analysis of the rectangular orthotropic simply supported plates reinforced with longitudinal stiff ribs. It was considered that the ribs with rectangular and circular cross-sections were parallel and equivalent to each other. It was further assumed that the ribs were rigidly fastened to the plate and that their cross sections were perpendicular to the undeformed middle surface of the plate. These reinforced plates were subjected to the uniaxial uniform distributed compressive loads. In the study, the effect of the numbers and the cross-sections of the ribs on the buckling load were investigated. The results are striking in that how the ribs effect the isotropic plates with low strength.

Key words: Buckling, reinforced plate, stiff ribs.

1. INTRODUCTION

Stiffened structures are the principal structural components of ships, submarines, aircrafts and similar structures, which operate under harsh environmental loading conditions and which need lighter and stronger structural materials. Buckling is one of the important failure mechanisms under these conditions. Therefore, with the increasing demand for lighter and stronger structures, the use of advanced composite materials is unavoidable and the research for ways to fully exploit their properties continues.

Several researchers over the past decades made an extensive study on buckling of composite plates. Kumar and Mukhopadhyay [1] developed a new stiffened plate element based on first order shear deformation theory, which is a combination of Allman's plane stress triangular element and Discrete Kirchhooff-Mindlin triangular plate bending element. Xu and Reifsnider [2] investigated composite compressive failure using a micromechanical model and found that a complete matrix slippage had reduced the composite longitudinal strength. Chattopadhyay and Gu [3] studied an exact elasticity solution for buckling of composite laminates. Tung and Surdenas [4] examined buckling of simply supported rectangular orthotropic plates under biaxial loading. An approximate solution for isotropic plates stiffened by a number of equivalent ribs was shown by Timoshenko [5].

Since composite materials have the ability to tailor many properties, such as a high ratio of stiffness, weight, strength, thermal properties, corrosion resistance, wear resistance, and fatigue life necessitates, many researches on composites have been performed and will be continuing in the future. Therefore, the main emphasis is on the manufacturing techniques aiming at the development of stronger and stiffer materials with lower density. One of the ways achieving this goal is to utilize metal fiber-metal matrix plates consisting of a low ductile, usually low strength metal matrix, and elastic, ductile and strong metal fibres. The strength and the stiffness of the fiber and the ductility of the matrix provide a new material with superior properties. In this study, buckling analysis of the plates reinforced with ribs was carried out with an analytical approach. The ribs used in this investigation have circular and rectangular cross-sections. The effects of the rib cross-sections on critical buckling load were studied. It is found that composite plates reinforced with stiff ribs have more resistance to buckling than that of pure matrix plates.

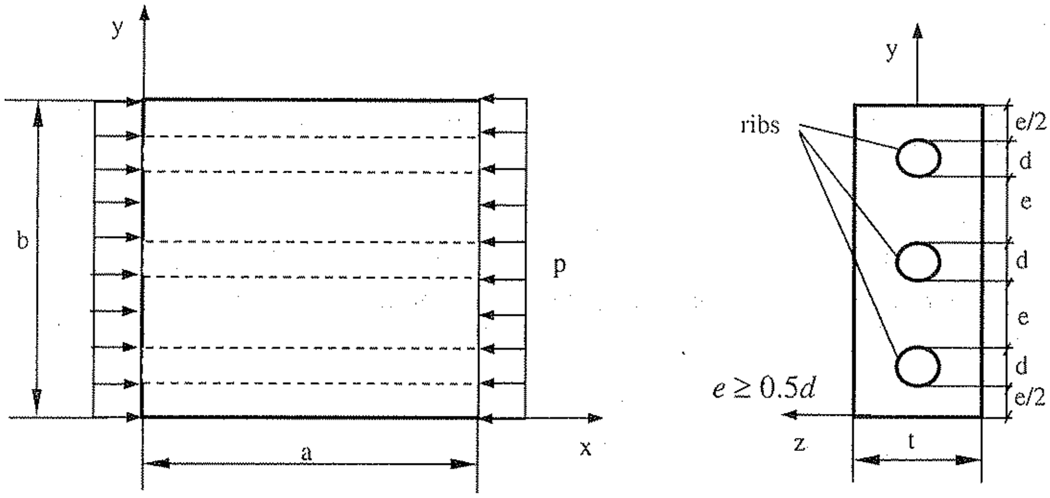

Figure 1. Loading case and geometry of a plate reinforced with cylindrical ribs.

