Mathematical & Computational Applications, Vol. 4, No. 2, pp. 169-174, 1999.
© Association for Scientific Research

OPTIMAL TRAJECTORY DETERMINATION FOR SAGITALLY SYMMETRIC MANUAL LIFTING TASKS

Omer Gundogdu and Kurt S. Anderson

Rensselaer Polytechnic Institute
Department of Mechanical Engineering, Aeronautical Engineering, & Mechanics
Troy, NY 12180

Abstract: A two dimensional, multi-link sagittally symmetric whole-body model was developed to simulate an optimal trajectory for manual material lifting. Non-linear optimal control techniques and genetic algorithms were utilized in the simulations to explore practical lifting patterns. The simulation results were then compared with the experimental data.

1. INTRODUCTION

Determination of muscle forces and joint moments is desirable for predicting possible low back injuries during manual material handling. Unfortunately, there is no device to directly measure muscle forces non-invasively [1]. Therefore, biomechanical modeling becomes a necessary tool for muscle stress analysis on the musculoskeletal system, particularly on the lumbar spine. These models also serve as an estimation tool for kinematics and kinetics of the motion [2].

Optimal control theory has recently been applied to the analysis of human locomotion by many researchers with the belief that it is a practical tool for explaining the control of the human musculoskeletal system, and as such it may be successfully used in predicting biodynamic behavior.

There are two primary reasons for using the optimal control techniques in the biodynamics modeling. Firstly, it is believed that locomotion obeys a certain "principle of optimality" [3-4]. Since optimal control theory aims to determine the control laws that will minimize (or maximize) a performance function subject to some physical constraints [5], it provides a means for determining actuator (muscle) torques. Secondly, a dynamic model should be established to predict the muscle forces and joint moments which produce the movement. However, the musculoskeletal system considered is highly redundant, i.e., the number of independent muscles acting on a particular joint exceeds the number of degrees of freedom of that joint. Moreover, many muscles can affect more than one joint at a time, which brings complex coupling to the system. So, there is no direct solution to the problem. Therefore, to overcome the above difficulties, optimal control techniques are popularly being used by researchers to estimate muscle forces [3,4,6].

2. EXPERIMENTS

Ten healthy male and ten healthy female subjects participated in this study after signing a consent form approved by the human subjects committee. Briefly, each subject lifted and lowered a two-handled box attached to the arm of the LIDOLift in the Biodynamics Laboratory of The Ohio State University. The lift cycle occurred in the sagittal plane of the subject. Each subject was instructed to lift and lower the box from as low as he/she could comfortably reach to waist height, for five continuous repetitions. Before the actual testing, the subject practiced at different loads, techniques, and movement times to gain familiarity with the equipment and testing protocol. Then, the tests were repeated for three (two for females) simulated loads, three techniques of lift, and three movement times of lift in a random order. The simulated masses for the study were 6.8, 13.6 and 20.5 kilograms (15, 30, and 45 pounds, respectively). Females did not perform the 20.5 kg lifts. The techniques were a self-selected, stoop (straight-knee), and squat (bent-knee) techniques of lifting. The movement times were 2, 4, and 6 seconds per cycle. The subject was paced to complete the lifts in these times by a metronome. Further analysis verified that the movement times were approximately 2, 4, and 6 seconds per lift. The 27 (18 for females) conditions within lift device were randomized for each subject.

The joint angular position data from the middle three cycles of each lifting condition were fit to 128 point curves and then averaged. This was performed so that a trial of any length time could be compared with any other trial. The angular position data were filtered with a 4th order Butterworth low-pass filter with a cutoff freq[illegible] Hz determined from residual analysis-[7]) and then numerically differentia[illegible] Taylor Series expansion to compute to angular velocity [8]. The same [illegible] repeated to compute the angular accelerations.

3. THEORY

[illegible] Model

[illegible] body was modeled as sagitally symmetric, two dimensional, and fi[illegible] [illegible] for the biomechanical simulation of manual lifting. These links po[illegible] the [illegible] mass, and inertia properties as estimated for their human [illegible]. Therefore, any movement or configuration could be described with fi[illegible] of these five links.

