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THE SENSITIVITY OF HALO NUCLEI TO THE NUCLEAR RADIUS

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Abstract : β-decay provides important information about the structure of halo nuclei. Because of the root-mean-square radius (R_{rms}) of the neutron halo distribution is very large, the phase-space factor for halo nuclei must be more sensitively depended on the radius. In this work the sensitivity of the nuclear radius to the phase-space factor was investigated for some halo nuclei and the improvement for phase-space factor for halo nuclei was reported. The results were applied to calculate the logft-values for the halo nuclei (^{6}He, ^{8}He, ^{11}Be, ^{14}Be, ^{9}Li, ^{11}Li). The modification proposed shows reasonable results for many cases.

1. INTRODUCTION

Nuclear spectroscopy for the nuclei near the neutron or proton driplines has been extensively studied in the latest years[1,2]. One of the most interesting discoveries is the neutron and proton halos[3,4]. ^{11}Li and ^{11}Be are the first nuclei observed as neutron halo and still the most interesting case. ^{6}He, ^{8}He, ^{9}Li and ^{14}Be etc. are the other some nuclei considered to have neutron halos. Although proton halo is not yet fully established, several data suggest the existence of a proton halo and ^{8}B, ^{9}C, ^{12}N, ^{17}F and ^{17}Ne are the examples for proton halo[5].

In nuclei near the dripline, the separation energy of the last nucleon(s) becomes extremely small. In such loosely bound nuclei the neutron or proton density distribution shows an extremely long tail, called halo. Although the density of a halo is very low, it leads to new properties in such nuclei. For example an increase of the nuclear radii with respect to atomic number A can be seen in Figure-1 for ^{6}He, ^{8}He, ^{11}Be, ^{14}Be, ^{9}Li and ^{11}Li [5]. The R_{rms} of the ^{11}Li density distribution is 3,12±0,3(1p orbital) and 3,05±0,3(2s orbital). While R_{rms} of the ^{9}Li core is 2,61±0,1 and 2,5±0,1, halo R_{rms} is 4,8±0,5 [5]. As seen, the R_{rms} of the halo neutron distribution is very large and almost two times larger than core R_{rms}. Since the separation energy of the last nucleon(s) becomes extremely small in halo nuclei the R_{rms} radii of them becomes larger.

β-decay of halo nuclei provides an important knowledge about nuclear structure and nuclei near the driplines. The observed logft-value for transition to the first excited state of ^{11}Be, logft=5,59, is much larger than the calculated values based on various available models, such as that with Cohen-Kurath interactions, logft=4,59-4,8[6]. Suziki and Otsuka [6] found that the formation of a neutron halo plays an important role in reducing the logft value and the another interesting observation in the β-decay of light neutron rich nuclei is the existence of strong super-allowed transition [7]. The radius of halo nucleus must be responded for this property and it must be a clear signature for the halo structure of a nucleus. Although the phase-space factor in the standard β-decay theory is related to radius this relation is very slightly.

In this work, by taking into account the difference between the calculated and observed logft values some halo nuclei, the sensitivity to nuclear radius of the phase-space factor was investigated for some halo nuclei and the new modification to phase-space factor was proposed. The modified phase-space factor was applied to halo nuclei and found the agreement results with the observed ones.

2. CALCULATIONS AND RESULTS

For allowed β-decay involving either electron or positron emission, one can show the phase-space factor, f, [8],

$$f=f(\pm Z, W_o) = m_e^{-5} c^{-9} \int F(\pm Z, W) p W (W - W_o)^2 \, dw \quad (1)$$

where, W_o is the difference between initial and final nuclear energies and p, W are the emitted electron's momentum and energy respectively. The quantity F(±Z,W) is the standard Fermi Function,

$$F(\pm Z,W)=2(1+\nu)(2pR)^{-2(1-\nu)} e^{+\pi y} |\Gamma(\nu+iy)|^2 (\Gamma(2y+1))^{-2}$$

where

$p=(W^2-1)^{1/2}$, $\alpha=e^2/hc=1/137$, $\nu=[1-(\alpha Z)^2]^{1/2}$ and $y=\alpha ZW/p$

