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ON GENERALIZED COMPLEMENTARITY PROBLEM

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Abstract - In this paper, we extend generalized implicit complementarity problem and study the existence of the solution of this new problem using variational inequality technique. An algorithm is given to find the approximate solution and prove that this approximate solution converges to the exact solution of the problem. We also study the sensitivity analysis for generalized variational inequality. Several special cases are also discussed.

1.INTRODUCTION

Due to applications of multivalued mappings in different branches of science,Engineering and Operation Researches, in 1976, Saigal [14], studied the variational inequality for multivalued mappings. Since then, different types of variational inequalities have been extented for multivalued mappings, see for example [2], [7], [9]. The theory of complementarity problem is very close to the theory of variational inequalities and it was introduced in early sixtees. In 1971, Karamardian [7] showed that if the set involved in variational inequality and complementarity problem is a convex cone then both problems have the same solution set.

In the last two decades it has also been extended for multivalued mappins, see [5], [2].

The study of the qualitative behaviour of the solution of the variational inequalities when the given operator and the feasible convex set vary with a parameter is known as sensitivity analysis, which is also important and meaningful. Sensitivity analysis provides us useful information for designing, planning various equilibrium systems, predicting the future changes of the equilibria as a result of the changes in the governing systems.Sensitivity analysis for variational inequalities has been studied by Tobin [15], Kyparises [8], Dafermos [3], Qui and Magnanti [13], Brokate and Siddigi [1] and Noor [10] by using different techniques.

Recently Chang and Huang [2] generalized implicit complementarity problem for multivalued mappings and gave an equivalent formulation of variational inequality which is called generalized multivalued variational inequality.

In this paper,we extend generalized multivalued complementarity problem and give an algorithm to find the approximate solution,which is more general than the algorithm of Chang and Huang [2]. In last section,we study the sensitivity analysis of extented generalizied multivalued variational inequality, by following the idea and techniques of Dafermos [3], which is based on the projection techniques.

2.PRELIMINARIES

Let H be a Hilbert space, whose inner product and norm are denoted by $(.,.)$ and $\|.\|$ respectively. Let K be a closed convex cone in H and K^* be dual cone of K, that is

$$K^* = \{ u \in H/(u,v) \geq 0, \text{ for all } v \in K \}.$$

If $D \subset H$ is a subset of H and $g : D \to H$ is a single-valued map, $T, A : D \to 2^H$ are multivalued mappings. Then we consider the problem of finding $u \in D$, $p \in A(u)$, $q \in T(u)$ such that

$$g(u) \in K, \;\; p+q \in K^*, \;\; (g(u), p+q)=0 \tag{2.1}$$

and we shall call it the *generalized complementarity problem.*

SPECIAL CASES:

(i) If $T,A : D \to H$ are single-valued mappings, then the problem (2.1) is equivalent to find $u \in D$ such that

$$g(u) \in K, \;\; A(u)+T(u) \in K^*, \;\; (g(u), A(u)+T(u))=0 \tag{2.2}$$

which is called the *generalized implicit complementarity problem.*

(ii) If $T(u)\equiv 0$ then (2.2) is equivalent to find $u \in D$ such that

$$g(u) \in K, \;\; A(u) \in K^*, \;\; (g(u), A(u))=0 \tag{2.3}$$

which is known as *implicit complementarity problem,* considered and studied by Isac [5] [6] and Noor [11] recently.

(iii) If $g(u)= u \in K$, then (2.3) is equivalent to find $u \in K$ such that

$$A(u) \in K^*, \; (u, A(u))=0 \tag{2.4}$$

which is known as complementarity problem and studied by Karamardian [7] and Habitler and Price [4].

In the sequel, we need the following definitions and concepts.

