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EFFECTS OF TIME DELAY ON CHAOTIC NEURONAL DISCHARGES

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Abstract- Effects of time delay on Hindmarsh-Rose(HR) model neuron are studied. For an individual neuron, with the scaling delay time and synaptic intensity, neuronal firing pattern's transform among tonic spiking, busting and resting firing state, and the neuronal chaotic spike could be controlled. Furthermore, two coupled HR neuronal system could be fully synchronized under certain coupled strength and delay time.

Keywords- Chaos, HR neuron, Delay time, Synchronization

1. INTRODUCTION

Time-delayed feedback mechanisms are relevant in many biological systems, generally, the systems operate under the influence of time-delayed feedback mechanisms, a single neuron might influence a recurrent loop though an auto-synapse, or coupled neuronal system though synaptic connections involving other neurons, the finite conduction velocity and the information processing time in synapses lead to transmission delay time [1]. Experimental results suggested that there was a transmission delay time in the crayfish stretch receptor organ [2]. Many feedback loops have been proposed to explain patterns of neural activity, a well known recurrent circuit is in the hippocampus CA3 region, which is known to be involved in associative memory recall [3]. At same times, synchronization of nonlinear systems is a fascinating subject that have been extensively studied on large variety of physical and biological systems [4-6], furthermore, physiological experiments have indicated the existence of synchronous activation of neurons in different areas of the brain of some animals [7].

Many neuroscientists and biologists, have done a lot of research on nonlinear dynamical system with time-delay, obtained a great deal of significant achievements, such as, controlling the activities of system by time-delay skillfully[8-10], specially, it is a typical way to control chaos by feedback of time-delay, and enhance synchronization of coupled dynamical systems.

The influence of time-delay on chaotic system has attracted particular attention extensively, an individual neuron can exhibit different intrinsic oscillatory activities introduced by external currents [11,12]. However, it is also interesting to analyze discharge activities induced by the time-delayed synaptic interaction between neurons without external current. In this case the properties of discharge activities of each neuron depend on the synaptic intensity ε and the time-delay τ. For this purpose, the aim of this work is to present the effect of a time delayed feedback loop on the firing patterns of HR model neuron, on one hand, recurrent feedback loops to control spikes from chaotic to periodic with fixed delay time and auto-synaptic intensity, on the other hand, fully synchronization of two coupled neuron under certain coupled strength and time delay.

2. TIME DELAYED HINDMARSH-ROSE MODEL NEURON

Here, one time-delay HR model neuron with external stimulus I is introduced, and the changes of discharge patterns of the neuron are studied on auto-synaptic intensity ε , the time-delay τ, as well as their dynamical behavior. The model neuron is described as follows,

$$\dot{x} = y - ax^3 + bx^2 - z + I + \varepsilon_{as}(x(t-\tau)\ , \quad \dot{y} = c - dx^2 - y\ , \quad \dot{z} = r[S(x-\chi) - z] \qquad (1)$$

where, x is the membrane potential of neuron, y is a recovery variable associated with the fast current for a sodium current or a potassium current, and z is a slowly changing adaptation current for a calcium current [13]. The values of parameters of model are: a=1.0, b=3.0, c=1.0, d=5.0, χ=-1.6, r=0.013, s=4.0. The first equation of model (1) describes the effect of feedback connection, x(t-τ) is the membrane potential at the earlier time t-τ, here ε_{as} is auto-synaptic intensity, τ is time-delay, I is external stimulus dc current.

3. CONTROLLING OF CHAOTIC DISCHARGES BY TIME DELAYED FEEDBACK LOOP

In this section, we main focuses on the controlling of chaotic discharges of individual HR model neuron with time delayed, shown in equations (1). The external stimulus I is set as 3.10 μA, as one external stimulus current is introduced to the first equation of neuron model to generate chaotic discharges, with ε_{as}=0 ms^{-1}, as well know, which makes the neuron to display chaotic discharges, it could been found out in the first return map of intervalspike interval (ISI), the typical chaotic characteristics indicated by chaotic saddle shown in Fig.1a.

Then, in the neuronal chaotic activities case, one feedback loop time-delay is added to the HR neuron, the neuron will show abundant activities, for examples, while the auto-synaptic intensity ε_{as} =0.4 ms^{-1} and time delay satisfies $\tau \in$ [30-45] ms, the neuronal discharges will change from chaotic to period-1, one typical return map of which shown Fig.1b, only one point in the plane; while ε_{as} = 0.2 ms^{-1} and $\tau \in$ [40, 60] ms. We also found that the neuronal chaotic spike could be controlled to period-2, period-3, period-4, and period-5 by different auto-synaptic intensity and time delay, all of which illustrated in Fig.1c,d,e,f, respectively.

