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# A NEW DECOMPOSITION METHOD FOR SOLVING SYSTEM OF NONLINEAR EQUATIONS

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**Abstract-** In this paper, an efficient decomposition method is constructed and used for solving system of nonlinear equations. The method based on the decomposition technique of Noor [M.A.Noor, K.I.Noor, Some iterative schemes for nonlinear equations, Appl. Math. Comput. 183(2006), 774-779]. This technique is revised to solve the system of nonlinear equations. Some illustrative examples have been presented, to demonstrate the proposed method and the results are compared with those derived from the previous methods. All test problems reveals the accuracy and fast convergence of the suggested method.

**Keywords-**Decomposition method, Iterative methods, System of nonlinear equations.

## 1. INTRODUCTION

Recently, several iterative methods have been made on the development for solving nonlinear equations and system of nonlinear equations. These methods have been improved using Taylor interpolating polynomials, quadrature formulas, homotopy perturbation method and decomposition techniques [1-14]. Chun [3] by improving Newton method has presented a new iterative method to solve nonlinear equations. His work is based on modification of the Abbasbandy's study [1]. Their methods have contained higher order differential derivatives displaying a serious drawback. To overcome this difficulty, Noor et al. [12] have considered an alternative decomposition technique which does not involve the derivative of the Adomian polynomial. Furthermore, Darvishi et al.[4,5] by using Adomian decomposition constructed new methods and Golbabai et al.[7] have applied the homotopy perturbation method to build a new family of Newton-like iterative methods for solving system of nonlinear equations.

In this paper, a new iterative method was constructed by using the Noor's decomposition technique [12,13]. This technique, however, needs to be revised to solve the system of nonlinear equations. Some illustrative examples have been presented, to demonstrate our method and the results are compared with those derived from the previous methods. All test problems reveals the accuracy and fast convergence of the new method.

## 2. ITERATIVE METHOD

Consider the system of nonlinear equations of the form

$$\mathbf{F}(\mathbf{x}) = \begin{cases} f(\mathbf{x}) = 0, \\ g(\mathbf{x}) = 0, \end{cases} \qquad \mathbf{x} = [x, y]^{\mathrm{T}} \in R^2 \tag{1}$$

where $f, g: R^2 \to R$ and $\mathbf{F}: R^2 \to R^2$. Suppose that $\mathbf{x}^+ = [\mu, \sigma]$ is a root of (1) and $\mathbf{u} = [\lambda, \gamma]^{\mathrm{T}}$ is an initial estimation sufficiently close to $\mathbf{x}^+$. Using Taylor's series around $\mathbf{u}$ for (1), we have

$$\mathbf{F}(\mathbf{x}) = \begin{cases} f(\mathbf{u}) + (x-\lambda) f_x(\mathbf{u}) + (y-\gamma) f_y(\mathbf{u}) + \frac{1}{2!}\left[(x-\lambda)^2 f_{xx}(\mathbf{u}) + 2(x-\lambda)(y-\gamma) f_{xy}(\mathbf{u}) + (y-\gamma)^2 f_{yy}(\mathbf{u})\right] + \cdots \\ g(\mathbf{u}) + (x-\lambda) g_x(\mathbf{u}) + (y-\gamma) g_y(\mathbf{u}) + \frac{1}{2!}\left[(x-\lambda)^2 g_{xx}(\mathbf{u}) + 2(x-\lambda)(y-\gamma) g_{xy}(\mathbf{u}) + (y-\gamma)^2 g_{yy}(\mathbf{u})\right] + \cdots \end{cases}, \tag{2}$$

$$\begin{bmatrix} f(\mathbf{u}) \\ g(\mathbf{u}) \end{bmatrix} + \begin{bmatrix} f_x(\mathbf{u}) & f_y(\mathbf{u}) \\ g_x(\mathbf{u}) & g_y(\mathbf{u}) \end{bmatrix} [\mathbf{x}-\mathbf{u}] + \frac{1}{2!}\left\{ \mathbf{e}_1 \otimes [\mathbf{x}-\mathbf{u}]^{\mathrm{T}} \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}-\mathbf{u}] + \mathbf{e}_2 \otimes [\mathbf{x}-\mathbf{u}]^{\mathrm{T}} \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}-\mathbf{u}] \right\} = \mathbf{0} \tag{3}$$

where $\otimes$ is the Kronecker product and $\mathbf{e}_i$ is a $2x1$ vector of zero except for a 1 in position $i$.

