processes

MDPI

Article

Multi-Objective Bus Timetable Coordination Considering Travel Time Uncertainty

Xueping Dou and Tongfei Li *

Beijing Key Laboratory of Traffic Engineering, Beijing University of Technology, Beijing 100124, China
* Correspondence: tfli@bjut.edu.cn

Abstract: This paper proposes a timetable coordination method for transfer problems in a bus transit system. With a given bus network, a stochastic mixed-integer linear programming (MILP) model has been formulated to obtain coordinated bus timetables with the objective of minimizing a weighted sum of the average value of total waiting time and its average absolute deviation value, allowing for random bus travel time. The vital decision variable is the terminal departure offset time of each target bus trip within a certain off-peak period. The robust MILP model can also be used to solve the first-bus transfer problem with the introduction of several new linear constraints. A solution method based on the Monte Carlo simulation has been developed to solve the MILP model. Numerical experiments have been conducted for different scenarios. The results indicate that bus timetables coordinated by the developed model are capable of substantially reducing waiting time for transfer and non-transfer passengers. In addition, the feasibility of simplifying a common sub-route into a single transfer stop in a timetable coordination problem has been explored based on numerical experiments.

Keywords: bus operation; multi-objective optimization; timetable coordination; waiting time; simulation

Citation: Dou, X.; Li, T. Multi-Objective Bus Timetable Coordination Considering Travel Time Uncertainty. *Processes* **2023**, *11*, 574. https://doi.org/10.3390/pr11020574

Academic Editor: Fabricio Napoles-Rivera

Received: 25 November 2022
Revised: 31 December 2022
Accepted: 20 January 2023
Published: 13 February 2023

1. Introduction

With the bus network expansion, especially in high-density cities, buses take on a more significant role for commuters because they not only serve direct routes, but also connect people to other routes. For example, Singapore generated on average more than a 4.0 million bus ridership per day in 2019, accounting for 53.3% of the island-wide average daily public transport ridership [1]. However, a major inconvenience in public transport is the need to make a transfer during the journey [2]. From passengers' perspective, ideally, there should be as many as routes as possible to provide direct bus services, but this is infeasible due to vast cost overruns of the bus operator. Transfers are integral to a public transit system that provides access to a large amount of potential destinations at an acceptable operating cost. An attractive bus transit system thus aims to benefit passengers by providing convenient transfers between different routes.

Timetable coordination is a proven strategy to reduce transfer waiting time and improve service connectivity given a bus network [3,4]. Timetable coordination reduces the difference between the arrival times of one bus and its connecting bus at a certain transfer stop via the adjustment of their scheduled terminal departure times. Without any coordination, transfers would increase passengers' total trip times substantially, especially during off-peak periods when bus service frequencies are low. The uncertainty of waiting time, due to the random bus arrival time, further annoys transfer passengers in practice because their tight weekday schedules are prone to be disrupted. This paper therefore deals with bus transfer problems arising within an off-peak period with the objective of the minimization of total waiting time of transfer and non-transfer passengers and its variability. The waiting time of non-transfer passengers is also considered in the transfer problem to avoid a notable increase in travel cost for non-transfer passengers due to timetable coordination. The first-bus transfer problem, that is, a special case of the off-peak period transfer problem from the theoretical viewpoint, is also addressed in this paper.