To verify effects of the bigger auto-synaptic intensity and time delay on the HR neuronal discharges pattern, it found out that the neuron will return to chaotic activities after time delay τ greater than 150 ms for the set auto-synaptic intensity ε_{as} = 0.1 ms^{-1}, on the other hand, the bigger auto-synaptic intensity (e.g., ε_{as} = 3.0 ms^{-1}) will inhibit neuronal spike completely.

Here, we show influence of the some particular values of the delay time of the feedback loop modifies the HR neuronal chaotic discharges, the goal is to investigate the effect of a time delay feedback loop in the dynamics of an individual neuron.

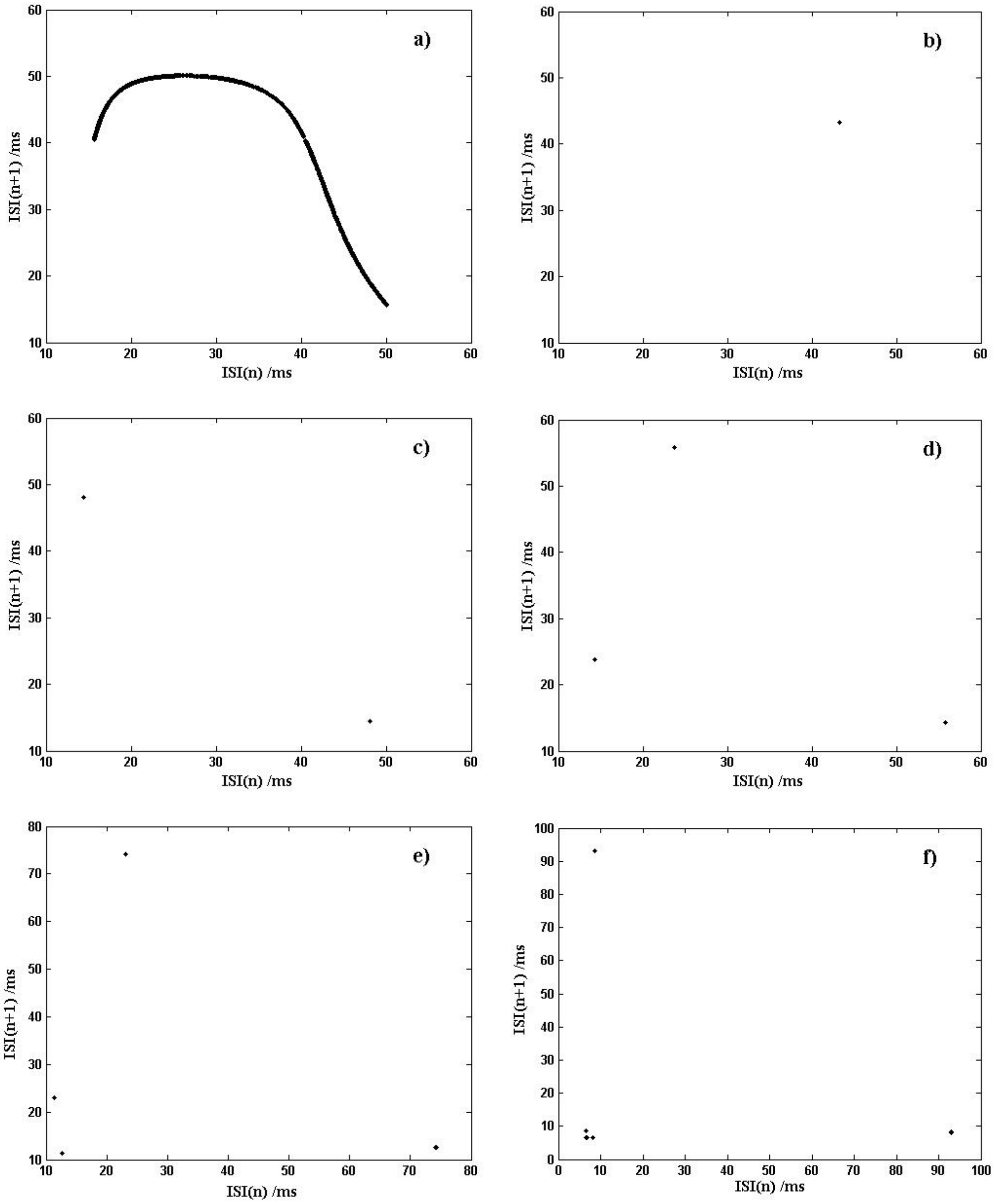

Fig.1 Diagram of the first return map of neuronal interspike interval (ISI) sequences, ISI(n) and ISI($n+1$) are value of nth, n+1th neuronal interspike interval, respectively. a) Chaotic discharges with ε_{as}=0 ms^{-1} ; b) Period-1 discharges with ε_{as}=0.4 ms^{-1}, τ=30 ms; c) Period-2 discharges with ε_{as}=0.2 ms^{-1}, τ=50 ms; d) Period-3 discharges with ε_{as}=0.1 ms^{-1}, τ=20 ms; e) Period-4 discharges with ε_{as}=0.4 ms^{-1}, τ=20 ms; f) Period-5 discharges with ε_{as}=0.8 ms^{-1}, τ=10 ms

4. TIME DELAY-ENHANCED SYNCHRONIZATION IN TWO COUPLED CHAOTIC HR NEURONS

Synchronization processes are of crucial importance for brain function. Well-coordinated synchrony within and between neuronal populations appears to be an important mechanism for neuronal signaling and information processing [14]. In this work, we consider the following two coupled HR neuronal system:

$$\begin{cases} \dot{x}_1 = y_1 - a{x_1}^3 + b{x_1}^2 - z_1 + I + \varepsilon(x_1 - x_2(t-\tau)) \\ \dot{y}_1 = c - d{x_1}^2 - y_1 \\ \dot{z}_1 = r[S(x_1 - \chi) - z_1] \\ \dot{x}_2 = y_2 - a{x_2}^3 + b{x_2}^2 - z_2 + I + \varepsilon(x_2 - x_1(t-\tau)) \\ \dot{y}_2 = c - d{x_2}^2 - y_2 \\ \dot{z}_2 = r[S(x_2 - \chi) - z_2] \end{cases} \tag{2}$$

where, ε is synaptic coupled intensity, $x_{1,2}$ is the membrane potential of the first and second neuron, respectively. And all of the right parameters are same to the single HR neuron shown in model equations (1), and the same external stimulus dc current is applied for two coupled, i.e., I =3.10 μA.