DEFINITION 2.1. [6]. Given a subset $D \subset H$, $g: D \to H$, a single-valued mapping and $A:D \to C(H)$ is a multi-valued mapping, where $C(H)$ denotes the family of all nonempty compact subsets of H. We say that

A is strongly monotone with respect to g, if there exists a constant $\alpha > 0$ such that

$(p-q, g(u)-g(v)) \geq \alpha \| g(u) - g(v) \|^2$, for all $u,v \in D$, for all $p \in A(u)$, for all $q \in A(v)$.

DEFINITION 2.2. Let $D \subset H$ be a subset, $g: D \to H$, a single-valued mapping and $T: D \to C(H)$, a multi-valued mapping, we say that T is h-Lipschitz continuous with respect to g, if there exists a constant $v > 0$ such that

$$h(T(u),T(v)) \leq v \, \|g(u) - g(v)\|, \qquad \text{for all } u,v \in D,$$

where $h(.,.)$ is the Hausdörff metric on $C(H)$.

LEMMA 2.1. If $K \subset g(D) \subset H$ is a closed convex cone in H, then $u \in D$, $p \in A(u)$, $q \in T(u)$ is a solution of the generalized complementarity problem (2.1) if and only if $u \in D$, $p \in A(u)$, $q \in T(u)$ satisfy the generalized variational inequality problem.

$$g(u) \in K,\ (g(v)-g(u),\ p+q) \geq 0, \qquad \text{for all } g(v) \in K \tag{2.5}$$

PROOF. Let $u \in D$, $p \in A(u)$, $q \in T(u)$ be the solution of the problem (2.1) that is,

$$g(u) \in K,\ p+q \in K^*,\ (g(u),\ p+q) = 0,$$

then for all $g(v) \in K$, we have

$$\begin{aligned}(g(v)-g(u),\ p+q) &= (g(v),\ p+q) - (g(u),\ p+q) \\ &= (g(v),\ p+q) \geq 0\end{aligned}$$

which shows that $u \in D$, $p \in A(u)$, $q \in T(u)$ satisfies the inequality (2.5).

Conversely suppose that $u \in D$, $p \in A(u)$, $q \in T(u)$ satisfy the inequality (2.5). Since $g(u) \in K$ and $K \subset g(D) \subset H$ is a convex cone, we know that $0 \in K$, $2g(u) \in K$, hence there exists an element $v \in D$ such that

$$2g(u) = g(v)$$

Putting $g(v)=0$ and $g(v)=2g(u)$ in inequality (2.5) respectively we have

$$(g(u),\ p+q) \leq 0 \tag{2.6}$$

$$(g(u),\ p+q) \geq 0 \tag{2.7}$$

Therefore, from(2.6) and (2.7), it follows that

$$(g(u),\ p+q) = 0$$

Next,we prove that $p+q \in K^*$. In fact, for any $v \in K \subset g(D)$, there exists a $w \in D$ such that

$g(w)=v \in K$. It follows from (2.5) that

$$\begin{aligned}0 \leq (g(w)-g(u),\ p+q) &= (g(w),\ p+q) - (g(u),\ p+q) \\ &= (g(w),\ p+q) \\ &= (v,\ p+q)\end{aligned}$$

which implies that $(v,\ p+q) \geq 0$, for all $v \in K$ and which shows that $p+q \in K^*$.
This completes the proof.

LEMMA 2.2. Let $D \subset H$ be a subset, then $u \in D$, $p \in A(u)$, $q \in T(u)$ is a solution of the generalized variational inequality problem (2.5) if and only if $u \in D$, $p \in A(u)$, $q \in T(u)$ satisfy the following relation.

$$g(u) = P_k[g(u) - \rho(p+q)]$$

where $\rho > 0$ is a constant and P_k is the projection of H on K.

PROOF. The proof is similar to the proof of Lemma 2.2 of Chang and Huang [2].

LEMMA 2.3. The projection P_k is nonexpansive, that is

$$\| P_k u - P_k v \| \leq \| u - v \|, \text{ for all } u, v \in H.$$

3.EXISTENCE THEORY

In this section, we give an algorithm to find the approximate solution of generalized complementarity problem (2.1). Further, we prove the existence of solutions for the problem (2.1) and the convergence of the iterative sequence constructed by Algorithms.