R is the nuclear radius and Γ is Gamma function. As seen, Fermi Function is related to radius however this relation is very slightly. The various approximations were made for F(±Z,W) expression. One of them is given by Daniel[9] as,

$$F(\pm Z,W)= 2\pi y / [1-\exp(-2\pi y)]$$

This expression don't depend on radius of nucleus. When it is inserted in Equation(1), phase-space factor is given,

$$f= \int_{mc^2}^{W_0} \sqrt{w^2-1}\, w\,(w-w_0)^2 \frac{2\pi y}{1-\exp(-2\pi y)}\, dw \quad (2)$$

In this work, firstly, f-values were calculated using above expression. The calculated and observed logft values were tabulated in Table-1.The disagreement between them are seen. Then we start with a simple assumption that the reason of the difference mentioned just above are the R_{rms} radii of the halo nuclei. Hence the new modification depended on R_{rms} have been described as,

$$(\log ft)_{exp.}=[A (R_n)^x]^{-1} (\log ft)_{theo.}$$

A and x parameters using the experimental logft-values, the calculated logft-values and the experimental neutron radius (R_N) which are given in Table-1 and Table-2. A and x values have been found approximately 0,09 and 3,14 respectively for the nuclei in the Tables. The radii of the halo nuclei have been calculated within the Hartree-Fock approximation as R_N(HFS) using with the Skyrme effective interaction SIII [10,11]. Parameters used in SIII in the present calculations are given in references[12]. The computer program , SK-HAFO(S_3), has been used to calculate the radii. In the our calculations we took the pairing effects into account in the BCS formalism in the approximation constant force.

Secondly, the expression, $[0,09 (R_n)^{3,14}]^{-1}$, has been convoluted with the well-known Fermi Function and again the logft-values have been calculated using the experimental, calculated HF-Skyrme and other workers R_n values. As a result, the Fermi phase-space factor can be taken as,

$$f=\frac{1}{0,09 R_N^{\pi}} \int_{mc^2}^{W_0} \sqrt{w^2-1}\, w\,(w-w_0)^2 \frac{2\pi y}{1-\exp(-2\pi y)}\, dw \quad (3)$$

3. CONCLUSION

In the nuclei that show the halo properties, since the separation energy of the last nucleon(s) becomes extremely small and R_{rms} radii becomes large, the logft– values should be reduced. The calculated results for the logft– values using the new modification are given in Table-1. These results support the strong depend on the radius of the phase-space factor for halo nuclei. The modification reduces the logft values by depending on R_{rms} values for every nuclei and is important to get close to the experimental logft- values.

We can conclude, with the present calculation, that the effect of the radius is important to reduce deviations from the observed logft value of the halo nuclei. The changing of experimental, theoretical and modified logft values with respect to HF-Skyrme R_{rms} are shown in Figure-2. Except for ^{14}Be, the calculated logft values using the this modification are approximately in reasonable agreement with the observed ones for the neutron halo nuclei. The disagreement for ^{14}Be can be resulted from the high percentage of error (19 %) in the measuring of R_{rms} value according to the other nuclei's that [19].

Finally, in this study, it can be seen that from Table-2 that the calculated radii using HF-Skyrme interaction are in good agreement with experiment, also as shown above, the modified logft values for HF-Skyrme, experimental and other worker's R_{rms} values are generally in the same range with the experimental logft values.

Table -1. The logft values calculated by using the new modification, theoretical and experimental logft values for neutron halo nuclei indicated.

