MULTI-OBJECTIVE OPTIMIZATION FOR DESIGN AND STABILITY OF A GENERAL CONTROL SYSTEM

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Abstract- With the advancement of new materials and techniques, smooth and high performance operation of critical systems such as flight simulators, satellite navigators and robot arms are desired. In these systems, often more than one objective has to be accomplished. In the present study, the *global criterion method* is used as the multi-objective optimization tool. A general mass-spring-dashpot system is analyzed. Damping ratio and natural frequency are chosen as design variables. System settling time, percent overshoot and time to reach to maximum peak are objective functions to be minimized. Numerical solutions with the given constraints are presented and design specifications for the control system are suggested. It is also shown that this optimization tool is a good candidate for multi-criterion problems.

1. INTRODUCTION

Due to recent developments in numerical tools and high precision high speed computational environment available to the design office, new systems have been devised and put in operation. Accurate and high performances are expected in addition to stable working conditions. To name such systems, flight simulators, satellite navigators and industrial robots can be mentioned. For example, a long duration for a space shuttle after its take off can not be tolerated as well as position errors for a pick-and-place robot used in automotive industry.

In most of these military and industrial systems, control objectives are more than one and mostly intensified at minimizing deviations from its target, i.e. overshoot, with minimum stable settling time. Then, the open loop or closed loop controls can be initiated.

Any such system can be idealized as a mass-spring-dashpot system and its stability conditions can be determined after transforming the system differential equation into Laplace domain from which locations of the poles would indicate the stability of the system. As it is well-known, when the damping ratio is zero or

negative, the system poles are located in the right-hand side of the *s-plane*, thus making the system unstable. When it is less than one, or equal to one or greater than one, the system in question is said to be underdamped or critically damped or overdamped, respectively.

For design purposes, when damping ratio and undamped natural frequency of a stable system can be calculated, the relations between the mass, the spring stiffness and the dashpot coefficients can be determined and accordingly appropriate design specifications can be recommended.

In the design optimization field, there have been techniques developed suitable for multiple objectives [1,2]. The global criterion method is one of them [3]. Similar to a single optimization problem but with many, design variables, corresponding constraints and objective functions are needed.

In this study, a general mass-spring-dash pot system is considered first. Then, its mathematical differential equation is transformed into *s-domain* and the undamped natural frequency and the damping ratio are formulated. By choosing these two as the design variables, numerical constraints are impinged on the system settling time, the percent overshoot and the time to reach maximum peak. The multi-objective functions are chosen as the minimization of the system settling time, the percent overshoot and the time to reach to maximum peak. Then, the *global criterion method* is utilized for a stable system and as a result, design specifications are suggested for this particular configuration.

2. THEORY

2.1 System Control Theory

As it is well conceived that any system can be decomposed into a combination of mass, spring and dashpot. A general such system is given in Figure 1.
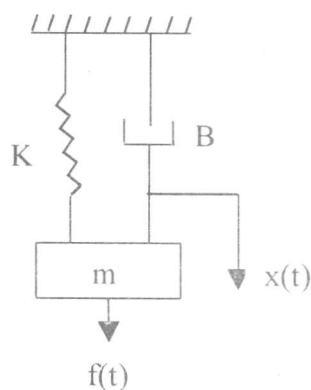

Figure1. A general system representation.

In this figure, the mass, the spring stiffness and the damping coefficients are denoted by m, K and B respectively. The time dependent forcing function is given by $f(t)$ and resulting output is shown as $x(t)$.