1.1. Related Studies

Since Klemt and Stemme [5] attempted to formulate a transfer optimization problem into a mixed-integer quadratic programming (MIQP) model, many studies have been dedicated to developing mixed-integer nonlinear programming (MINLP) models to find the optimal terminal departure times of buses that minimized passenger transfer waiting time. Heuristic algorithms were widely adopted to solve these optimization models. Schröder and Solchenbach [6] tackled a quadratic semi-assignment problem to improve transfer quality with small alterations to existing timetables. Khani and Shafahi [7] presented a MINLP model to decrease transfer waiting time which was calculated based on headways and departure times of intersecting routes. A genetic algorithm was suggested to obtain an optimum solution. Parbo et al. [8] proposed a bi-level bus timetabling approach to minimize the weighted waiting time. Their approach considered the route choices of passengers to obtain accurate passenger weights in the timetable optimization. Due to the difficulty in solving nonlinear models, types of linear formulations arise to coordinate the arrival times of buses from various routes. Shafahi and Khani [9] concentrated on setting the offset times of routes with homogeneous headways in a general transit network, taking the advantage of the fixed headway to calculate the average waiting time for each transfer. Saharidis et al. [10] deduced a bus rescheduling model that attempted to minimize passenger waiting times at transfer nodes when unexpected passenger demand happens. Abdolmaleki et al. [11] formulated the transfer waiting time minimization problem as an optimization problem with congruence constraints. Some researchers investigated multi-objective timetable coordination models where the objective that the minimization of transfer waiting time was addressed together with other objectives. Chakroborty et al. [12] aimed to minimize the total transfer time of transfer passengers and the initial waiting time of boarding passengers. A genetic algorithm was utilized to obtain optimal timetables. Chen et al. [13] presented coordinated timetabling models which attempted to minimize the weighted sum of passenger waiting time and bus operating cost.

Some other scholars, however, aimed to maximize simultaneous arrivals of transit units at a transfer node which was defined as synchronization. Chu et al. [14] put forward a model for planning bus timetables with detailed attention to the synchronization of transfers between routes and the satisfaction of capacity constraints. Ceder and Tal [15] proposed a mixed-integer programming model with an objective of minimization of synchronization and utilized a genetic algorithm to obtain its solution. To avoid bus bunching along the network, Ibarra-Rojas and Rios-Solis [16] redefined synchronization as the arrival of two trips with a separation time within a tight time window. They formulated an integer programming model to maximize the number of synchronizations via setting the rational departure times of trips, and further extended the model for a multi-period bus timetabling problem [17].

All of the models discussed in the above literature assumed that the travel time of buses were constants; namely, the randomness of practical bus operation has not been captured in the formulation. Bookbinder and Désilets [18], however, built a transfer optimization model that incorporated the effects of randomness, shedding insight into real operational issues that need to be addressed. Cevallos and Zhao [19] presented a method, which adding offset times to the scheduled bus terminal departure times for optimization of transfer times in a bus network, took into consideration the randomness of bus arrivals. They attempted to find an optimum solution using a genetic algorithm approach. Wu et al. [20] developed a stochastic integer programming model for the bus timetable design problem to minimize the total waiting time for three types of passengers: transfer passengers, boarding passengers, and through passengers. Kim and Schonfeld [21] formulated a probabilistic optimization model, which was put forward to deal with stochastic variability in travel times and waiting times, for integrating conventional and flexible services with timed transfers. With the advancement of vehicular communication, transfer issues in modular and autonomous bus systems have been investigated in recent years [22,23].

The first-train transfer problem in a rail transit system has received attention recently. Guo et al. [24] proposed a timetable coordination model to reduce the connection time for the first trains in a metro network, considering the importance of rail routes and transfer stations. Kang et al. [25–27] built a mixed-integer programming model for minimizing train arrival time difference and the amount of missed trains. For a more holistic and detailed review of the transfer issue in a public transport system, the reader is referred to the surveys of Liu et al. [28] and Gkiotsalitis et al. [29].

Three gaps are identified in the previous studies. Firstly, most of the timetable coordination models in the above literature review assumed a deterministic travel time. However, this assumption is, to some extent, unrealistic because bus travel time is largely influenced by road traffic conditions. Secondly, a limited number of studies considering random bus arrivals attempted to minimize the expected/average value of total waiting time in the objective functions. These functions, however, cannot ensure the robustness of the total waiting time reduction. Thirdly, previous studies usually investigated the off-peak transfer and first transfer problems separately. Moreover, the first-transfer problem in a bus network seemed not to be ever explored in the literature.

