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ANALYSIS OF A BULK-SERVICE QUEUEING-SYSTEM SUBJECT TO INTERRUPTIONS

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Abstract

The present study depicts the behavior of mean queue length on failure service parameters and shows that the mean queue length is proportional to failure parameter while it varies inversely with repair parameters

Key words : Threshold phenomenon, stochastic behavior, poisson arrivals, bulk service steady state

1. INTRODUCTION

Interruptions with customers service due to occasional break-down in the system are the general phenomena which bring an impact on systems efficiency, mean queue length and customer waiting time in the system. Such systems with random break-down have been studied by various authors such as Avi-Itzhak and Noor [1], Graver [3], Madan [5,6], Thiruvengadan [8].Recently, Jaiswal, Sharma and Karmeshu [4] studied a stochastic analysis of a non-linear machine interference model and found that model exhibits threshold effect in mean queue size where the number of failed machine arises inordinately.

The present paper gives an analytic study of Madan [6] work. The study depicts the behavior of mean queue length on failure and service parameters and show that the mean queue length is proportional to failure parameter while it varies inversely to repair parameters.

2. DESCRIPTION OF MODEL

(i) Arrivals one by one in Poisson fashion with mean arrival rate λ ($\lambda>0$).

(ii) The service of units is rendered in batches of fixed size b (>1) and service times of successive batches follow exponential distribution with mean service rate $1/\mu$ ($\mu>0$).

(iii) As soon as the service of a batch is completed another batch of size minimum (b,n) where n (≥ 0) is the queue length is taken for service however, if $n=0$ at such an instant then the service channel remains idle till such time when a new unit arrives.

(iv) Each time a break-down is encountered, the service of batch is suspended and service channel is immedeately sent for repairs as soon as the service channel instantly resumes service of the same batch of units whose service was suspended.

(v) Queue discipline FcFS.

(vi) The operative times of service channel and its repair times are exponentially distributed with mean operative time $1/\alpha$ and the mean repair time $1/\beta$ ($\alpha,\beta>0$).

3. NOTATION AND MATHEMATICAL FORMULATION OF QUEUE MODEL

Assume :

(i) W_n : Steady state probability that the service channel is working state and there are n ($n\geq 0$) units in queues excluding a batch in service.

(ii) R_n : Steady state probability that the service channel is under repaires and there are n ($n\geq 0$) units in queue excluding the batch of suspended service units.

(iii) Q : Steady state probability that there is no unit either in the queue or in service i.e. the Service Channel, through operative is idle.

4. STEADY STATE DIFFERENCE EQUATIONS OF THE SYSTEM

$$(\lambda+\mu+\alpha)W_n = \lambda W_{n-1} \tag{1}$$

$$(\lambda+\mu+\alpha)W_0 = \lambda Q + \mu\sum_{k=0}^{p} W + \beta R_0 \tag{2}$$

$$(\lambda+\beta)R_n = \lambda R_{n-1} + \alpha W_n \qquad (n>0) \tag{3}$$

$$(\lambda+\beta)R_0 = \alpha(Q+W_0) \tag{4}$$

$$(\lambda+\alpha)Q = \mu W_0 \tag{5}$$

Define the probability generating functions

$$W(x)= \sum_{n=0}^{\infty} W_n x^n, \qquad R(x)= \sum_{n=0}^{\infty} R_n x^n \tag{6}$$

In order to get the solutions of equations (1) to (5) using existing methods and applying Rouche Theorem, proceeding on similar lines as Choudhary and Templeton [2] and using normal conditions $W(1)+R(1)+Q=1$, we have the probability that service channel, through operative Q is idle.

$$W(x)= \frac{\{-\lambda^2 x + \lambda(\lambda+\alpha+\beta)+\alpha\beta\}Q}{\{\lambda^2 x^2 - \lambda(\lambda+\mu+\alpha+\beta)x + \mu(\lambda+\beta)\}} \tag{7}$$

$$R(x)= \frac{\alpha\{-\lambda x + (\lambda+\alpha+\mu)\}Q}{\{\lambda^2 x^2 - \lambda(\lambda+\mu+\alpha+\beta)x + \mu(\lambda+\beta)\}} \tag{8}$$

(vi) The operative times of service channel and its repair times are exponentially distributed with mean operative time $1/\alpha$ and the mean repair time $1/\beta$ ($\alpha,\beta>0$).