ALGORITHM 3.1. Let $D \subset H$ be a subset, $g: D \to H$ be a single valued mapping, $T, A: D \to C(H)$ be two multi-valued mappings, where $C(H)$ denote the family of all nonempty compact subsets of H. Let $K \subset g(D) \subset H$ be a closed convex cone. For any given $u_0 \in D$, take $p_0 \in A(u_0)$, $q_0 \in T(u_0)$ and

$$w_1 = P_k[g(u_0) - \rho(p_0 + q_0)]$$

Since $K \subset g(D)$, we can choose $u_1 \in D$ such that

$$g(u_1) = w_1 = P_k[g(u_0) - \rho(p_0 + q_0)]$$

Since $q_0 \in T(u_0) \in C(H)$ and $p_0 \in A(u_0) \in C(H)$, by Nadler [9], there exists a $q_1 \in T(u_1)$ and $p_1 \in A(u_1)$ such that

$$\|p_0 - p_1\| \leq h(A(u_0), A(u_1))$$

$$\|q_0 - q_1\| \leq h(T(u_0), T(u_1))$$

we can choose $u_2 \in D$ such that

$$g(u_2) = w_2 = P_k[g(u_1) - \rho(p_1 + q_1)]$$

By induction, we can obtain sequences $\{u_n\}, \{p_n\}$ and $\{q_n\}$ as follows.

$$p_n \in A(u_n), \ \|p_n - p_{n+1}\| \leq h(A(u_n), A(u_{n+1}))$$

$$q_n \in T(u_n), \ \|q_n - q_{n+1}\| \leq h(T(u_n), T(u_{n+1}))$$

$$g(u_{n+1}) = w_{n+1} = p_k[g(u_n - \rho(p_n + q_n)] \qquad n = 0, 1, 2, ... \qquad (3.1)$$

where $\rho > 0$ is a constant.

If T: D→H and A: D→H are single-valued mappings, then from Algorithm 3.1 ,we can obtain the following:

ALGORITHM 3.2. For a given $u_0 \in D$, we can obtain a sequence $\{u_n\}$ as follows.

$$g(u_{n+1})= P_k[g(u_n)- \rho(A(u_n) +T(u_n))]$$

$$n=0, 1, 2, \ldots.$$

where $\rho > 0$ is a constant.

If T≡0, then from Algorithm 3.2, we have the following.

ALGORITHM 3.3. For a given $u_0 \in D$, we can obtain a sequence $\{u_n\}$ as follows.

$$g(u_{n+1})= P_k[g(u_n)-\rho A(u_n)] \ ; n=0, 1, 2, \ldots.$$

where $\rho > 0$ is a constant.

If g is an identity mapping, then Algorithm 3.3 becomes

ALGORITHM 3.4. For a given $u_0 \in K$, we can obtain a sequence as follows.

$$u_{n+1}= P_k[u_n- \rho A(u_n)],\ n=0, 1, 2, \ldots.$$

where $\rho > 0$ is a constant.

THEOREM 3.1. Let D be a subset, g:D→H ; A,T: D→C(H) and K ⊂g(D)⊂H be a closed convex cone. Suppose that A is strongly monotone with respect to g and h-Lipschitz continuous with respect to g and T is h-Lipschitz continuous with respect to g. If

$$0<\rho< \frac{2(\alpha-\nu)}{\beta^2-\nu^2}; \qquad \rho\nu<1,\ \nu<\alpha \tag{3.2}$$

where α and β are the strongly monotone and h- Lipschitz constants of A respectively and ν is h-Lipschitz constant of T.

Then there exists $u \in D$, $p \in A(u)$, $q \in T(u)$, which are the solution of the generalized complementarity problem (2.1) and

$$g(u_n)\to g(u),\ (n\to \infty)$$

$$p_n\to p\ (n\to \infty), \qquad q_n\to q\ (n\to \infty)$$

where $\{u_n\}$, $\{p_n\}$ and $\{q_n\}$ are the three sequences generated by the Algorithm (3.1).