1.2. Objective and Contributions

The objective of this study is to present an off-peak period timetable coordination method that minimizes the total waiting time and its variability via altering the existing terminal departure times of buses. Note that only small changes to the terminal departure times are allowed in order to mitigate possible operating cost addition (e.g., a larger fleet size due to excessive timetable alterations). In other words, this study seeks to reduce total waiting time via small perturbations in the existing timetable instead of redesigning a totally new timetable.

The contributions of this study can be summarized as follows. Firstly, the travel time of each bus between two consecutive stops is rationally assumed to be a continuous random variable to better describe the practical circumstances of bus operation. Secondly, a robust optimization model is formulated to enhance the reliability of convenient transfers. The model is transformed into a mixed-integer linear programming (MILP) model, which can also be used to solve the first-bus transfer problem via further adding linear constraints on first bus arrival times. A Monte Carlo simulation-based algorithm is proposed to obtain the optimum solution with an optimization solver. Thirdly, a preliminary assessment, based on numerical experiments, is conducted for the feasibility of simplifying each overlapped sub-route into a single transfer node in a bus network.

The remainder of this paper is organized as follows. Section 2 describes the notations, assumptions and problem description. The objective function and its corresponding constraints for off-peak period transfer optimization problem are deduced and linearized in Section 3. The additional constraints for the first-bus transfer optimization problem are also addressed. Section 4 presents the algorithm incorporating Monte Carlo simulation, which is numerically validated in Section 5. Finally, Section 6 concludes this paper and suggests future research topics.

2. Problem Description

Consider an urban bus network defined by the directed graph $G(\boldsymbol{R}, \boldsymbol{K})$, where $\boldsymbol{R}$ is the set of predetermined bus routes, and $\boldsymbol{K}$ is the set of fixed bus stops. r denotes a particular bus route that belongs to set $\boldsymbol{R}$. k is a particular bus stop in the set $\boldsymbol{K}_r$ which groups all bus stops located along route r. The bus trips for route r are denoted according to their scheduled terminal departure times, namely, the jth departing trip is referred to as bus j in the set $\boldsymbol{J}_r$ which consists of all bus trips during the planning time horizon.

In the network $G(\boldsymbol{R}, \boldsymbol{K})$, passengers on bus j of route r can alight at bus stop k, walk to bus stop k', and then board the first bus arriving at stop k' of route r' after his/her arrival at that node. Note that it is assumed that bus capacity is always sufficient in this study. This assumption is realistic for off-peak period transfer problems. The necessary walking time between stop k and k' is denoted by $W^{kk'}$, which can be easily estimated by a field investigation or simulation test.

It is assumed that there is a fixed and known timetable for each bus route. This timetable is referred to as the existing timetable. The relevant parameter related to the existing timetable is the scheduled terminal departure time of bus j of route r, denoted by D_{jr}. Note that buses are assumed to be always dispatched on time. Namely, D_{jr} is just the practical terminal departure time. In reality, the travel time between stop k and $k+1$ of bus j of route r, denoted by T^k_{jr}, is a random parameter that follows a certain distribution. The characteristics of this distribution can be accurately derived from historical automatic vehicle location (AVL) data. It is assumed that sufficient AVL data is available that records practical bus operation in the absence of any control strategy. It should be noted that the definition of travel time here includes passenger boarding and alighting time.

The timetable coordination problem addressed in this paper is thus to determine the optimal departure offset times for target routes to coordinate the arrival times between bus j and its connecting bus, leading to a better bus service perceived by transfer and non-transfer passengers. The decision variable is thus the offset time of scheduled terminal departure time of bus j of route r, denoted by x_{jr}. Since only minor alterations for existing timetables are allowed, lower and upper bounds of terminal departure offset time of route r, denoted by $S_r^{\min}$ and $S_r^{\max}$, are limited as follows:

$$2|S_r^{\max}| \le D_{(j+1)r} - D_{jr},\ \ \forall\ r \in \boldsymbol{R},\ j \in \boldsymbol{J}_r. \tag{1}$$

$$2\left|S_r^{\min}\right| \le D_{(j+1)r} - D_{jr},\ \ \forall\ r \in \boldsymbol{R},\ j \in \boldsymbol{J}_r. \tag{2}$$

Equations (1) and (2) imply that the absolute values of parameters $S_r^{\min}$ and $S_r^{\max}$ should be less than the half of the minimum headway of route r during the planning time horizon. In practice, the predetermined parameters $S_r^{\min}$ and $S_r^{\max}$ can be flexibly adjusted based on the real situation.