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$$Q = \frac{1-(\lambda/\mu)(1+\alpha/\beta)}{(1+\alpha/\mu)(1+\alpha/\beta)} \tag{9}$$

The probability that service channel is under repairs irrespective of number of units in the system is given by

$$R(1)= \frac{\alpha(\mu+\alpha)}{\mu\beta-\lambda(\alpha+\beta)} \qquad Q = \frac{\alpha(\mu+\alpha)}{(\alpha+\mu)(\alpha+\beta)} \tag{10}$$

And the probability that service channel is in working state is given by

$$W(1)= \frac{\{\lambda(\alpha+\beta)+\alpha\beta\}}{\mu\beta-\lambda(\alpha+\beta)} \qquad Q = \frac{\lambda(\alpha+\beta)+\alpha\beta}{(\alpha+\mu)(\alpha+\beta)} \tag{11}$$

and the necessary condition for the study to exist is that

$$(\lambda/\mu)\ (1+\alpha/\beta) < 1 \tag{12}$$

The expression in the RHS of equation (9) is the utilization factor of the system can be denoted by ρ. So

$$\rho = \frac{\lambda(\alpha+\beta)+\alpha\beta}{(\alpha+\mu)(\alpha+\beta)} \tag{13}$$

5. STEADY STATE SOLUTION

The denominator in RHS of equation (7) has 2 zeros given by

$$x_1, x_2 = \frac{(\lambda+\mu+\alpha+\beta) \pm \left\{(\lambda+\mu+\alpha+\beta)^2 - 4\mu(\lambda+\beta)\right\}}{2\lambda} \tag{14}$$

such that

$$x_1 + x_2 = \frac{(\lambda+\mu+\alpha+\beta)}{\lambda} \qquad \text{and} \qquad x_1, x_2 = \frac{\mu(\lambda+\beta)}{\lambda^2} \tag{15}$$

Also the N^r of RHS of (7) can be written as

$$\lambda^2(x'-x)Q$$

where

$$x' = \frac{\lambda(\lambda+\alpha+\beta)+\alpha\beta}{\lambda^2}.$$

So (7) and (8) in modified form can be expressed as

$$W(x)=\frac{x'}{x_1x_2}\left\{1+\left(\frac{1}{x_1}+\frac{1}{x_2}-\frac{1}{x'}\right)x+\left[\frac{1}{x_1^2}+\frac{1}{x_1x_2}+\frac{1}{x_2^2}-\frac{1}{x'}(\frac{1}{x_1}+\frac{1}{x_1})\right]x^2\right.$$
$$\left.+\left[\frac{1}{x_1^3}+\frac{1}{x_1^2x_2}+\frac{1}{x_1x_2^2}+\frac{1}{x_2^3}-\frac{1}{x'}(\frac{1}{x_1^2}+\frac{1}{x_1x_2}+\frac{1}{x_2^2})\right]x^3+\ldots\right\}Q \quad (16)$$

$$R(x)=\frac{\alpha x''}{x_1x_2}\left\{1+\left(\frac{1}{x_1}+\frac{1}{x_2}-\frac{1}{x''}\right)x+\left[\frac{1}{x_1^2}+\frac{1}{x_1x_2}+\frac{1}{x_2^2}-\frac{1}{x''}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)\right]x^2+\right.$$
$$\left.\left[\frac{1}{x_1^3}+\frac{1}{x_1^2x_2}+\frac{1}{x_1x_2^2}+\frac{1}{x_2^3}-\frac{1}{x''}\left(\frac{1}{x_1^2}+\frac{1}{x_1x_2}+\frac{1}{x_2^2}\right)\right]x^3+------\right\}Q \quad (17)$$

The solution W_n and R_n for n= 0,1,2,..... can be obtained by picking up the coefficients of various powers of x in R.H.S. expressions of (16) and (17) general solutions can be expressed as

$$W_n=\frac{x'}{x_1x_2}\left[\sum_{i=0}^{n}\frac{1}{x_1^{n-i}x_2^i}-\frac{1}{x'}\sum_{i=0}^{n-1}\frac{1}{x_1^{n-i-1}x_2^i}\right]Q \quad (18)$$

$$R_n=\frac{\alpha x''}{x_1x_2}\left[\sum_{i=0}^{n}\frac{1}{x_1^{n-i}x_2^i}-\frac{1}{x''}\sum_{i=0}^{n-1}\frac{1}{x_1^{n-i-1}x_2^i}\right]Q \quad (19)$$

The solutions given in equations (9), (18), (19) can be seen for at least n=0,1,2,..... etc. to be satisfying the systems equations (1) through (5) for b=1.

6. THE MEAN QUEUE LENGTHS

Let L_1 and L_2 denote the average queue length in the steady state when the service channel is busy and when it is under repairs, then

$$L_1=\left|d/dx\ W(x)\right|_{x=1}=\frac{(\frac{\lambda^2}{\mu^2})\left\{(1+\frac{\alpha}{\beta})^2+\frac{\alpha}{\beta}(\frac{\mu}{\beta}-1)\right\}+\frac{\lambda\alpha}{\mu}\left\{\frac{1}{\beta}+\frac{1}{\mu}(1+\frac{\alpha}{\beta})\right\}}{(1+\frac{\alpha}{\mu})(1+\frac{\alpha}{\beta})\left\{1-\frac{\lambda}{\mu}(1+\frac{\alpha}{\beta})\right\}} \quad (20)$$

$$\frac{\lambda}{\mu}(1+\frac{\alpha}{\beta})<1$$

$$L_2=\left|d/dx\ R(x)\right|_{x=1}=\frac{(\frac{\lambda^2\alpha}{\mu^2\beta})(1-\frac{\mu}{\beta})-\frac{\lambda\alpha^2}{\mu^2\beta}+\frac{\lambda\alpha}{\beta^2}(1+\frac{\mu}{\beta})^2}{(1+\frac{\alpha}{\mu})(1+\frac{\alpha}{\beta})\left\{1-\frac{\lambda}{\mu}(1+\frac{\alpha}{\beta})\right\}} \quad (21)$$