2. MATHEMATICAL FORMULATION

A plate reinforced with many parallel ribs may be considered as orthotropic and homogeneous. If it is reinforced with only a few ribs (one or two), it can not be considered orthotropic and homogeneous. In this work, we considered a homogeneous and orthotropic rectangular plate with principal directions parallel to the sides strengthened by some parallel stiff ribs. The geometry of the plate reinforced with ribs circular and rectangular-cross sections are shown in Figure 1 and 2. The orthotropic rectangular plates with two edges simply supported *(y=0, y=b)* are exposed to uniformly distributed normal compressive forces on these edges.

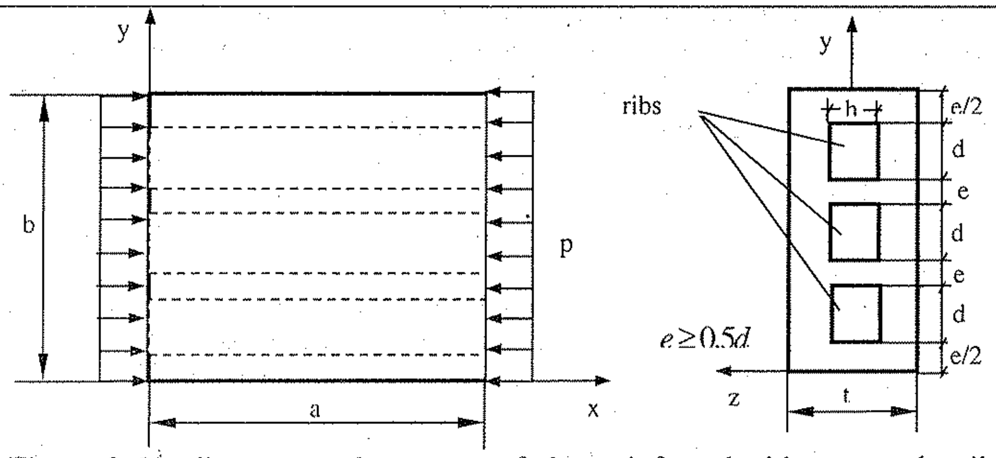

Figure 2. Loading case and geometry of plate reinforced with rectangular ribs

Taking into consideration the combined work done on the plate and on the rib, the buckling analysis was carried out [8]. While the plate is subjected to a compressive load p, each rib is subjected to compressive force P_k on its ends. Therefore, the rib force can readily be found because it is postulated that the compressive strain ε_x in the plate is the same as the compressive strain ε_{xk} in any rib. Obviously,

$$\varepsilon_x = -\frac{p}{E_1}, \qquad \varepsilon_{xk} = -\frac{P_k}{E_f A_f} \qquad \text{and} \quad \varepsilon_x = \varepsilon_{xk} \Rightarrow P_k = p\frac{E_f A_f}{E_1} \tag{1}$$

where E_1 is the Young's modulus of the plate parallel to the ribs, E_f, A_f and $E_f A_f$ the Young's moduli, the cross-sectional area, and the axial rigidity for the ribs, respectively. In the solution of the problem, we assumed that the deflection of the plate, the deflection and the twist of the ribs were small deflections. Denoting by $w(x,y)$ the deflection of the plate, by $W_k(x)$, $\theta_k(x)$ the deflection and rotation of the ribs, we have by virtue of the rigid connection between the plate and the ribs:

$$W_k(x) = w(x,\eta_k), \quad \theta_k(x) = \left(\frac{\partial w}{\partial y}\right)_{y=\eta_k} \tag{2}$$