Here, in order to verify the degree of synchronization of the two coupled neuron under the variation of time delay and synaptic intensity, one real-time error of response of the two coupled neuron is defined as:

$$e(t) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \tag{3}$$

With the changing of time delay and synaptic intensity, the original chaotic spike state could be synchronized, including anti-synchronization. As one example, with fixed $\varepsilon = 0.4\ ms^{-1}$, different time delay, the coupled neuronal system (see Eq. (2)) shows different firing pattern, while delay time smaller than 18 ms, two neuron still shown irregular spiking activities. When delay time grown up to 18 ms, two neurons changed irregular to period-1 anti-synchronization firing state, shown in Fig.2a, and this process takes 1600 ms. Seen from Fig.2b, after synchronized, we could found that the performance of error of synchronization *e*(*t*) regular oscillation against time. With delay time increasing to 20 ms, the coupled neuron could been completely synchronized period-2 firing state, shown in Fig.2d, the time course of membrane potential overlap each other as shown in enlarged Fig.2f, and the value of *e*(*t*) decreases to zero at 1300 ms, which is shorter than anti-synchronization process at τ =18 ms, the different phase space of anti-synchronization and synchronization are shown in Fig.2c,f, respectively. Along with time increasing two neurons will back to irregular again. All of these suggest that synchronization is sensitive depend on delay time for fixed synaptic coupled intensity.

At same time, there are many anti-synchronization and synchronization process for different synaptic coupled intensity as well as delay time, such as $\varepsilon = 3.0\ ms^{-1}$, one bigger different synaptic coupled intensity, the time synchronization will increasing along with delay time changed from 20 to 50 ms. Another synaptic coupled intensity ε =0.9 ms-1, we also observed the sequence of spikes as delay time increases, two neuron changed from period-3 anti-synchronization (τ=50 ms) to completed synchronization (τ =60 ms) firing state.