[illegible] of freedom revolute joints were assumed at each joints, nam[illegible] [illegible] elbow. Spinal column was considered as one rigid [illegible] included in the mass of back. The hands were also m[illegible] [illegible] relative motion with respect to forearms were ne[illegible] [illegible]

3.2. Dynamic Model

For a typical rigid link i in an n-link open chain mechanism, the joint reaction forces and joint moments can be obtained by utilizing the Newton-Euler recursive formulation in the following form

$$F_{x,i} = -F_{x,i+1} + m_i a_{x,i} \tag{3.2.1}$$

$$F_{y,i} = m_i g - F_{y,i+1} + m_i a_{y,i} \tag{3.2.2}$$

$$M_i = M_{i+1} + (F_{y,i+1} l_{i+1} + F_{y,i} l_i)Cos(\theta) - (F_{x,i+1} l_{i+1} + F_{x,i} l_i)Sin(\theta) + I_{zz}\ddot{\theta} \tag{3.2.3}$$

where $F_{x,i}$ and $F_{x,j}$ are forces at the joints i and i+1 in x-direction, likewise $F_{y,i}$ and $F_{y,j}$ forces at the joints i and i+1 in y-direction, M_i and M_{i+1} moments in opposite directions acting at joints i and i+1, m_i the mass of the link, l_i and l_{i+1} lengths from center of mass to joints i and i+1, respectively, $a_{x,i}$ and $a_{y,i}$ accelerations in x- and y- directions, g gravitational acceleration, I_{zz} mass moment of inertia about z axes (perpendicular to both x and y), and θ, θ', and θ'' angular displacement, velocity, and acceleration of the link, respectively.

3.3. Optimization

One of the most significant problems in optimization of biomechanical systems is the choice of a proper cost function reflecting most of the aspects of locomotion. In this paper, it was chosen to minimize "integration over the time of sum of the square of the ratio of the predicted joint moments to the corresponding joint dynamic strength". The joint strengths were considered to be measures of joint capacities under different postures and joint angular velocities.

$$J = \int_0^{t_f} \sum_{i=1}^{5} \left[\frac{M_i(\theta,\dot{\theta},\ddot{\theta})}{S_i(\theta,\dot{\theta})} \right]^2 dt \tag{3.3.1}$$

where t_f is the lifting duration, M_i moments and S_i joint dynamic strengths for the ith joint. The dynamic strength values were used in the objective function as opposed to static ones to better replicate the joint behavior and to improve the simulation. They were defined to be functions of joint angular positions and velocities for each joint i [13] in the following form

$$S_i(\theta,\dot{\theta}) = \beta_{i0} + \beta_{i1}\theta_i + \beta_{i2}\dot{\theta}_i + \beta_{i3}\theta_i^2 + \beta_{i4}\dot{\theta}_i^2 + \beta_{i5}\theta_i\dot{\theta}_i \tag{3.3.2}$$

The coefficients β_1 through β_5 determined based on experimental results were directly taken from [13].

3.4. Numerical Formulation of the Problem

The problem is highly nonlinear and an infinite dimensional one. One of the approaches to solve two point boundary value problems at that nature is to approximate the states and/or controls by a polynomial and/or a Fourier series [10-11]. For this study, joint angles were approximated as seventh order polynomial in the form

$$\theta_i = \sum_{j=0}^{7} a_{i,j}\, t^j \tag{3.4.1}$$

for the i[th] joint. Since the boundary conditions (initial and final angular positions, angular velocities, and angular accelerations) were known for a lifting experiment, six of the coefficients can be determined. The other two coefficients were added to the polynomials to introduce extra degree of freedom for optimization. By substituting these polynomials and their derivatives into equation (3.3.1), the problem becomes a finite dimensional parameter optimization of the form

$$J = \int_0^{t_f} f_1(a_{i,j}, t)dt \tag{3.4.2}$$

where i is the joint number, and j coefficient index of the polynomial. Since the lifting duration is known, the problem can further be simplified by discretization in integration time steps Δt as

$$\Delta t = \frac{t}{k} \tag{3.4.3}$$

where t is time, k is the number of integration steps. Then, the problem becomes minimizing another function in coefficients and integration step size

$$J = f_2(a_{i,j})\Delta t \tag{3.4.4}$$

A genetic algorithm implementing Goldberg's [12] algorithm in Matlab was used for optimizations. Once the coefficients in the polynomial are estimated, the optimized path for a lifting task can easily be determined. A sample result for a randomly selected subject (mass of 94.7 kg, height of 1.85 m) was given for his hip joint which is the most critical one in lifting in Fig. 1.