The potential energy of the system increases by the amount

$$V_b = \frac{1}{2}\int_0^a\int_0^b\left[D_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_1\nu_2\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + D_2\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_k\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]dx\,dy$$

$$+\frac{1}{2}\sum_{k=1}^{n}\left[(E_f I_f)_k\int_0^a (W_k'')^2 dx + (G_f J_f)_k \int_0^a (\theta_k')^2 dx\right] \tag{3}$$

D_1, D_2 and D_k are the bending stiffnesses for the plate and given as

$$D_1 = 2\int_0^{\frac{h}{2}}\frac{E_1 z^2}{1-\nu_1\nu_2}dz, \qquad D_2 = 2\int_0^{\frac{h}{2}}\frac{E_2 z^2}{1-\nu_1\nu_2}dz \qquad \text{and} \qquad D_k = 2\int_0^{\frac{h}{2}} G_{12}dz$$

where E_1, E_2 and G_{12} are Young's moduli and shear modulus of the composite plate in the x-y directions, are given as

$$E_1 = E_f V_f + E_m V_m, \qquad E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \qquad \text{and} \qquad G_{12} = \frac{G_f G_m}{G_f V_m + G_m V_f}$$

and $(E_f I_f)_k$ and $(E_f J_f)_k$ are the bending and torsional rigidities for the ribs, respectively. It decreases by the amount of work done by the external forces

$$A = \frac{p}{2}\int_0^a\int_0^b\left(\frac{\partial w}{\partial x}\right)^2 dx\,dy + \frac{1}{2}\sum_{k=1}^{n}P_k\int_0^a (W_k')^2 dx \tag{4}$$

Conditions at the supported (loaded) edges of the plate and at the ends of the ribs are satisfied by assuming ($m=1,2,3..$)

$$w = f(y)\sin\frac{m\pi}{a}x \ \text{and} \qquad W_k = f(\eta_k)\sin\frac{m\pi}{a}x, \qquad \theta_k = f'(\eta_k)\sin\frac{m\pi}{a}x$$

where the function f in the form of a series with undetermined coefficients for a plate with supported edge a can be written as

$$f=\sum_n A_{mn} f_n(y)=A_{mn}\sin\frac{n\pi}{b}y$$

where f_n are continuous functions that satisfy conditions at $y=0,\quad y=b$.

From equation $V_b=A$ we get

$$p=\frac{\pi^2\sqrt{D_1D_2}}{b^2(1+2\delta)}\left[\left(\sqrt{\frac{D_1}{D_2}}+C_1\right)\left(\frac{m}{c}\right)^2+2\left(\frac{D_3}{\sqrt{D_1D_2}}+\chi\right)n^2+\sqrt{\frac{D_2}{D_1}}n^4\left(\frac{c}{m}\right)^2\right] \tag{5}$$

where if $n=1,3,5,\ldots$ then, $C_1=2\nu$, $\chi=0$ and if $n=2,4,6,\ldots$ then $\delta=1$, $C_1=0$. The critical load P_{cr} is the smallest of all the values found from formula (5). Obviously the smallest value from (5) is obtained for *n=1*, which corresponds to a half-wave, or for *n=2*, which corresponds to two half-waves in the *y*-direction. It is impossible to decide which of the two values (*n=1* or *2*) must be used. This depends on ratios $\sqrt{D_1/D_2}$, $D_3/\sqrt{D_1D_2}$, χ, δ, ν. The following notations are introduced for brevity:

$$k_{m1}=\frac{1}{1+2\delta}\left[\left(\sqrt{\frac{D_1}{D_2}}+2\nu\right)\left(\frac{m}{c}\right)^2+\frac{2D_3}{\sqrt{D_1D_2}}+\sqrt{\frac{D_2}{D_1}}\left(\frac{c}{m}\right)^2\right],\qquad r_1=\sqrt[4]{\frac{D_1}{D_2}+2\nu\sqrt{\frac{D_1}{D_2}}},$$