PROOF. From Algorithm 3.1 and Lemma 2.3, we have

$$\|w_{n+1}-w_n\| = \| g(u_{n+1})-g(u_n)\|$$

$$= \| P_k[g(u_n)-\rho(p_n+q_n)]-P_k[g(u_{n-1})-\rho(p_{n-1}+q_{n-1})]\|$$

$$\leq \| g(u_n)-g(u_{n-1})-\rho(p_n-p_{n-1})\| + \rho \|(q_n-q_{n-1})\|$$

Since A is strongly monotone with respect to g, we have

$$\| g(u_n)-g(u_{n-1})-\rho(p_n-p_{n-1})\|^2 = \| g(u_n)-g(u_{n-1})\|^2 + \rho^2 \| p_n-p_{n-1}\|^2 - 2\rho(g(u_n)-g(u_{n-1}),\ p_n-p_{n-1})$$

$$\leq \| g(u_n)-g(u_{n-1})\|^2 - 2\rho\alpha \| g(u_n)-g(u_{n-1})\|^2 + \rho^2 \| p_n-p_{n-1}\|^2$$

Since A is h-Lipschitz continuous with respect to g, from (3.1), we have

$$\|p_n-p_{n-1}\| \leq h(A(u_n), A(u_{n-1}))$$

$$\leq \beta \| g(u_n)-g(u_{n-1})\| \tag{3.3}$$

Therefore,

$$\| g(u_n)-g(u_{n-1})-\rho(p_n-p_{n-1})\|^2 \leq (1-2\rho\alpha+\rho^2\beta^2) \| g(u_n)-g(u_{n-1})\|^2$$

Further, since T is h-Lipschitz continuous with respect to g, from (3.1), we have

$$\|q_n-q_{n-1}\| \leq h(T(u_n), T(u_{n-1}))$$

$$\leq \nu \| g(u_n)-g(u_{n-1})\| \tag{3.4}$$

Thus, we have

$$\|w_{n+1}-w_n\|\le \theta \,\| g(u_n)- g(u_{n-1})\|= \theta \,\|w_n- w_{n-1}\| \tag{3.5}$$

where

$$\theta=\sqrt{\left(1-2\rho\alpha+\rho^2\beta^2\right)}+\rho\nu< 1 \qquad\qquad [\text{By } (3.2)].$$

From (3.5), we know that $\{w_n\}$ is a Cauchy sequence in K. Letting $w_n \to w \in K$ $(n\to\infty)$. Hence there exist an $u \in D$ such that $g(u)= w \in K$ $(\therefore K \subset g(D))$. By (3.3) and (3.4), we have

$$\|p_n- p_{n-1}\|\le \beta \,\| g(u_n)- g(u_{n-1}) \|$$

$$=\beta \,\|w_n- w_{n-1}\|$$

and

$$\| q_n- q_{n-1}\|\le \nu\|w_n- w_{n-1}\|$$

This implies that $\{p_n\}$ and $\{q_n\}$ are also Cauchy sequences in H.