We can rewrite (3) as

$$\mathbf{x} = \mathbf{c} + \mathrm{N}(\mathbf{x}), \tag{4}$$

where

$$\mathbf{c} = \mathbf{u} - \mathbf{J}^{-1}(\mathbf{u})\mathbf{F}(\mathbf{u}), \tag{5}$$

$$\mathrm{N}(\mathbf{x}) = -\frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ \mathbf{e}_1 \otimes [\mathbf{x}-\mathbf{u}]^{\mathrm{T}} \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}-\mathbf{u}] + \mathbf{e}_2 \otimes [\mathbf{x}-\mathbf{u}]^{\mathrm{T}} \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}-\mathbf{u}] \right\}, \tag{6}$$

and

$$\mathbf{J}^{-1}(\mathbf{u}) = \mathbf{J}^{-1}(f, g)(\mathbf{u}) = \begin{bmatrix} f_x(\mathbf{u}) & f_y(\mathbf{u}) \\ g_x(\mathbf{u}) & g_y(\mathbf{u}) \end{bmatrix}^{-1}. \tag{7}$$

To satisfy that $\mathrm{N}(\mathbf{x})$ is nonlinear and vector form, the following decomposition method constructed by Noor & Noor [12] was used in the vector form

$$\mathbf{x} = \sum_{i=0}^{\infty} \mathbf{x}_i. \tag{8}$$

The nonlinear operator $\mathrm{N}(\mathbf{x})$ can be decomposed as

$$\mathrm{N}\left(\sum_{i=0}^{\infty} \mathbf{x}_i\right) = \mathrm{N}(\mathbf{x}_o) + \sum_{i=1}^{\infty} \left\{ \mathrm{N}\left(\sum_{j=0}^{i} \mathbf{x}_j\right) \right\}. \tag{9}$$

Combining (4), (8), and (9), the iterative scheme can be obtained as

$$\begin{aligned}
&\mathbf{x}_o = \mathbf{c},\\
&\mathbf{x}_1 = N(\mathbf{x}_o),\\
&\mathbf{x}_2 = N(\mathbf{x}_o + \mathbf{x}_1),\\
&\vdots\\
&\mathbf{x}_{n+1} = N(\mathbf{x}_o + \mathbf{x}_1 + \cdots + \mathbf{x}_n),\ n = 1,2\ldots.
\end{aligned} \tag{10}$$

Then

$$\mathbf{x}_1 + \mathbf{x}_2 + \cdots + \mathbf{x}_n = N(\mathbf{x}_o) + N(\mathbf{x}_o + \mathbf{x}_1) + \cdots + N(\mathbf{x}_o + \mathbf{x}_1 + \cdots + \mathbf{x}_n),\ \ n = 1,2,\ldots \tag{11}$$

and

$$\mathbf{x} = \mathbf{c} + \sum_{i=1}^{\infty} \mathbf{x}_i\ . \tag{12}$$

From (5), (6), (8), and (10), it follows

$$\mathbf{x}_o = \begin{bmatrix} x_o \\ y_o \end{bmatrix} = \mathbf{c} = \mathbf{u} - \mathbf{J}^{-1}(\mathbf{u})\mathbf{F}(\mathbf{u}) \tag{13}$$

and

$$\mathbf{x}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = N(\mathbf{x}_o) = -\frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ e_1 \otimes [\mathbf{x}_o - \mathbf{u}]^T \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o - \mathbf{u}] + e_2 \otimes [\mathbf{x}_o - \mathbf{u}]^T \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o - \mathbf{u}] \right\}. \tag{14}$$

Joining (8), (10), and (13),

$$\mathbf{x} = \mathbf{c} = \mathbf{x}_o = \mathbf{u} - \mathbf{J}^{-1}(\mathbf{u})\mathbf{F}(\mathbf{u})$$

is obtained. This gives us Newton's method for solving the nonlinear system $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ as computing the approximate solution $\mathbf{x}_{n+1}$ for a given $\mathbf{x}_o$, by the iterative scheme

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \mathbf{J}^{-1}(\mathbf{x}_n)\mathbf{F}(\mathbf{x}_n),\quad n \geq 0.$$

Using (8), (10), and (14), we obtain

$$\mathbf{x} = \mathbf{x}_o + N(\mathbf{x}_o)$$

$$\mathbf{x} = \mathbf{u} - \mathbf{J}^{-1}(\mathbf{u})\mathbf{F}(\mathbf{u}) - \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ e_1 \otimes [\mathbf{x} - \mathbf{u}]^T \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x} - \mathbf{u}] + e_2 \otimes [\mathbf{x} - \mathbf{u}]^T \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x} - \mathbf{u}] \right\}$$

This formulation produces the following iterative methods for solving system of nonlinear equation (1).