$$k_{m2}=\sqrt{\frac{D_1}{D_2}}\left(\frac{m}{c}\right)^2+8\left(\frac{D_3}{\sqrt{D_1D_2}}+\chi\right)+16\sqrt{\frac{D_2}{D_1}}\left(\frac{c}{m}\right)^2,\qquad \overline{k_2}=8\left(\frac{D_3}{\sqrt{D_1D_2}}+\chi+1\right)$$

$$\overline{k_1}=\frac{2}{1+2\delta}\left(\sqrt{1+2\nu\sqrt{\frac{D_1}{D_2}}}+\frac{2D_3}{\sqrt{D_1D_2}}\right),\qquad r_2=\sqrt[4]{\frac{D_1}{D_2}},\qquad \chi=\frac{(G_fJ_f)}{b\sqrt{D_1D_2}}$$

The formula for critical load is of the form

$$p_{cr}=\frac{\pi^2\sqrt{D_1D_2}}{b^2}k \tag{6}$$

The results of the present investigation may be presented as fallows:

1. If the ratio of sides of the plate satisfies condition $c=mr_1$ then $k=\overline{k_1}$
2. If the ratio of sides satisfies $c=0.5mr_2$ then $k=\overline{k_2}$
3. For larger ratios of sides $c\geq 3$ then

$k=\overline{k_1}$ if $\overline{k_1}<\overline{k_2}$ or $k=\overline{k_2}$ if $\overline{k_1}>\overline{k_2}$

These notations may be conveniently presented in Table 1, which includes values of *c* and the corresponding values of *k* and *m* for the cases of *n=1* and *n=2*.

If the rigidity ratio D_1/D_2, χ, and ν are known, the critical load for a plate with a given side *c* can be determined. In order to find the corresponding *k*, we establish the place in the table where the given value of *c* is found for both *n=1* and *n=2*. After determining the lines where *c* is found, the corresponding values of *k* are taken from the column; of the two values, the smaller must be selected. Which value is smaller depends on the plate parameters.

Table 1

n	c	m	k	n	c	m	k
1	$0<c<r_1$	1	k_{11}	2	$0<c<0.5r_2$	1	k_{12}
1	$c=r_1$	1	$\overline{k_1}$	2	$c=0.5r_2$	1	$\overline{k_2}$
1	$r_1<c<\sqrt{2}r_1$	1	k_{11}	2	$0.5r_2<c<0.5\sqrt{2}r_2$	1	k_{12}
1	$\sqrt{2}r_1<c<2r_1$	2	k_{21}	2	$0.5\sqrt{2}r_2<c<r_2$	2	k_{22}
1	$c=2r_1$	2	$\overline{k_1}$	2	$c=r_2$	2	$\overline{k_2}$
1	$2r_1<c<\sqrt{6}r_1$	2	k_{21}	2	$r_2<c<0.5\sqrt{6}r_2$	2	k_{22}
1	$\sqrt{6}r_1<c<3r_1$	3	k_{31}	2	$0.5\sqrt{6}r_2<c<1.5r_2$	3	k_{32}
1	$c=3r_1$	3	$\overline{k_1}$	2	$c=1.5r_2$	3	$\overline{k_2}$

3. NUMERICAL RESULTS AND DISCUSSION

To observe the effect of the stiff ribs on the critical buckling load, the buckling of the pure matrix and rib materials should be examined. For the isotropic materials, buckling formulation [7] is given as

$$N_{cr}=\frac{\pi^2 E t^3}{12b^2(1-\nu^2)}k \qquad k=\left(\frac{b}{a}+\frac{a}{b}\right)^2 \tag{7}$$

where E, *and* ν are elasticity moduli and Poisson's ratio of isotropic material, and a, b and t are the dimensions of the isotropic plates. For pure aluminium and steel plates of 250x250x5 mm, the critical buckling loads are found as 506 N/mm and 1419.4 N/mm, respectively. While aluminium is light but low resisting, steel is heavy but strong. If those two materials are appropriately composited, both light and stronger composite materials can be produced.