For a clear and consistent presentation, all of the parameters in this paper are presented with Greek or uppercase letters, whereas all of the variables are expressed in lowercase letters. All time parameters and variables are analyzed in units of minutes.

3. Model Formulation

This study aims to leverage timetable coordination to minimize the total waiting time for transfer and non-transfer passengers in a bus network. Specifically, the total waiting time consists of the waiting time which stems from the bus-to-bus transfer process (i.e., transfer waiting time) and the time that a passenger has to wait to board a bus at his/her origin stop (i.e., initial waiting time). Hence, the estimation of transfer waiting time and initial waiting time forms the basis to build an optimization model for the optimal bus timetable coordination.

3.1. Transfer Waiting Time

Due to the uncertainty of bus travel/arrival times, both the waiting times of transfer and non-transfer passengers are random variables. Let $\boldsymbol{T} = \left(T^k_{jr}:\ r \in \boldsymbol{R};\ j \in \boldsymbol{J}_r;\ k \in \boldsymbol{K}_r\right)$ denote the random vector consisting of the random travel times for the target bus routes. Therefore, the deterministic transfer waiting time for a specific scenario l after timetable coordination is calculated by

$$w_l(\boldsymbol{T}) = \sum_{r\in\boldsymbol{R}} \sum_{r'\in\boldsymbol{R}, r'\neq r} \sum_{k\in\boldsymbol{K}_r} \sum_{k'\in\boldsymbol{K}_{r'}} \sum_{j\in\boldsymbol{J}_r} \sum_{j'\in\boldsymbol{J}_{r'}} t^{kk'}_{jrj'r'l},\ \ \forall\ l \in \boldsymbol{L}. \tag{3}$$

where $t^{kk'}_{jrj'r'l}$ is the waiting time if passengers from bus j of route r alight at stop k and board bus j' of route r' at stop k' for a specific scenario l; $\boldsymbol{L}$ is the set of all possible scenarios of the random travel time vector $\boldsymbol{T}$. However, only when bus j' is the first bus arriving at stop k' after transfer passengers' arrivals (i.e., bus j' is the connecting bus of bus j), $t_{jrj'r'l}^{kk'}$ is the practical transfer waiting time of the passengers from bus j. The connecting bus identification is thus the cornerstone of transfer waiting time calculation and reduction.

In effect, a connecting bus for a specific scenario l can be identified with the following constraints.

$$t_{jrj'r'l}^{kk'} \geq 0, \\ \forall\, l \in \boldsymbol{L};\ r,\ r' \in \boldsymbol{R},\ r \neq r';\ k \in \boldsymbol{K}_r;\ k' \in \boldsymbol{K}_{r'};\ j \in \boldsymbol{J}_r;\ j' \in\ \boldsymbol{J}_{r'}. \tag{4}$$

$$t_{jrj'r'l}^{kk'} \geq \begin{cases} a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'}, & j' = 1; \\ a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'} - M \cdot y_{jr(j'-1)r'l}^{kk'}, & \textit{otherwise}. \end{cases} \\ \forall\, l \in \boldsymbol{L};\ r,\ r' \in \boldsymbol{R},\ r \neq r';\ k \in \boldsymbol{K}_r;\ k' \in \boldsymbol{K}_{r'};\ j \in \boldsymbol{J}_r;\ j' \in\ \boldsymbol{J}_{r'}. \tag{5}$$