Let $p_n\to p$ $(n\to \infty)$ and $q_n\to q$ $(n\to \infty)$ and let

$$w'=P_k[g(u)-\rho(p+q)]$$

It follows from Algorithm (3.1) and Lemma 2.3 that

$$\|w_{n+1}- w'\|=\| P_k [g(u_n)-\rho(p_n+q_n)]- p_k [g(u)-\rho(p+q)] \|$$

$$\le \| g(u_n)-g(u) \|+\rho \,\|p_n-p\|+\rho \,\|q_n-q\|$$

$$\le \| g(u_n)-g(u) \|+\rho\beta \,\| g(u_n)-g(u) \|+\rho\nu\| g(u_n)-g(u) \|$$

which implies that $w=w'$, that is

$$g(u)=P_k[g(u)-\rho(p+q)] \tag{3.7}$$

Next we prove that $p\in A(u)$, $q\in T(u)$. In fact, we have

$$d(p,A(u))\le \|p-p_n\|+ d(p_n,A(u))$$

$$\le \|p-p_u\|+ h(A(u_n),A(u))$$

$$\leq \|p-p_n\|+\beta\| g(u_n)-g(u)\|$$

$$\leq \beta\| w-w_n\|+\beta\| w_n-w\|=0$$

where $d(p,A(u))=\inf\{\| p-z \|: z\in A(u)\}$,

But from the above inequality,

$$d(p,A(u))=0 \Rightarrow p\in A(u).$$

Similarly, we have $q\in T(u)$.

By Lemma 2.1, Lemma 2.2 and (3.7), it follows that $u\in D$, $p\in A(u)$, $q\in T(u)$ are the solution of the generalized complementarity problem (2.1) and,

$$g(u_n)\to g(u), \quad (n\to\infty)$$

$$p_n\to p \quad (n\to\infty), \qquad q_n\to q \quad (n\to\infty)$$

This completes the proof.

4.SENSITIVITY ANALYSIS

In this section we study the sensitivity analysis of the generalized variational inequality of the type (2.5). To formulate the problem, let M be an open subset of H in which the parameter λ takes values and assume that $\{K_\lambda: \lambda\in M\}$ is a family of closed convex subsets of H. The parametric generalized variational inequality is to find $u\in D$, $(p,\lambda)\in A(u, \lambda)$, $(q,\lambda)\in T(u, \lambda)$ such that

$$g(u)\in K_\lambda$$

and

$$(g(v)-g(u),(p, \lambda)+(q, \lambda))\geq 0 \tag{4.1}$$

$$\text{for all } g(v)\in K$$

where $A(u,\lambda)$ and $T(u,\lambda)$ are multivalued mappings, which are defined on the set of (u,λ) with $\lambda\in M$. We also assume that for some $\bar{\lambda}\in M$, the problem (4.1) admits a solution $\bar{u}$.

We want to investigate those conditions under which, for each λ in a neighbourhood of $\bar{\lambda}$, the problem (4.1) has a unique solution $u(\lambda)$ near $\bar{u}$ and the function $u(\lambda)$ is continuous and differentiable. We assume that B is the closure of a ball in H centered at $\bar{u}$.

We need the following concepts.

DEFINITION 4.1. The single-valued operator $A(u,\lambda)$ defined on $B\times M$ is said to be,

(i) *Strongly monotone*, if there exists a constant $\alpha>0$ such that

$$(A(u,\lambda)-A(v,\lambda),u-v)\geq\alpha\ \|u-v\|^2,$$

$$\text{for all } \lambda\in M,\ u,v\in B;$$

(ii) *Lipschitz continuous*, if there exist a contant $\beta>0$ such that

$$\|A(u,\lambda)-A(v,\lambda)\|\leq\beta\|u-v\|,$$

$$\text{for all } \lambda\in M,\ u,v\in B;$$

In particular, it follows that $\alpha\leq\beta$.

DEFINITION 4.2 A multivalued mapping $T(u,\lambda)$ defined on $B\times M$ to $C(H)$ is said to be h-*Lipschitz continuous*, if there exist a contant $v>0$ such that

$$h(T(u,\lambda),\ T(v,\lambda))\leq v\|u-v\|$$

for all $u,v\in B$, where $h(.,.)$ is the Hausdörff metric on $C(H)$.

DEFINITION 4.3 A multivalued mapping $A(u,\lambda)$ defined on $B\times M$ to $C(H)$ is said to be *strongly monotone* if there exist a contant $\alpha>0$ such that

$$((p_1,\ \lambda)-(p_2,\lambda),u-v)\geq\ \alpha\ \|u-v\|^2$$

for all $u,v\in B$, $(p_1,\lambda)\in A(u,\lambda)$ and $(p_2,\lambda)\in A(v,\lambda)$.