In this study, aluminium plates are reinforced with stiff steel ribs with rectangular and circular cross-sections. The effects of those ribs on buckling load are given in tables, where the buckling loads are non-dimensionized by dividing the buckling loads of composite plates to that of the pure matrix material (aluminium). Material properties are taken as E_f=200 GPa and $\nu_f=0.27$ for steel, and E_m=70 Gpa and $\nu_m=0.30$ for aluminium.

Tables 2-5 give the non-dimensionized buckling loads for the square composite plates reinforced with rectangular cross-sectional ribs under uniform uniaxial compression. In tables, first lines (d=1, 1.5, 1.75 ....20 mm.) represent the widthes of the rectangular ribs, and the first column (nr = 8, 16, 20,...) the numbers of ribs.

In these tables, it was found that while the rib widthes were getting larger, their numbers were diminishing in each table (for example: for d=1 mm, nr=166; for d=4 mm, nr=41; for d=10 mm, nr=16). Similarly, from table to table, while the rib thicknesses were groving, less ribs were used in composite plates (for example, for h=1 mm, nr=166; for h=1.25 mm, nr=133; and for h=2.5 mm, nr=66). nr shows numbers of ribs

Table 2. Non-dimensionized buckling loads for square composite plate reinforced with rectangular ribs under uniform uniaxial compression. (h=1 mm, t/h=5)

nr	1.000	1.500	1.750	2.000	2.500	3.000	4.000	6.000	8.000	10.000
16	1.283	1.378	1.418	1.455	1.519	1.575	1.667	1.806	1.915	2.008
20	1.333	1.437	1.480	1.519	1.587	1.646	1.742	1.890	2.008	
27	1.408	1.523	1.570	1.612	1.684	1.746	1.850	2.014		
41	1.527	1.654	1.705	1.751	1.830	1.899	2.019			
55	1.618	1.753	1.807	1.857	1.944	2.022				
66	1.677	1.818	1.875	1.927	2.022					
83	1.755	1.904	1.967	2.024						
95	1.803	1.959	2.025							
111	1.860	2.026								
166	2.024									

Table 3. Non-dimensionized buckling load for square composite plate reinforced with rectangular ribs under uniform uniaxial compression. (h=1.25 mm, t/h=4)

nr	1.000	1.500	1.750	2.000	2.500	3.000	4.000	6.000	8.000	10.000
13	1.336	1.441	1.484	1.523	1.592	1.650	1.747	1.895	2.014	2.119
16	1.388	1.500	1.546	1.587	1.658	1.719	1.820	1.979	2.111	
22	1.474	1.596	1.646	1.690	1.766	1.832	1.943	2.126		
33	1.596	1.730	1.783	1.832	1.917	1.992	2.126			
44	1.690	1.832	1.890	1.943	2.039	2.126				
53	1.753	1.902	1.965	2.022	2.128					
66	1.832	1.992	2.062	2.126						
76	1.885	2.055	2.130							
88	1.943	2.126								
133	2.130									

Table 4. Non-dimensionized buckling load for square composite plate reinforced with rectangular ribs under uniform uniaxial compression. (h=1.667mm, t/h=3)

nr	1.000	1.500	1.750	2.000	2.500	3.000	4.000	6.000	8.000
10	1.415	1.530	1.577	1.619	1.692	1.754	1.858	2.023	2.163
12	1.465	1.586	1.635	1.678	1.754	1.819	1.929	2.109	2.267
16	1.549	1.678	1.730	1.776	1.858	1.929	2.052	2.267	
25	1.692	1.834	1.892	1.945	2.042	2.129	2.292		
33	1.787	1.941	2.006	2.067	2.179	2.286			
40	1.858	2.023	2.095	2.163					
50	1.945	2.129	2.212	2.292					
57	2.000	2.199	2.290						
66	2.067	2.286							
100	2.292								