The mathematical system differential equation can be written as

$$M\ddot{x} + B\dot{x} + Kx = f(t) \tag{1}$$

and, applying Laplace transformation to Eq.1 results the transfer function as

$$G = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{2}$$

where ω_n is the undamped natural frequency and ξ is the damping ratio of the system. These constants can be expressed in terms of systems mass, spring stiffness and damping coefficients as given below.

$$\omega_n = \sqrt{K/M} \tag{3}$$

$$\xi = \frac{B}{2\sqrt{KM}} \tag{4}$$

Stability of the system is determined by the location of the poles, i.e. zeroes of the characteristic equations. Figure 2 illustrates the cases for various damping ratios
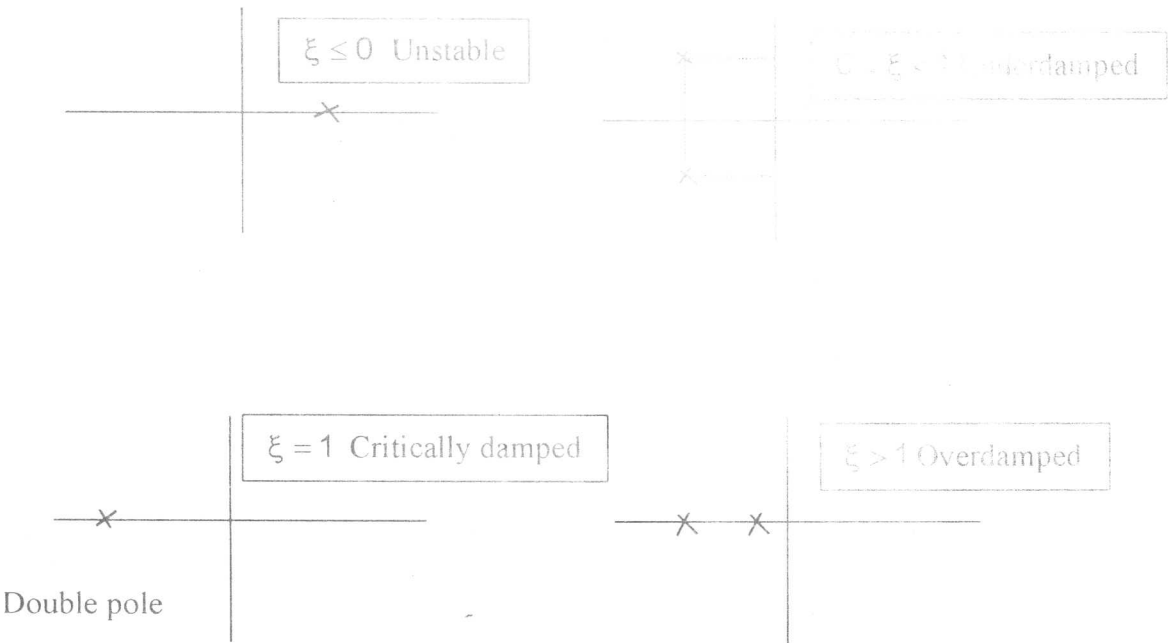

Figure 2. Stability cases for different values of damping ratios.

A response graph for output function of X(t) can be given for stable systems with different damping ratios. A such plot is shown in Figure 3 illustrating the overshoot from the steady state response, *Ts*, system settling time and *Tp*, time to reach maximum peak.
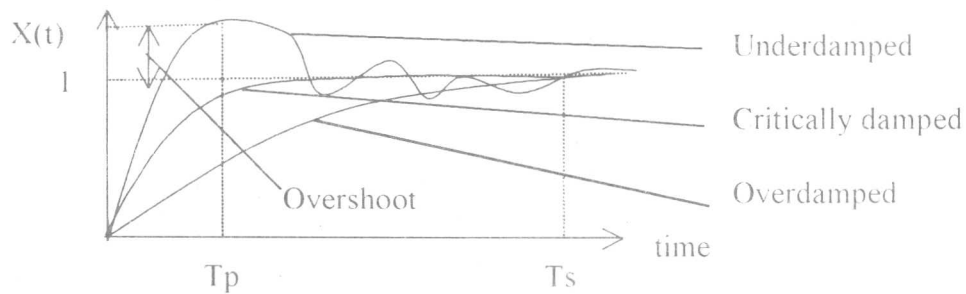

Figure 3. Illustration of system characteristics for various damping conditions.