$$M \cdot \left(y_{jrj'r'l}^{kk'} - 1\right) \leq a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'} < M \cdot y_{jrj'r'l}^{kk'}, \\ \forall\, l \in \boldsymbol{L};\ r,\ r' \in \boldsymbol{R},\ r \neq r';\ k \in \boldsymbol{K}_r;\ k' \in \boldsymbol{K}_{r'};\ j \in \boldsymbol{J}_r;\ j' \in\ \boldsymbol{J}_{r'}. \tag{6}$$

$$y_{jrj'r'l}^{kk'} \in \{0,\ 1\}, \\ \forall\, l \in \boldsymbol{L};\ r,\ r' \in \boldsymbol{R},\ r \neq r';\ k \in \boldsymbol{K}_r;\ k' \in \boldsymbol{K}_{r'};\ j \in \boldsymbol{J}_r;\ j' \in\ \boldsymbol{J}_{r'}. \tag{7}$$

where $a_{j'r'l}^{k'}$ is the arrival time at stop k' of bus j' of route r' for scenario l after timetable coordination, a_{jrl}^{k} is the arrival time at stop k of bus j of route r for scenario l after timetable coordination, M is a sufficiently large positive value, and $y_{jrj'r'l}^{kk'}$ is a binary variable which equals one when the arrival time a_{jrl}^{k} is earlier than the arrival time $a_{j'r'l}^{k'}$ by at least the necessary walking time $W^{kk'}$ and zero otherwise. In another word, the binary variable $y_{jrj'r'l}^{kk'}$ will be one only when the passengers from bus j of route r are likely to take bus j' of route r' at stop k'.

Constraints (4)–(7) illustrate three different situations:

1. If $a_{(j'-1)r'l}^{k'} - a_{jrl}^{k} - W^{kk'} < 0$ and $a_{j'r'l}^{kk'} - a_{jrl}^{kk'} - W^{kk'} < 0$ (i.e., $y_{jr(j'-1)r'l}^{kk'} = 0$ and $y_{jrj'r'l}^{kk'} = 0$), bus j' of route r' is obviously not the connecting bus of bus j of route r for scenario l. As $y_{jr(j'-1)r'l}^{kk'} = 0$ and $y_{jrj'r'l}^{kk'} = 0$, then $t_{jrj'r'l}^{kk'} \geq 0$. In effect, due to the minimization objective function (detailed in Section 3.3), $t_{jrj'r'l}^{kk'}$ will be forced to be zero in this case.
2. If $a_{(j'-1)r'l}^{k'} - a_{jrl}^{k} - W^{kk'} < 0$ and $a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'} \geq 0$ (i.e., $y_{jr(j'-1)r'l}^{kk'} = 0$ and $y_{jrj'r'l}^{kk'} = 1$), bus j' of route r' is the connecting bus of bus j of route r. As $y_{jr(j'-1)r'l}^{kk'} = 0$ and $y_{jrj'r'l}^{kk'} = 1$, then $t_{jrj'r'l}^{kk'} \geq a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'}$. Due to the minimization objective function, $t_{jrj'r'l}^{kk'}$ will be forced to be $a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'}$ in this case, which is the practical waiting time of transfer passengers from bus j of route r.
3. If $a_{(j'-1)r'l}^{k'} - a_{jrl}^{k} - W^{kk'} \geq 0$ and $a_{j'r'l}^{k'} - a_{jrl}^{k} - W^{kk'} > 0$ (i.e., $y_{jr(j'-1)r'l}^{kk'} = 1$ and $y_{jrj'r'l}^{kk'} = 1$), bus j' of route r' is obviously not the connecting bus of bus j of route r for scenario l. As $y_{jr(j'-1)r'l}^{kk'} = 1$ and $y_{jrj'r'l}^{kk'} = 1$, then $t_{jrj'r'l}^{kk'} \geq 0$. In effect, due to the minimization objective function, $t_{jrj'r'l}^{kk'}$ will be forced to be zeros in this case.