Table 5. Non-dimensionized buckling load for square composite plate reinforced with rectangular ribs under uniform uniaxial compression. (h=2.5mm, t/h=2)

nr	1.000	1.500	1.750	2.000	2.500	3.000	4.000	6.000	8.000
8	1.582	1.713	1.766	1.814	1.898	1.972	2.102	2.334	2.556
11	1.684	1.825	1.883	1.936	2.031	2.118	2.278	2.584	
16	1.814	1.972	2.039	2.102	2.221	2.334	2.556		
22	1.936	2.118	2.199	2.278	2.431	2.584			
26	2.006	2.207	2.299	2.390	2.570				
33	2.118	2.355	2.469	2.584					
38	2.192	2.459	2.590						
44	2.278	2.584							
66	2.584								

In Table 6, the non-dimensionized buckling loads are given for the square orthotropic plates reinforced with cylindirical ribs under uniform uniaxial compression. In the same table, the first line (d=1, 1.25, 1.667, 2, 2.5, 3.125, 4 mm.) represents the diameter of the circular ribs, and the first column (nr = 10, 25, 40,...) the numbers of ribs. The other quantities are non-dimensionized buckling loads calculated by dividing the buckling loads of composite plates to that of the pure matrix material (aluminium).

Also in this table, it was found that the number of ribs were decreasing (for example, for d=1 mm, nr=140; for d=2 mm, nr=83; for d=4 mm, nr=40) as the enlargement of rib diameters. As seen from the table, the plates reinforced with fewer ribs and larger diameter have a higher resistance to buckling than those with too many ribs and small diameter.

Table 6. Non-dimensionized buckling load for square composite plate reinforced with cylindiric ribs under uniform uniaxial compression.

nr	1.000	1.250	1.667	2.000	2.500	3.125	4.000
10	1.160	1.231	1.353	1.448	1.578	1.720	1.891
25	1.329	1.443	1.614	1.733	1.893	2.080	2.353
40	1.450	1.581	1.771	1.905	2.096	2.347	2.771
50	1.514	1.653	1.852	1,997	2.214	2.517	
75	1.640	1.791	2.016	2.194			
83	1.673	1.828	2.062	2.253			
90	1.700	1.858	2.101				
100	1.736	1.898	2.154				
111	1.772	1.940					
140	1.857						
166	1.923						

4. CONCLUSIONS

In this study, analytical buckling analysis of rectangular orthotropic simply supported plates reinforced with longitudinal stiff ribs were investigated. Parallel and equivalent ribs with rectangular and circular cross-sections were assumed in the composite plates, which are rigidly fastened to the plate and their cross sections are perpendicular to the undeformed middle surface of the plate. Being subjected to the uniaxial uniform distributed compressive loads, in the study, the effect of the numbers and the cross-sections of the ribs on the buckling load were investigated.

The results show that the composite plates reinforced with stiff ribs are more resistive to buckling than that of pure matrix plates. Particularly, plates with prismatic ribs are stronger in terms of buckling resistivity than the ones with cylinderical ribs. In the cases of both cylindrical and prismatic ribs, as the number of ribs increase, the buckling resistivity increases.

If the entries in Tables 2-5 are examined, it is observed that the plates reinforced with prismatic ribs according to the buckling loads may have too many ribs with smaller diameters or fewer ribs with larger diameters. As for the plates with circular ribs, the plates reinforced with fewer ribs and larger diameter have a higher resistance to buckling than those with too many ribs and small diameter. Therefore, for the plates with cylindrical ribs, it might be recommended to use fewer ribs with larger diameter rather than too many ribs with smaller diameter.

One disadvantage of using ribs with small cross-sections is that there will be no contribution of the rib stiffening to buckling resistivity due to the decrease in rib stiffness. However, there are some possible difficulties in production and in fastening of aluminium plate with steel ribs when ribs with larger cross-sections are used.

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