From Lemma (2.2), we conclude that the problem (4.1) can be transformed to a fixed point problem of the map.

$$F(u,\lambda)=u-g(u)+\ P_{k_\lambda}[g(u)-\rho((p,\lambda)+(q,\lambda))] \tag{4.2}$$

for all $\lambda\in M$, $\rho>0$ is a constant, where P_{k_λ} is the projection of H on the family of closed convex sets K_λ.

Since we are interested in the case, when the solution of the problem (4.1) lies the interior of B, so we consider the map $F^*(u,\lambda)$ defined by

$$F^*(u,\lambda)=u-g(u)+P_{k_{\lambda\cap B}}\ [g(u)-\rho((p,\lambda)+(q,\lambda))] \tag{4.3}$$

$$\text{for all } (u,\lambda)\in B\times M.$$

We have to show that the map $F^*(u,\lambda)$ has a fixed point, which by (4.2) is also a solution of (4.1). First of all we prove that the map $F^*(u,\lambda)$ is a contraction map with respect to u, uniformly in $\lambda\in M$, by using strongly monotonicity and h-Lipschitz continuity of the operator $A(u,\lambda)$, strongly monotonicity and Lipschitz continuity of $g(u)$ and h-Lipschitz continuity of $T(u,\lambda)$.

LEMMA 4.1. For all u_1, $u_2 \in B$ and $\lambda \in M$, we have

$$\|F^*(u_1,\lambda)-F^*(u_2,\lambda)\| \leq \theta\|u_1-u_2\|$$

where

$$\theta = k+\rho\nu+\sqrt{1-2\rho\alpha+\rho^2\beta^2} \qquad \text{for } \nu(1-k)<\alpha, \qquad k<1$$

$$\alpha > \nu(1-k)+ \sqrt{\left((\beta^2-\nu^2)k(2-k)\right)} \qquad \text{and}$$

$$\left|\rho - \frac{\alpha+\beta(k-1)}{\beta^2-\nu^2}\right| < \frac{\sqrt{\left(\alpha+\nu(k-1)^2-(\beta^2-\nu^2)k(2-k)\right)}}{\beta^2-\nu^2}$$

with $k=2\sqrt{\left(1-2\xi+\sigma^2\right)}$

where ξ is the strongly monotone constant of $g(u)$.

PROOF. Using (4.3), we have

$$\|F^*(u_1,\lambda)-F^*(u_2,\lambda)\| = \|\{u_1-g(u_1)+P_{k_\lambda}[g(u_1)-\rho((p_1,\lambda)+(q_1,\lambda))]\}$$

$$-\{u_2-g(u_2)+P_{k_\lambda}[g(u_2)-\rho((p_2,\lambda)+(q_2,\lambda))]\}\|$$

and using the fact that the projection operator is nonexpansive, we have

$$\|F^*(u_1,\lambda)-F^*(u_2,\lambda)\| \leq 2\|u_1-u_2-(g(u_1)-g(u_2))\|+\|u_1-u_2-\rho((p_1,\lambda)-(p_2,\lambda))\|+\rho\|((q_1,\lambda)-(q_2,\lambda))\|$$

Now, the operator $g(u)$ is both strongly monotone and Lipschitz continuous and the operator $A(u,\lambda)$ is strongly monotone and h-Lipschitz continuous, so by the method of Noor [10]

$$\|u_1-u_2-(g(u_1)-g(u_2))\|^2 = \|u_1-u_2\|^2+\|g(u_1)-g(u_2)\|^2-2(u_1-u_2,g(u_1)-g(u_2))$$

$$\leq (1-2\xi+\sigma^2)\|u_1-u_2\|^2 \tag{4.4}$$

where σ is the Lipschitz constant $g(u)$. And

$$\|u_1-u_2-\rho((p_1,\lambda)-(p_2,\lambda))\|^2 = \|u_1-u_2\|^2+\rho^2\|(p_1,\lambda)-(p_2,\lambda)\|^2-2\rho(u_1-u_2,(p_1,\lambda)-(p_2,\lambda))$$