Nuclei	Logft(exp)	Logft(theo.)	Modified Logft		
			With Exp. R_{rms}	With HF-Skyrme R_{rms}	WithOther workers R_{rms}
^{6}He	2.9 [13] 2.7 – 2.76 [14]	2.92	2.66	2.93	2.64
^{8}He	4 ± 0.4 [15]	4.28	4	3.99	4.10
^{9}Li	4.90 –5.16 [14] 5.00± 0.05 [15]	5.10	4.88	4.89	4.96
^{11}Li	< 4 [5]	4.58	4	4.20	3.27
^{11}Be	5.79 [14]	6.56	6.2	6.30	5.84
^{14}Be	3.6 – 4.0 [16]	3.80	3.25	3.21	3.37

Table -2. Neutron radii (R_N) of halo nuclei indicated as experimental, HF-Skyrme and other workers respectively

Nuclei	R_N (exp)	R_N (HF-S)	R_N (Other workers)
^{6}He	2.61± 0.03 [17]	2.131	2.65 [20]
^{8}He	2.64 ± 0.03 [17]	2.654	2.60 [18]
^{9}Li	2.53 [18]	2.507	2.39 [19]
^{11}Li	3.21± 0.17 [17-19]	2.816	5.65 [21]
^{11}Be	2.73 ± 0.05 [22]	2.595	3.65 [21]
^{14}Be	3.22 ± 0.19 [19]	3.320	2.95 [19]

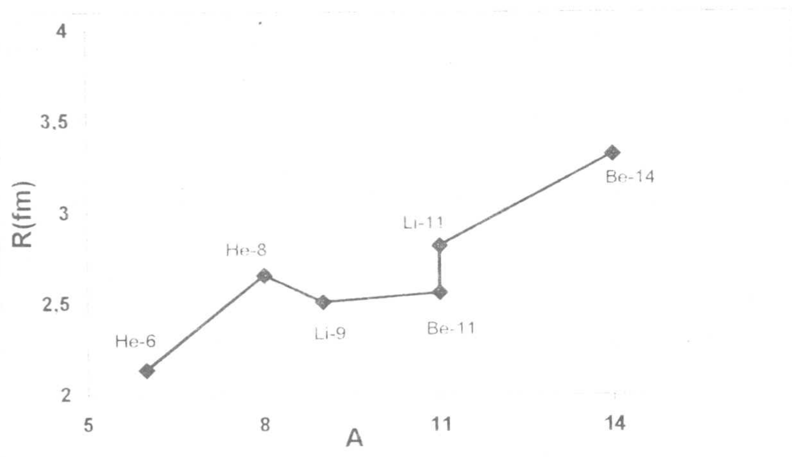

Figure 1. RMS radii of neutron halo nuclei determined from the HF-Skyrme interaction. Sudden increases of neutron radii are seen for nuclei near neutron dripline.

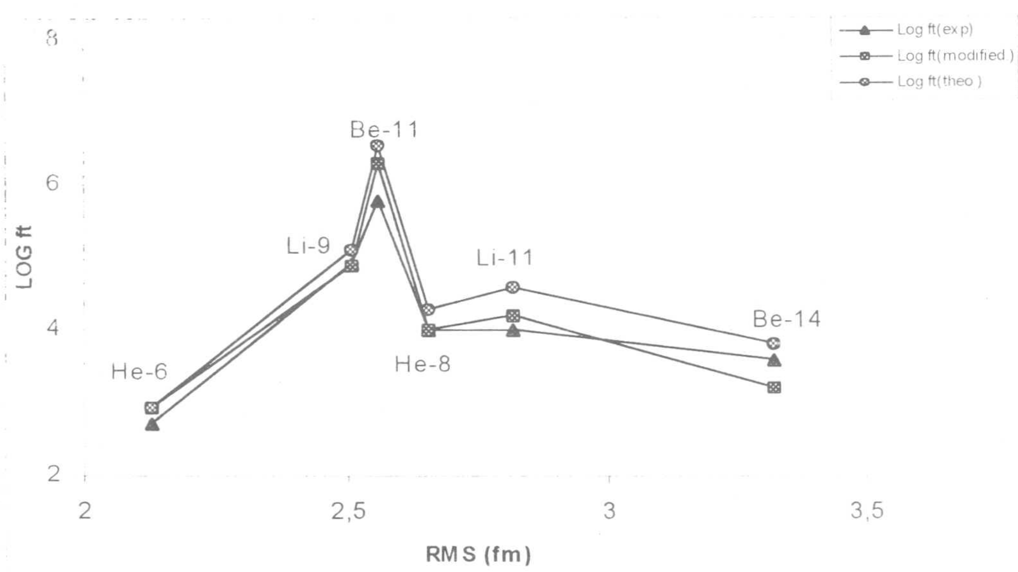

Figure 2. Experimental, theoretical and modified Log ft values with respect to R_{rms} radii of neutron halo nuclei determined from the HF-Skyrme interaction.