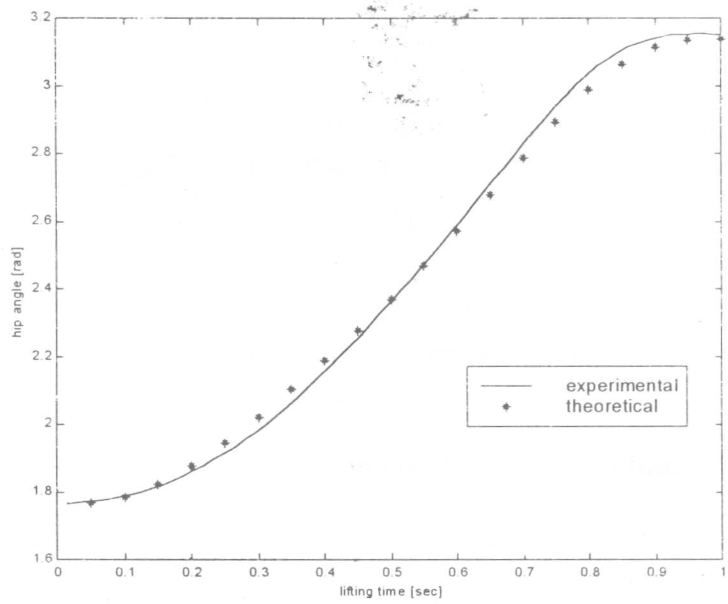

Fig. 1. Comparison of an experimental and a theoretical motion trajectory for the hip joint

4. DISCUSSION AND CONCLUSION

A two dimensional, sagitally symmetric model was established to simulate manual lifting. Joint torques for the five rigid link describing human body in two dimension were obtained with the use of Newton-Euler formulation. Then, the aforementioned objective function was formed based on these torques.
As mentioned before, dynamic strength values were used in the objective function with the belief that they are dependent not only on joint angular position but also on joint angular velocities [13]. Using strength values specifically obtained for the same subjects rather then normalized ones in the literature [14] brought additional power to our model.
It was another strength of this paper to use Genetic Algorithms (GA) to optimize objective function as opposed to other researchers [2,9,10,14] who used generalized reduced gradient algorithms. Since GAs search from population of points, not a single point, they have better chance to catch global optimum as compared to other heuristic methods, although they don't guarantee global optimum. Furthermore, they don't require any derivative information, they just use objective function evaluations, which brings another ease to researchers because getting derivative information is cumbersome in many cases, especially for highly nonlinear systems such as biomechanic ones [12].
Physical constraints on the system were the geometrical ones, for example a joint can only operate in a certain range of angles, cannot exceed its upper or lower bound. This upper and lower bounds for each joints implemented in the objective as penalty functions.
Simulation results were compared with experimental findings and a good correspondence was observed. A randomly chosen sample result is given in Fig. 1 for the hip angle which is the most critical one in lifting operations.

5. ACKNOWLEGEMENT

We would like to express our sincere appreciation to Dr. M. Parnianpour, K.A. Khalaf, and P. Sparto for providing us their raw data, support and giving valuable comments during the preparation of this paper.

6. REFERENCES

[1] M. G. Pandy, B. A. Garner, and F. C. Anderson. Optimal control of non-ballistic muscular movements: A constraint-based performance criterion for rising from a chair, Trans. ASME J. Biomechanical Eng., 117, 15-26,1995

[2] S. H. Hsiang, and M. M. Ayoub, Development of methodology in biomechanical simulation of manual lifting, Int. J. Industrial Ergonomics, 13, 271-288, 1994.

[3] C. K. Chow, and D. H. Jacobson, Studies of human locomotion via optimal programming, Mathematical Biosciences, 10, 239-306, 1971.

[4] B. Ma, An improved algorithm for solving constrained optimal control problems, Ph.D. Thesis, University of Maryland, 1994.

[5] D. E. Kirk, Optimal Control Theory, Prentice Hall, New Jersey, 1970

[6] M. G. Pandy, F. C. Anderson, and D. G. Hull, A parameter optimization approach for optimal control of large-scale musculoskeletal systems, Trans. ASME J. Biomechanical Eng., **114**, 450-460, 1992

[7] D. A. Winter, Biomechanics and motor control of human movement, John-Wiley & Sons, Inc., New York, 1990.

[8] S. C. Chapra, and R. P. Canale, Numerical Methods for Engineers, McGraw-Hill, new York, 1998

[9] K. A. Khalaf, L. Wade, M. Parnianpour, and S. R. Simon, Biomechanical simulation of manual multi-link coordinated lifting, Fifteenth Southern Biomechanical Eng. Conference, 197-198, Dayton, Ohio, 1996

[10]W. A. Gruver, and E. Sachs, Algorithmic methods in optimal control, Pitman Advanced Publishing Program, Boston, 1980.

[11]M. L. Nagurka, and V. Yen, Fourier-based optimal control of nonlinear dynamic systems, J. Dynamic Systems, Measurement, and Control, **112**, 17-26, 1990.

[12]D. E. Goldberg, Genetic Algorithms in Search, optimization, and Machine Learning, Addison Wesley, Reading, Massachusetts, 1989.

[13]K. A. Khalaf, Development of experimental and analytical models for biomechanical simulation and ergonomic assessment of manual material handling tasks, Ph.D. Thesis, The Ohio State University, 1998.

[14]M. S. Hsiang, Simulation of manual materials handling, Ph.D. Thesis, Texas Tech University, 1992.