The arrival time after timetable coordination for a particular scenario l can be easily calculated by

$$a_{jrl}^{k} = D_{jr} + x_{jr} + \sum_{i \in \boldsymbol{K}_r, i<k} T_{jrl}^{i},\ \ \forall\, l \in \boldsymbol{L},\ r \in \boldsymbol{R},\ k \in \boldsymbol{K}_r,\ j \in \boldsymbol{J}_r. \tag{8}$$

In Equation (8), $i < k$ indicates that stop i is the stop passed by buses before their arrival at stop k. When stop k is the origin terminal, the arrival time at stop k is just the terminal departure time, namely, $\left(D_{jr} + x_{jr}\right)$.

3.2. Initial Waiting Time

The deterministic initial waiting time $\widetilde{w}_l(\mathrm{T})$ for a specific scenario l after timetable coordination is defined as

$$\widetilde{w}_l(\boldsymbol{T}) = \sum_{r\in \boldsymbol{R}} \sum_{k\in \boldsymbol{K}_r} \sum_{j\in \boldsymbol{J}_r, j\geq 2} \frac{a^k_{jrl} - a^k_{(j-1)rl}}{2}, \ \forall\, l \in \boldsymbol{L}. \tag{9}$$

As presented in Equation (9), the average waiting time of each non-transfer passenger is equal to a half of bus headway (i.e., the time interval between the arrivals of two consecutive bus trips), which is widely used to estimate initial waiting time of passengers at transit stops [30].

3.3. Mixed-Integer Linear Programming Model

The objective function of the timetable coordination model is formulated as

$$\min z = \sum_{l\in \boldsymbol{L}} \eta_l \cdot p_l(\boldsymbol{T}) + \lambda \cdot \sum_{l\in \boldsymbol{L}} \eta_l \cdot \left| p_l(\boldsymbol{T}) - \sum_{l'\in \boldsymbol{L}} \eta_{l'} \cdot p_{l'}(\boldsymbol{T}) \right| \tag{10}$$

where the objective function value z is a weighted sum of the expected value of the random rate of increase in total passenger waiting time and its expected absolute deviation value; $p_l(\boldsymbol{T})$ is a deterministic rate of increase in total waiting time for a specific scenario $l \in \boldsymbol{L}$; η_l is the occurrence probability of scenario l; and λ is a non-negative weight coefficient to balance the expected value of the random rate of increase in total waiting time, expressed by the first term on the right-hand side of Equation (10), and the expected absolute deviation value of the random rate of increase in total waiting time, shown by the second term on the right-hand term of Equation (10). The occurrence probabilities should fulfill the fundamental condition: $\sum\limits_{l\in \boldsymbol{L}} \eta_l = 1$. The absolute deviation is a frequently used approach for measuring the variability of random variables since it is linear programming solvable. In other words, the objective function (10) aims to simultaneously minimize the rate of increase in total waiting time and its variability, leading to more rational timetable alterations.

The deterministic rate of increase in total waiting time $p_l(\boldsymbol{T})$ for a particular scenario l is defined as

$$p_l(\boldsymbol{T}) = \beta \cdot wp_l(\boldsymbol{T}) + \widetilde{\beta} \cdot \widetilde{w}p_l(\boldsymbol{T}), \ \forall\, l \in \boldsymbol{L}. \tag{11}$$

where $wp_l(\boldsymbol{T})$ is the deterministic rate of increase in transfer waiting time in scenario l, $\widetilde{w}p_l(\boldsymbol{T})$ is the deterministic rate of increase in initial waiting time in scenario l, β is the importance of minimizing transfer waiting time, and $\widetilde{\beta}$ is the significance of minimizing initial waiting time. The values of positive parameters β and $\widetilde{\beta}$ are predetermined by the authority and operator, and fulfill $\beta + \widetilde{\beta} = 1$. As presented in Equation (11), the rate of increase in total waiting time is a weighted sum of the rates of increase in transfer waiting time and initial waiting time.