Let L_q denotes the average queue length under steady state irrespective of whether the service channel is busy or under repairs, then

$$L_q = L_1 + L_2 - L_q = \frac{\frac{\lambda^2}{\mu^2}(1+\frac{\alpha}{\beta})^2 + \frac{\lambda\alpha}{\mu}(\frac{1}{\beta}+\frac{1}{\mu}) + \frac{\lambda\alpha}{\beta^2}(1+\frac{\alpha}{\mu})^2}{(1+\frac{\alpha}{\mu})(1+\frac{\alpha}{\beta})\left\{1-\frac{\lambda}{\mu}(1+\frac{\alpha}{\beta})\right\}} \tag{22}$$

When the Service Channel is not subject to failures we have on letting =0 in equation (22),

$$L_q = \frac{\lambda^2/\mu^2}{1-\lambda/\mu} \qquad \lambda/\mu < 1 \tag{23}$$

7. DEPENDENCE OF MEAN QUEUE LENGTH ON FAILURE AND REPAIR RATES

We evaluate mean queue length L_q using equation (22) choosing some arbitrary values of λ, μ, α and β such that these values satisfy the condition (20). In order to check the effect of failure and repair parameters, the following results were obtained by varying failure parmeter α, keeping λ, μ, β fixed.

λ	μ	α	β	L_q
3	5	50	100	5.48 90 93
		30		2.41 22 88
		20		1.89 78 57
		10		1.53 52 94
		05		1.32 93 44
		02		1.13 29 38
		01		1.03 37 99
		00		0.90 00 00

fixed and in second case varying repair parameter β, keeping λ, μ, α fixed.

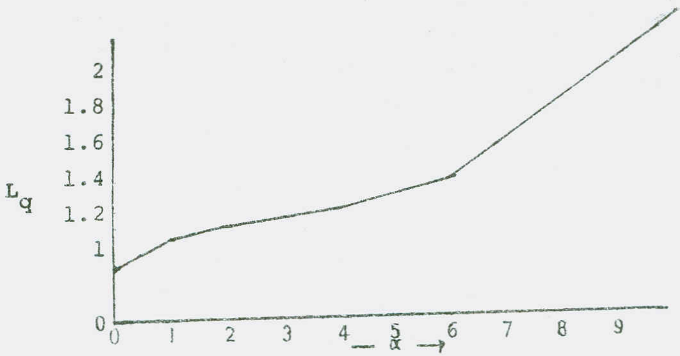
λ	μ	α	β	L_q
3	5	10	100	1.53 52 94
			200	1.48 16 73
			300	1.36 45 16
			500	1.33 71 94
			800	1.32 27 26
			1,000	1. 31 88 43
			10,000	1. 30 17 55
			1,00,000	1.30 01 75
			10,00,000	1.30 00 18

The dependence of the mean queue length on and graphically shown in figure (1) and (2) we show that as the failure rate decreases, the mean queue length goes on reducing and tends to 1.0, to the steady state value which can be directly obtained by equation (22) (figure 1).

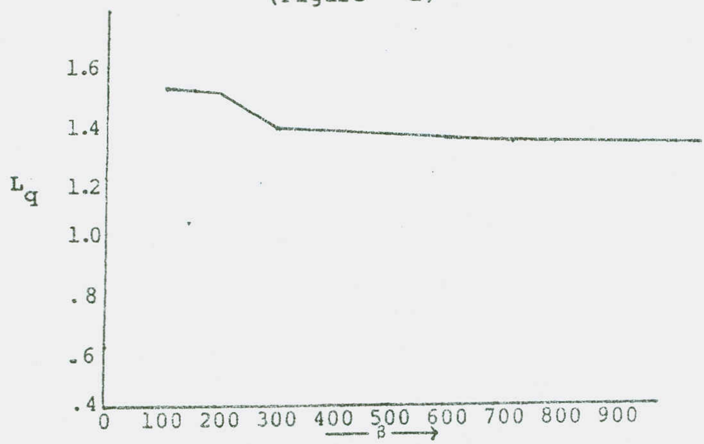
Again figure 2 depicts that when the value of β increase, the mean queue length goes on decreasing and tends to 1.3.

8. CONCLUDING REMARK

The study can be extended by taking more repair centres.



(Figure - 1)



(Figure - 2)

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