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Correction

Correction: Kunc, O.; Fritzen, F. Finite Strain Homogenization Using a Reduced Basis and Efficient Sampling. *Math. Comput. Appl.* 2019, *24*, 56

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The authors wish to make a correction to Formula (42) of the paper [1]. The correct formula reads

$$\overline{C}_{ijkl}(\overline{\boldsymbol{F}}) = \overline{C}_{ijkl}(\overline{\boldsymbol{R}}\,\overline{\boldsymbol{U}}) = \sum_{m,n=1}^{3} \overline{R}_{im}\overline{C}_{mjnl}(\overline{\boldsymbol{U}})\overline{R}_{kn} \qquad (i,j,k,l = 1,2,3). \tag{1}$$

Correspondingly, a correction to Equations (A1)–(A4) of Appendix A of [1] is now provided. To this end, Green's strain tensor $\overline{\boldsymbol{E}} = \frac{1}{2}(\overline{\boldsymbol{F}}^{\mathsf{T}}\overline{\boldsymbol{F}} - \boldsymbol{I})$, the corresponding stored energy density function $\overline{W}^{\mathrm{E}}(\overline{\boldsymbol{E}}) = \overline{W}(\overline{\boldsymbol{F}})$, the second Piola–Kirchhoff stress $\overline{\boldsymbol{S}} = \partial\overline{W}^{\mathrm{E}}/\partial\overline{\boldsymbol{E}}|_{\overline{\boldsymbol{E}}}$, and the corresponding stiffness tensor $\overline{\mathbb{C}}^{\mathrm{E}} = \partial^2\overline{W}^{\mathrm{E}}/(\partial\overline{\boldsymbol{E}})^2|_{\overline{\boldsymbol{E}}}$ are introduced. Starting from the well-known relationship $\overline{\boldsymbol{P}} = \overline{\boldsymbol{F}}\,\overline{\boldsymbol{S}}$ between $\overline{\boldsymbol{S}}$ and the first Piola–Kirchhoff stress $\overline{\boldsymbol{P}} = \partial\overline{W}/\partial\overline{\boldsymbol{F}}|_{\overline{\boldsymbol{F}}}$ (see for instance [2]), we express the components of $\overline{\mathbb{C}}$ in terms of those of $\overline{\boldsymbol{S}}$ and of $\overline{\mathbb{C}}^{\mathrm{E}}$:

$$\overline{C}_{ijkl} = \frac{\partial^2\overline{W}}{\partial\overline{F}_{ij}\partial\overline{F}_{kl}} = \frac{\partial\overline{P}_{ij}}{\partial\overline{F}_{kl}} = \sum_{m=1}^{3}\frac{\partial\overline{F}_{im}\overline{S}_{mj}}{\partial\overline{F}_{kl}} = \sum_{m=1}^{3}\left(\delta_{ik}\delta_{lm}\overline{S}_{mj} + \overline{F}_{im}\frac{\partial\overline{S}_{mj}}{\partial\overline{F}_{kl}}\right) \tag{2}$$

$$= \delta_{ik}\overline{S}_{lj} + \sum_{m,n,o=1}^{3}\overline{F}_{im}\frac{\partial\overline{S}_{mj}}{\partial\overline{E}_{no}}\frac{\partial\overline{E}_{no}}{\partial\overline{F}_{kl}} \tag{3}$$

$$= \delta_{ik}\overline{S}_{lj} + \sum_{m,n,o=1}^{3}\overline{F}_{im}\overline{C}^{\mathrm{E}}_{mjno}\frac{\partial\overline{E}_{no}}{\partial\overline{F}_{kl}} \tag{4}$$

$$= \delta_{ik}\overline{S}_{lj} + \sum_{m,p=1}^{3}\overline{F}_{im}\overline{C}^{\mathrm{E}}_{mjpl}\overline{F}_{kp}\,. \tag{5}$$

In the last step, the minor symmetry $\overline{C}^{\mathrm{E}}_{mjno} = \overline{C}^{\mathrm{E}}_{mjon}$ has been exploited, and $i,j,k,l = 1,2,3$ above and throughout. From this, the inverse relation

$$\overline{C}^{\mathrm{E}}_{ijkl} = -\left(\overline{U}^{-2}\right)_{ik}\overline{S}_{lj} + \sum_{m,n=1}^{3}\left(\overline{F}^{-1}\right)_{im}\overline{C}_{mjnl}\left(\overline{F}^{-\mathsf{T}}\right)_{nk} \tag{6}$$

can be derived. The fact that Green's strain tensor is frame invariant, i.e., $\overline{\boldsymbol{E}}(\overline{\boldsymbol{R}}\,\overline{\boldsymbol{U}}) = \overline{\boldsymbol{E}}(\overline{\boldsymbol{U}})$, implies that both the left hand side $\overline{C}^{\mathrm{E}}_{ijkl} = \overline{C}^{\mathrm{E}}_{ijkl}(\overline{\boldsymbol{E}})$ and the second Piola–Kirchhoff stress $\overline{S}_{lj} = \overline{S}_{lj}(\overline{\boldsymbol{E}})$ are independent of $\overline{\boldsymbol{R}}$. This is in contrast to $\overline{C}_{mjnl} = \overline{C}_{mjnl}(\overline{\boldsymbol{R}}\,\overline{\boldsymbol{U}})$ from which follows that

$$\sum_{m,n=1}^{3}\left(\overline{F}^{-1}\right)_{im}\overline{C}_{mjnl}(\overline{\boldsymbol{R}}\,\overline{\boldsymbol{U}})\left(\overline{F}^{-\mathsf{T}}\right)_{nk} = \sum_{m,n=1}^{3}\left(\overline{U}^{-1}\right)_{im}\overline{C}_{mjnl}(\overline{\boldsymbol{U}})\left(\overline{U}^{-\mathsf{T}}\right)_{nk}, \tag{7}$$

By contraction of the indices i and k with the second index of $\overline{\boldsymbol{F}}$ and the first index of $\overline{\boldsymbol{F}}^{\top}$, respectively, Equation (1) follows.

The above changes do not affect the scientific results.

References

1. Kunc, O.; Fritzen, F. Finite Strain Homogenization Using a Reduced Basis and Efficient Sampling. *Math. Comput. Appl.* **2019**, *24*, 56. [CrossRef]
2. Bertram, A. *Elasticity and Plasticity of Large Deformations*; Springer: Berlin/Heidelberg, Germany, 2008. [CrossRef]