The deterministic rate of increase in transfer waiting time $wp_l(\boldsymbol{T})$ for a specific scenario l is defined by

$$wp_l(\boldsymbol{T}) = \frac{w_l(\boldsymbol{T}) - w^0_l(\boldsymbol{T})}{w^0_l(\boldsymbol{T})}, \ \forall\, l \in \boldsymbol{L}. \tag{12}$$

where $w_l(\boldsymbol{T})$ and $w^0_l(\boldsymbol{T})$ are the deterministic transfer waiting times under the coordinated and original timetables, respectively.

The deterministic rate of increase in initial waiting time $\widetilde{w}p_l(\boldsymbol{T})$ for a specific scenario l is denoted as

$$\widetilde{w}p_l(\boldsymbol{T}) = \frac{\widetilde{w}_l(\boldsymbol{T}) - \widetilde{w}_l^0(\boldsymbol{T})}{\widetilde{w}_l^0(\boldsymbol{T})}, \ \ \forall\ l \in \boldsymbol{L}. \tag{13}$$

where $\widetilde{w}_l(\boldsymbol{T})$ and $\widetilde{w}_l^0(\boldsymbol{T})$ are the deterministic initial waiting times under the coordinated and original timetables, respectively.

Remark 1. *The deterministic transfer waiting time $w_l^0(\boldsymbol{T})$ (initial waiting time $\widetilde{w}_l^0(\boldsymbol{T})$) for a specific scenario l under the original timetable can be obtained with the method introduced in Section 3.1 (Section 3.2) and the introduction of constraints $x_{jr} = 0, \ \ \forall\ j \in \boldsymbol{J}_r,\ r \in \boldsymbol{R}$. (That is, no alteration is allowed to set to existing timetables).*

Remark 2. *The rate of increase in transfer (initial) waiting time, rather than the increase in transfer (initial) waiting time, is adopted in the objective function to ensure that the impact of timetable coordination on waiting time of transfer passengers is comparable to the effect of timetable alteration on waiting time of non-transfer passengers.*

Constraints (14) and (15) present the domain of terminal departure offset time x_{jr} of bus j of route r. Due to the unique characteristics of timetabling problems, the offset time allocated to the terminal departure time is constrained to be an integer variable expressed in units of minutes, as seen in Equation (15) where $\mathbb{Z}$ denotes the set of integers.

$$S_r^{\min} < x_{jr} < S_r^{\max}, \ \ \forall\ r \in \boldsymbol{R},\ j \in \boldsymbol{J}_r. \tag{14}$$

$$x_{jr} \in \mathbb{Z}, \ \ \forall\ r \in \boldsymbol{R},\ j \in \boldsymbol{J}_r. \tag{15}$$

In conclusion, the off-peak period transfer optimization problems can be formulated as a MINLP model including the nonlinear objective function (10) and linear constraints (3)–(9) and (11)–(15).

In effect, the objective function (10) can be linearized into Equation (18) by adding auxiliary non-negative variables $pu_l(\boldsymbol{T})$ and $pl_l(\boldsymbol{T})$.

$$pu_l(\boldsymbol{T}),\ pl_l(\boldsymbol{T}) \geq 0, \ \ \forall\ l \in \boldsymbol{L}. \tag{16}$$

$$p_l(\boldsymbol{T}) - \sum_{l' \in \boldsymbol{L}} \eta_{l'} \cdot p_{l'}(\boldsymbol{T}) = pu_l(\boldsymbol{T}) - pl_l(\boldsymbol{T}), \ \ \forall\ l \in \boldsymbol{L}. \tag{17}$$

$$\min z = \sum_{l \in \boldsymbol{L}} \eta_l \cdot p_l(\boldsymbol{T}) + \lambda \cdot \sum_{l \in \boldsymbol{L}} \eta_l \cdot (pu_l(\boldsymbol{T}) + pl_l(\boldsymbol{T})) \tag{18}$$

To this end, the optimal terminal departure offset times can be derived by a MILP model incorporating the objective function (18) and constraints (3)–(9) and (11)–(17).