**Algorithm 2.1.** For a given $\mathbf{x}_o = \begin{bmatrix} x_o \\ y_o \end{bmatrix}$ compute the approximate solution $\mathbf{x}_{n+1} = \begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix}$ by the iterative schemes

$$\mathbf{w}_n = \begin{bmatrix} v_n \\ z_n \end{bmatrix} = -\mathbf{J}^{-1}(\mathbf{x}_n)\mathbf{F}(\mathbf{x}_n),$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \mathbf{w}_n - \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{x}_n)\left\{ e_1 \otimes [\mathbf{w}_n - \mathbf{x}_n]^T \begin{bmatrix} f_{xx}(\mathbf{x}_n) & f_{xy}(\mathbf{x}_n) \\ f_{yx}(\mathbf{x}_n) & f_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{w}_n - \mathbf{x}_n] \right.$$
$$\left. + \mathbf{e}_2 \otimes [\mathbf{w}_n - \mathbf{x}_n]^T \begin{bmatrix} g_{xx}(\mathbf{x}_n) & g_{xy}(\mathbf{x}_n) \\ g_{yx}(\mathbf{x}_n) & g_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{w}_n - \mathbf{x}_n] \right\}, \quad n \geq 0.$$

The above algorithm was defined by Golbabai and Javidi [7] using the homotopy perturbation method. It has been shown that the Noor and Noor decomposition is very simple and easier than the homotopy perturbation and Adomian decomposition methods for solving the nonlinear systems.

Now, Algorithm 2.1 can be improved by using (6) and (10) as following:

$$\mathbf{x}_2 = N(\mathbf{x}_o + \mathbf{x}_1)$$

$$\mathbf{x}_2 = -\frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ e_1 \otimes [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}]^T \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}] + e_2 \otimes [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}]^T \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}] \right\}. \tag{15}$$

Substituting the equations (13), (14), and (15) into (8), we can obtain

$$\mathbf{x} = \mathbf{x}_o + N(\mathbf{x}_o) + N(\mathbf{x}_o + \mathbf{x}_1)$$

$$\mathbf{x} = \mathbf{u} - \mathbf{J}^{-1}(\mathbf{u})\mathbf{F}(\mathbf{u}) - \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ e_1 \otimes [\mathbf{x}_o - \mathbf{u}]^T \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o - \mathbf{u}] + e_2 \otimes [\mathbf{x}_o - \mathbf{u}]^T \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o - \mathbf{u}] \right\}$$
$$- \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{u})\left\{ e_1 \otimes [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}]^T \begin{bmatrix} f_{xx}(\mathbf{u}) & f_{xy}(\mathbf{u}) \\ f_{yx}(\mathbf{u}) & f_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}] + e_2 \otimes [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}]^T \begin{bmatrix} g_{xx}(\mathbf{u}) & g_{xy}(\mathbf{u}) \\ g_{yx}(\mathbf{u}) & g_{yy}(\mathbf{u}) \end{bmatrix} [\mathbf{x}_o + \mathbf{x}_1 - \mathbf{u}] \right\}.$$

This formulation allows us to propose the following iterative method for solving system of nonlinear equation (1).

**Algorithm 2.2.** For a given $\mathbf{x}_o$, compute the approximate solution $\mathbf{x}_{n+1}$ by the iterative schemes

$$\mathbf{y}_n = \mathbf{x}_n - \mathbf{J}^{-1}(\mathbf{x}_n)\mathbf{F}(\mathbf{x}_n), \quad n = 0,1,2,\ldots$$

$$\mathbf{t}_n = \begin{bmatrix} p_n \\ q_n \end{bmatrix} = -\frac{1}{2!}\mathbf{J}^{-1}(\mathbf{x}_n)\left\{ e_1 \otimes [\mathbf{y}_n - \mathbf{x}_n]^T \begin{bmatrix} f_{xx}(\mathbf{x}_n) & f_{xy}(\mathbf{x}_n) \\ f_{yx}(\mathbf{x}_n) & f_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n - \mathbf{x}_n] + e_2 \otimes [\mathbf{y}_n - \mathbf{x}_n]^T \begin{bmatrix} g_{xx}(\mathbf{x}_n) & g_{xy}(\mathbf{x}_n) \\ g_{yx}(\mathbf{x}_n) & g_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n - \mathbf{x}_n] \right\}$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \mathbf{J}^{-1}(\mathbf{x}_n)\mathbf{F}(\mathbf{x}_n)$$
$$- \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{x}_n)\left\{ e_1 \otimes [\mathbf{y}_n - \mathbf{x}_n]^T \begin{bmatrix} f_{xx}(\mathbf{x}_n) & f_{xy}(\mathbf{x}_n) \\ f_{yx}(\mathbf{x}_n) & f_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n - \mathbf{x}_n] + e_2 \otimes [\mathbf{y}_n - \mathbf{x}_n]^T \begin{bmatrix} g_{xx}(\mathbf{x}_n) & g_{xy}(\mathbf{x}_n) \\ g_{yx}(\mathbf{x}_n) & g_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n - \mathbf{x}_n] \right\}$$
$$- \frac{1}{2!}\mathbf{J}^{-1}(\mathbf{x}_n)\left\{ e_1 \otimes [\mathbf{y}_n + \mathbf{t}_n - \mathbf{x}_n]^T \begin{bmatrix} f_{xx}(\mathbf{x}_n) & f_{xy}(\mathbf{x}_n) \\ f_{yx}(\mathbf{x}_n) & f_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n + \mathbf{t}_n - \mathbf{x}_n] + e_2 \otimes [\mathbf{y}_n + \mathbf{t}_n - \mathbf{x}_n]^T \begin{bmatrix} g_{xx}(\mathbf{x}_n) & g_{xy}(\mathbf{x}_n) \\ g_{yx}(\mathbf{x}_n) & g_{yy}(\mathbf{x}_n) \end{bmatrix} [\mathbf{y}_n + \mathbf{t}_n - \mathbf{x}_n] \right\}. \tag{16}$$