$$\|(p_1,\lambda)-(p_2,\lambda)\| \leq h(A(u_1,\lambda),A(u_2,\lambda))$$

$$\leq \beta\|u_1-u_2\|$$

$$(u_1-u_2,(p_1,\lambda)-(p_2,\lambda)) \geq \alpha\|u_1-u_2\|^2$$

Therefore

$$\|u_1-u_2-\rho((p_1,\lambda)-(p_2,\lambda))\|^2\leq\|u_1-u_2\|^2+\rho^2\beta^2\|u_1-u_2\|^2-2\rho\alpha\|u_1-u_2\|^2$$

$$=(1-2\rho\alpha+\rho^2\beta^2)\|u_1-u_2\|^2 \tag{4.5}$$

where β and α are h-Lipschitz continuous constant and strongly monotone constants of $A(u,\lambda)$ respectively.

$$\|(q_1,\lambda)-(q_2,\lambda)\|\leq h(T(u_1,\lambda),T(u_2,\lambda)\|$$

$$\leq v\|u_1-u_2\| \tag{4.6}$$

since $T(u,\lambda)$ is h-Lipschitz continuous.

Using (4.4) to (4.6), we have

$$\|F^*(u_1,\lambda)-F^*(u_2,\lambda)\|\leq \theta\|u_1-u_2\|$$

where

$$\theta=k+\rho v+\sqrt{1-2\rho\alpha+\rho^2\beta^2} \quad \text{with} \quad k=2\sqrt{(1-2\xi+\sigma^2)}$$

Now, using the techniques of Noor [12], one can show that $\theta<1$ from which it follows that the map $F^*(u,\lambda)$ defined by (4.3) is a contraction map. We now proceed to show that (i) $u(\lambda)$ depends continuously upon λ and (ii) for λ near $\bar{\lambda}$, $u(\lambda)$ is in fact a fixed point of $F(u,\lambda)$, i.e. a solution of the variational inequality (4.1). We also know by assumption, the function $\bar{u}$, for $\lambda=\bar{\lambda}$ is a solution of the parametric generalized variational inequality problem (4.1). We see that $\bar{u}$ is a fixed point of $F(u,\bar{\lambda})$ and it is also a fixed point of $F^*(u,\bar{\lambda})$, consequently we have $u(\bar{\lambda})=\bar{u}=F^*(u(\bar{\lambda}),\bar{\lambda})$.

We now show that the solution $u(\lambda)$ of the problem (4.1) is continuous (Lipschitz continuous).

LEMMA 4.2. If the multivalued operators $T(\bar{u},\lambda)$, $A(\bar{u},\lambda)$, the single-valued operator $g(\bar{u})$ and the map

$$\lambda\rightarrow P_{K_{\lambda\cap S}}[g(\bar{u})-\rho((\bar{p},\bar{\lambda})+(\bar{q},\bar{\lambda}))]$$

are continuous (Lipschitz continuous), in λ at $\bar{\lambda}$, then the solution $u(\lambda)$ of (4.1) is continuous (Lipschitz continuous) at $\lambda=\bar{\lambda}$.

PROOF: Fix $\lambda\in M$. Then using the triangle inequality and Lemma (4.1), we have

$$\|u(\lambda)-u(\bar{\lambda})\|=\|F^*(u(\lambda),\lambda)-F^*(u(\bar{\lambda}),\lambda)\|+\|F^*(u(\bar{\lambda}),\lambda)-F^*(u(\bar{\lambda}),\bar{\lambda})\|$$

$$\leq\theta\|u(\lambda)-u(\bar{\lambda})\|+\|F^*(u(\bar{\lambda}),\lambda)-F^*(u(\bar{\lambda}),\bar{\lambda})\| \tag{4.7}$$