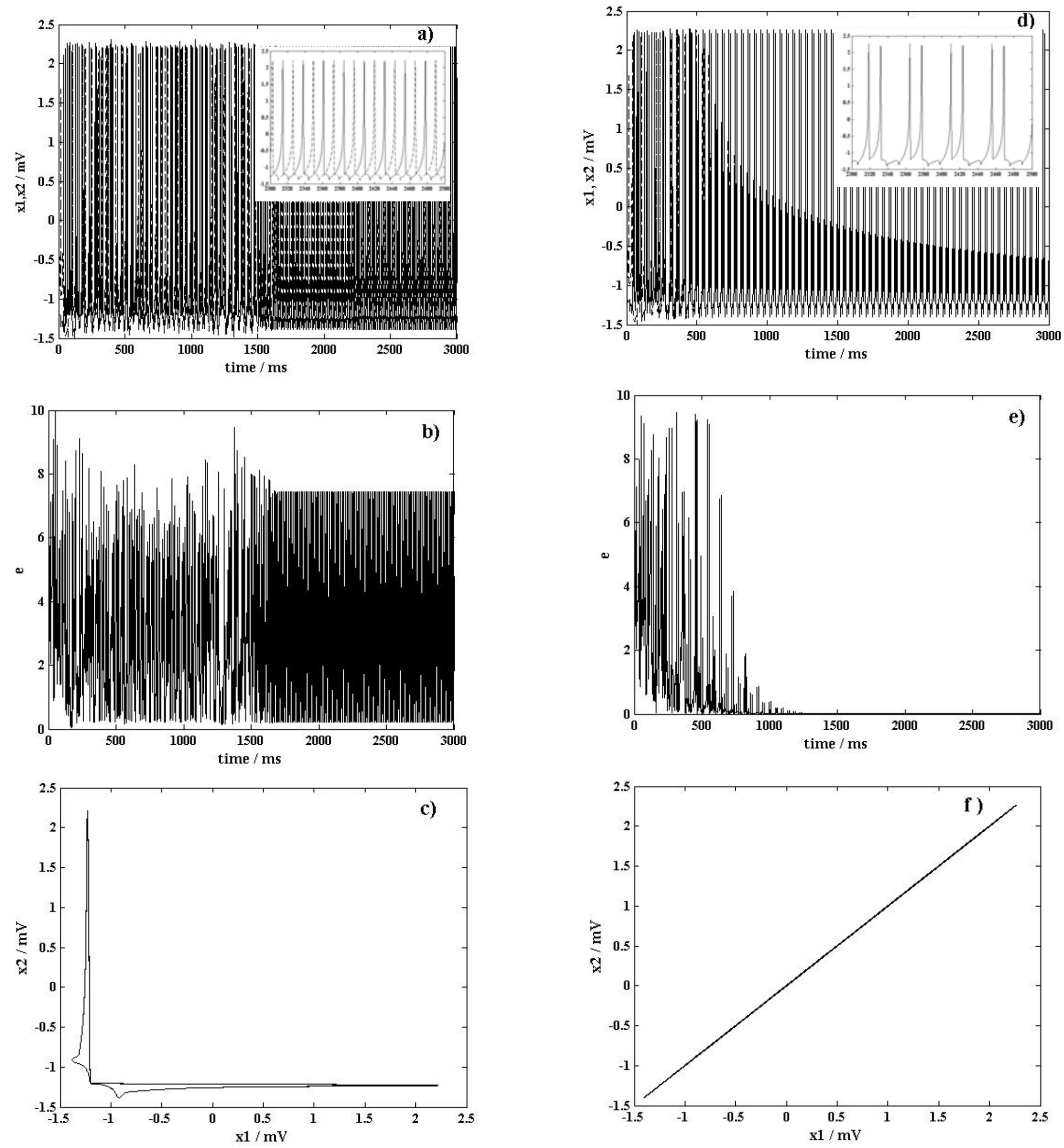

Fig.2 Time series and membrane potential correlation between coupled two neurons: anti-synchronization and complete synchronization at small coupling strengths ε = 0.4 ms^{-1}. a) x_1(dot line) and x_2 (solid line) versus t at τ =18 ms; b) e(t) versus t at τ =18 ms; c) x_1 versus x_2 at τ =18 ms; d) x_1(dot line) and x_2 (solid line) versus t at τ =20 ms; e) e(t) versus t at τ =20 ms; f) x_1 versus x_2 at τ =20 ms

5. CONCLUDING REMARKS

In summary, we have studied the effects of a time-delayed on a single and two coupled Hindmarsh-Rose model neuron. The results indicate that while auto-synaptic intensity between (0 1), the neuron could be controlled from chaotic to periodic or bursting firing by different delay time feedback. We also observed the anti-synchronization and complete synchronization with the different delay time and coupling strengths, and it will take different length of time depends on time delay up to synchronal state. These finding may have significance in synchronizing large groups of nearby cells in the cortex via weak synaptic input from other areas.

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