## 3. APPLICATIONS

In this section, two examples are presented to illustrate the efficiency of the method developed in this study. By applying Algorithm 2.2(A2.2), the results are compared with standard Adomian decomposition method (SADM), revised Adomian decomposition method (RADM) [6], the algorithms A1 and A2 obtained by using the homotopy perturbation method by Golbabai and Javidi in [7], and Newton-Raphson method(N-R). The computational results for two examples are presented in Table 1 and Table 2. These examples exhibit the accuracy and convergence of the developed method, numerically.

*Example 1*: Consider the following system of nonlinear equations [7,10] using initial approximation $\mathbf{x}_o = [0.8, 0.8]^T$:

$$\begin{aligned} &x^2 - 10x + y^2 + 8 = 0, \\ &xy^2 + x - 10y + 8 = 0. \end{aligned} \tag{17}$$

The exact solution of the problem is $(1,1)$.

*Example 2*: Consider the other system of nonlinear equations [7] :

$$\begin{aligned} x^2y - 1 + x - y &= 0, \\ x^2 + y^2 - 2 &= 0. \end{aligned} \tag{18}$$

We solve this system by (16) using initial approximation $\mathbf{x}_o = [0.5, 0.5]^T$. The exact solution of this problem is also $(1,1)$.

Table 1: Numerical results showing example1 with $x_o = 0.8$ and $y_o = 0.8$

| Method | Number of Iterations | x,y | Obtained solution |
|---|---|---|---|
| A 2.2 | 2 | x | 1.00003866167173 |
| | | y | 1.00004979886010 |
| A 2.2 | 3 | x | 1.00000000027180 |
| | | y | 1.00000000076022 |
| SADM | 5 | x | 0.99607593000000 |
| | | y | 0.99556077000000 |
| RADM | 5 | x | 0.99778000000000 |
| | | y | 0.99785300000000 |
| A1 | 2 | x | 0.99994464092480 |
| | | y | 0.99992464311298 |
| A1 | 3 | x | 0.99999999884257 |
| | | y | 0.99999999823479 |
| N-R | 2 | x | 0.99992606251646 |
| | | y | 0.99990539102108 |
| N-R | 3 | x | 0.99999999731318 |
| | | y | 0.99999999646110 |

Table 2: Numerical results showing example 2 with $x_o = 0.5$ and $y_o = 0.5$

| Method | Number of Iterations | x,y | Obtained solution |
|---|---|---|---|
| A 2.2 | 3 | x | 0.95850974858149 |
| | | y | 0.99220393210356 |
| A 2.2 | 4 | x | 0.99916931633484 |
| | | y | 0.99987041741491 |
| A2 | 2 | x | 0.90503300648706 |
| | | y | 0.96134766510584 |
| A2 | 3 | x | 0.99975332008120 |
| | | y | 1.00191363624924 |
| N-R | 2 | x | 1.06935578416718 |
| | | y | 0.98691638090214 |
| N-R | 3 | x | 1.00084842985638 |
| | | y | 1.00160442120459 |

## 4. CONCLUSION

A new iterative method is constructed and applied to solving the system of nonlinear equations. The suggested method is shown to be more convergent than the Adomian decomposition, revised Adomian decomposition methods [6], and Newton-Raphson method and is useful and effective as the homotopy perturbation method [7]. Finally, our proposed method can be used as an alternative method for solving system of nonlinear equations, and in engineering problems related to nonlinear systems.

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