REFERENCES

1) P.G. Hansen, Nuclear structure at the drip lines, Nucl. Phys. A **553**, 89c, 1993.
2) B. Jonson, Halo Nuclei, Nucl. Phys. A **574**, 151c-166c,1994.
3) I. Tanihata , H. Hamagaki ,et all., Measurements of Interaction Cross Sections and Nuclear Radii in the Light p-Shell Region, Phys. Rev. Lett. **55**, 2676, 1985.
4) P.G. Hansen and B. Jonson, The Neutron Halo of Extremely Neutron-Rich Nuclei, Europhys. Lett. **4**, 409-414 ,1987.
5) I. Tanihata, Neutron halo nuclei, J.Phys. G. Nucl. Part. Phys. **22**, 157-198, 1996.
6) T. Suziki and T. Otsuka , Configuration of the two-neutron halo of ^{11}Li and Gamow-Teller transition , Phys. Rev. C **50**, R555,1994.
7) M.J.G. Borge, P.G. Hansen, et all., Super-allowed beta decay of nuclei at the drip-line, Z. Phys. A **340**, 255-261, 1991.
8) John N. Bahcall, Phase-space Integrals for Beta Decay and Nuclear Matrix Elements, Nucl. Phys. **75**, 10-16, 1966.
9) H.Daniel, A Method for Rapid Measuring Q Values of β^- Decay, Nucl. Instr. and Methods **171**, 289-291, 1980.
10) D. Vautherin and D.M. Brink, Hartree–Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei, Phys. Rev. C **5**, 626, 1972.
11) D. Vautherin, Hartree–Fock Calculations with Skyrme's Interaction. II. Axially Deformed Nuclei , Phys. Rev. C **7**, 296, 1973.
12) J. Friedrich and P.G. Reinhard, Skyrme–force parametrization: Least–Squares fit to nuclear ground-state properties, Phys. Rev. C **33**, 335, 1986.
13) Emil Jan Konopinski, Beta – Decay, Reviews of Modern Phys. **15**, 209, 1943.
14) W.T.Chou, E.K.Warburton and B.A.Brown, Gamow-Teller beta-decay rates for A≤18 nuclei, Phys. Rev. C **47**, 163, 1993.
15) Paul S. Hauge and S.Maripuu, Shell-Model Calculations for A = 6-14 Nuclei with a Realistic Interaction, Phys. Rev. C **8**, 1609, 1973.
16) M. Belbot, J.J Kolata, M. Zahar, et all., β decay of the neutron-rich isotope ^{14}Be, Phys. Rev. C **56**, 3038, 1997.
17) I. Tanihata, Nuclear Structure Studies Using High-Energy Radioactive Nuclear Beams, Nucl. Phys. A **478**, 795 c, 1988.
18) H.Sato and Y.Okuhara, High-energy Nucleus-nucleus Scattering and the Matter Radius of the Unstable Nucleus, Phys. Lett. B **162**, 217, 1985.
19) I. Tanihata, et all., Measuring of Interaction Cross Sections Using Isotope Beams of Be and B and Isospin Dependence of the Nuclear Radii, Phys. Lett. B **206**, 592, 1988.
20) Y. Suziki and K. Ikeda, Cluster-orbital shell model and its application to the He isotopes, Phys. Rev. C **38**, 410, 1988.
21) M.V. Zhukov, B.V. Danilin, et all., β decay of ^{11}Li to the deuteron channel and halo analog states in ^{11}Be, Phys. Rev. C **52**, 2461, 1995.
22) K.Riisager, A.S. Jensen and P. Moller, Two-body halos, Nuc. Phys. A **548**, 393-413, 1992.