From (4.3) and the fact that the projection map is nonexpansive, we have

$$\|F^*(u(\bar{\lambda}),\lambda)-F^*(u(\bar{\lambda}),\bar{\lambda})\|=\|P_{k_{\lambda\cap B}}[g(u(\bar{\lambda}))-\rho((p(\bar{\lambda}),\lambda)+(q(\bar{\lambda}),\lambda))]\|$$

$$-\|P_{k_{\lambda\cap B}}[g(u(\bar{\lambda}))-\rho((p(\bar{\lambda}),\bar{\lambda})+(q(\bar{\lambda}),\bar{\lambda}))]\|$$

$$\leq\rho\|(p(\bar{\lambda}),\lambda)-(p(\bar{\lambda}),\lambda)\|+\rho\|(q(\bar{\lambda}),\lambda)-(q(\bar{\lambda}),\lambda)\|$$

$$+\|P_{k_{\lambda\cap B}}[g(u(\bar{\lambda}))-\rho((p(\bar{\lambda}),\bar{\lambda})+(q(\bar{\lambda}),\bar{\lambda}))]$$

$$-P_{k_{\bar{\lambda}\cap B}}[g(u(\bar{\lambda}))-\rho((p(\bar{\lambda}),\bar{\lambda})+(q(\bar{\lambda}),\bar{\lambda}))]\| \qquad (4.8)$$

Now from Remark (4.1) and combining (4.7) and (4.8), we have

$$\|u(\lambda)-\bar{u}\|\leq\frac{\rho}{1-\theta}\|(p(\bar{\lambda}),\lambda)-(p(\bar{\lambda}),\bar{\lambda})\|+\frac{\rho}{1-\theta}\|(q(\bar{\lambda}),\lambda)-(q(\bar{\lambda}),\bar{\lambda})\|$$

$$+\frac{\rho}{1-\theta}\|(q(\bar{\lambda}),\lambda)-(q(\bar{\lambda}),\bar{\lambda})\|+\frac{1}{1-\theta}\|P_{k_{\lambda\cap B}}[g(\bar{u})-\rho((p(\bar{\lambda}),\bar{\lambda})+(q(\bar{\lambda}),\bar{\lambda}))]$$

$$-P_{k_{\bar{\lambda}\cap B}}[g(\bar{u})-\rho((p(\bar{\lambda}),\bar{\lambda})+(q(\bar{\lambda}),\bar{\lambda}))]\|$$

From which the required result follows.

The following result follows by similar arguments as given in Lemma 4.2.

LEMMA 4.3. Under the assumption of Lemma 4.2, there exists a neighbourhood $N\subset M$ of $\bar{\lambda}$ such that $\lambda\in N$, $u(\lambda)$ is the unique solution of problem (4.1) in the interior of B.

Combining the above results we arrive at the following:

THEOREM 4.1. Let $\bar{u}$ be the solution of the parametric generalized variational inequality problem (4.1) at $\lambda=\bar{\lambda}$, the multivalued $T(\bar{u},\lambda)$ be h-Lipschitz continuous and the multivalued mapping $A(\bar{u},\lambda)$ is strongly monotone and locally h-Lipschitz continuous, the map $g(\bar{u})$ be locally strongly monotone and locally h-Lipschitz continuous.

Suppose that $T(\bar{u},\lambda)$, $A(\bar{u},\lambda)$, $g(\bar{u})$ and the map

$$\lambda\rightarrow P_{k_{\lambda\cap B}}[g(\bar{u})-\rho((\bar{p},\bar{\lambda})+(\bar{q},\bar{\lambda}))]$$

are continuous (Lipschitz continuous) at $\lambda=\bar{\lambda}$, then there exists a neighbourhood $N\subset M$ of λ such that for $\lambda\in N$, the problem(4.1) has a unique solution $u(\lambda)$ in the interior of B, $u(\bar{\lambda})=\bar{u}$ and $u(\lambda)$ are continuous (Lipschitz continuous) at $\lambda=\bar{\lambda}$.

REMARK 4.2. The function $u(\lambda)$ as defined in theorem 4.1 is continuously differantiable on some neighbourhood N of λ. For this see Dafermos [3].

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