It can be easily noted from Figure 3 that for no overshoot, system must be critically damped or overdamped. In aerospace or robotic applications where precision is a key design factor , the system settling time must be within a second accompanying a tolerable 1% overshoot above the steady state target.

From the theory of linear control [4-7], the system settling time, *Ts*, the percent overshoot, *PO*, and time to reach maximum peak *Tp* can be expressed as

$$Ts = \frac{4}{\xi \omega_n} \tag{5}$$

$$PO = e^{\xi \pi / \sqrt{1-\xi^2}} * 100 \tag{6}$$

$$Tp = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \tag{7}$$

2.2. Multi-Criterion Optimization Theory

As opposed to a classical design optimization problem where single cost function is to be optimized, the multi-criterion theory considers more than one objective function. This is mostly the case in advanced systems operating in military or in high technology areas, where many aspects of a single design are expected to operate at its best compromised conditions. A well outlined reference on multi-criterion optimization is due to Osyczka, [2].

In this section, it is not intended to give full theory but mention some of them to interested readers. In the "*weighted method*", similar to weighted residual method of finite elements, some weights are assigned to objective functions

according to their importance. This method may fail for nonconvex sets. In the *"epsilon-constraint"* method, one or remaining objective functions are expressed as constraints to the problem. A clear disadvantage is that the problem may not yield feasible solutions for certain values of epsilon. In the *"lexicographic method"*, a hierarchical approach is employed according to the priorities of the objective functions. Another method is the *"minimum deviation method"* where best and worst values are to be calculated for each cost function. The *"min-max method"*, *"goal programming"*, *"genetic algorithm methods"* can also be utilized for the solution of multi-dimensional optimization problems. Global criterion method is explained in the following section.

2.3. Global Criterion Method

Let $f_i^*(x^*)$ be the ith 'ideal' solution or optimum value of cost function f_i considering only the ith objective. Then the optimization problem can be written

$$\text{minimize} \qquad F = \sum_{i=1}^{k} \left[\frac{f_i^*(x^*) - f_i(x)}{f_i^*(x^*)} \right]^T \qquad\qquad (8)$$

subject to constraints $g_j \leq 0 \quad j=1,...m,$

where underscore x represents vector solutions. In must be noted that the feasible designs can be selected for different positive number values exp.

3. PROBLEM SPECIFICATION

A sample problem is chosen for an application where the system should settle down in 2 seconds, the maximum value should be reached within 1.5 seconds and minimum deviation from target should not exceed 10%. The purpose is to optimally design the system according to these specifications, i.e. determine the mass, the spring and the dashpot characteristics.

Undamped natural frequency and damping ratio are picked as the backbone design variables after an examination of the system.

The objective functions are the overall minimization of percent overshoot, time to reach maximum peak and system settling time. Then the optimization problem can be written as,

$x_1 = \chi$ Damping ratio, $x_2 = \varphi_n$ Undamped natural frequency (rad/s)
minimize

$$Ts \rightarrow f_1(x_1,x_2) = \frac{4}{x_1 x_2}, (PO) \rightarrow f_2(x_1) = e^{-\pi x_1 / \sqrt{1-x_1^2}} *100, Tp \rightarrow f_3(x_1,x_2) = \frac{\pi}{x_2\sqrt{1-x_1^2}}$$

4. RESULTS

The numerical results to the multi-criterion design problem are presented in Table 1, for **p** values of 1 to 3.

Table 1

Value of p	ξ	ωn (rad/s)	Ts (s)	Tp (s)	PO(%)
1	0.59	7.78	0.86	0.50	10
2	0.77	9.99	0.51	0.49	2.1
3	0.59	7.78	0.86	0.50	10

The corresponding design suggestions are given in Table 2 along with constraint values at optimal design points.

Table 2

Value of **p**	K/M	B/2√KM	Constraint Ts (s)	Constraint Tp (s)	Constraint PO(%)
1,3	60.6	0.59	-0.56	~0	5.4E-4
2	99.9	0.78	-0.74	~0	-0.79

From these results, optimal design points can be determined as the values corresponding to **p** = 2. Thus, appropriate values for the mass, spring and damper can be chosen accordingly.

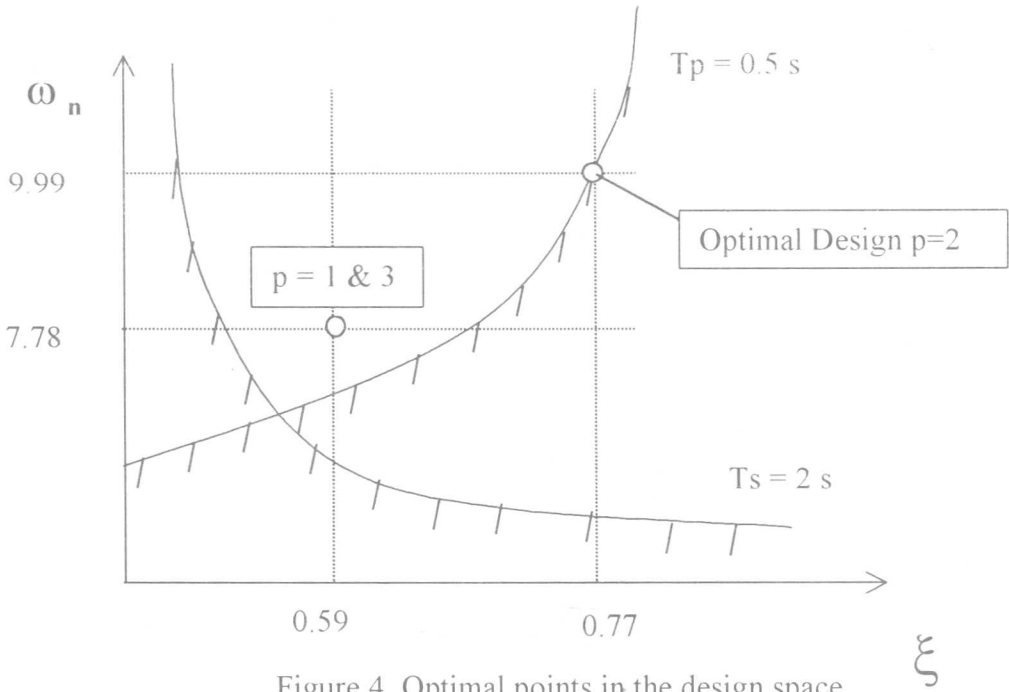The Figure 4 illustrates the optimal points for **p** values of 1 and 2.

Figure 4. Optimal points in the design space.

5. CONCLUSIONS AND FUTURE WORK

The mass-spring-dashpot system is analyzed under the conditions of multi-optimality requirements. A powerful method of "Global Criterion Method" is used for the multi-criterion design optimization problem. This problem required minimization of three objective, i.e., cost functions of system settling time, time to reach maximum peak and percent overshoot from target, with two design variables of undamped natural frequency and damping ratios

It is shown that the optimization method and the suggested procedure for the control system problem are vigorous. It can be noted that no direct values for the mass, the spring and the dashpot are inferred. However, the relations between them are presented to give flexibility to the designer in choosing the system characteristics. An extreme case for zero system settling time and zero time to reach maximum peak are also experimented to show the versatility of the procedure and the program. In that case, the natural frequency is increased to its thousand fold suggesting a million for the spring to mass ratio.

For future work, it is suggested that this procedure can be applied to different systems. To give examples, the designers in the area of automobile suspension or robot manipulators can easily benefit from the presented outcomes.

6. ACKNOWLEDGEMENTS

7. REFERENCES

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