3.4. Constraints for First-Bus Transfer Problem

The model built in Section 3.3 is also applicable to solve the first-bus transfer problem in a public transit system when replacing bus j by the first bus (i.e., let $j = 1$). In addition, constraints on first bus arrival times should be considered:

$$T_{\min} \leq a_{1rl}^k \leq T_{\max}, \ \ \forall\ l \in \boldsymbol{L},\ r \in \boldsymbol{R},\ k \in \boldsymbol{K}_r. \tag{19}$$

where $T_{\min}$ and $T_{\max}$ are the earliest allowable arrival time and latest allowable arrival time, respectively. $T_{\min}$ and $T_{\max}$ are predetermined by the authority and operator. Equation (19) ensures that the arrival time at each stop of the first bus after timetable coordination should comply with the earliest and latest service start times.

4. Solution Method

It is typically impossible to solve a stochastic program exactly unless the random variable has only a small number of possible realizations. However, by the law of large numbers, the expected value of a random variable can be approximated by taking the sample mean of some number of independent samples of the variable. Hence, one standard method of approximately solving a stochastic program is to use a Monte Carlo sampling procedure to generate N independent observations and then solve the approximating problem [31].

Let $\boldsymbol{T}^n \in \{\boldsymbol{T} : n = 1, \ldots, N\}$ be an independently and identically distributed random sample of N realizations of the random travel time vector $\boldsymbol{T}$. The expected value can be approximated by the sample average function, namely,

$$\sum_{l \in \boldsymbol{L}} \eta_l \cdot p_l(\boldsymbol{T}) \approx \frac{1}{N} \sum_{n=1}^{N} p(\boldsymbol{T}^n) \tag{20}$$

$$\sum_{l \in \boldsymbol{L}} \eta_l \cdot (pu_l(\boldsymbol{T}) + pl_l(\boldsymbol{T})) \approx \frac{1}{N} \sum_{n=1}^{N} (pu(\boldsymbol{T}^n) + pl(\boldsymbol{T}^n)) \tag{21}$$

Using the sample means, the proposed robust optimization model can be approximated by the following determined MILP model, which can be efficiently solved by any optimization solver that implements the branch and bound method, such as CPLEX. Furthermore, Mak et al. [31] have presented statistical analysis results regarding the quality of the approximated solution to the sample size. These results can guide schedulers in selecting an appropriate sample size for a given case.

[MILP]

Objective:	Equation (18)
Subject to:	Equations (3)–(9), (11)–(17), (20), and (21)

5. Numerical Experiments

5.1. Parameter Setting

A bus network consisting of three bus routes ($r = 1, 2, 3$) and eight transfer stops, as shown in Figure 1, is used to evaluate the applicability and performance of the model proposed in Section 3. Only the transfers occurring at the same stop (i.e., $k = k'$) are considered in the numerical experiments for simplification. The walking times $W^{kk'}$ are assumed to be identical and set to 0.5 min. The planning time horizon is 06:00:00–06:30:00 (the time is represented in the 24-h HH:MM:SS format).

Previous studies have suggested various probability distributions for transit travel times. Although there appears to not be a unique rule to determine which specific distribution is best, positively skewed distributions, such as lognormal, have been more preferred than other distributions [32]. A lognormal distribution $LN(\mu_{jr}^k, \sigma_{jr}^k)$ with truncated tails $\left[L_{jr}^k, U_{jr}^k\right]$, is thus employed as the probability distribution of the travel time T_{jr}^k in the numerical experiments. μ_{jr}^k and σ_{jr}^k denote the mean and standard deviation of the lognormal distribution, respectively. L_{jr}^k and U_{jr}^k denote the lower and upper bounds of travel time T_{jr}^k, respectively. For simplicity, it is assumed that all bus trips have the same travel time distribution. The parameters of the distribution are defined as follows: $\sigma_{jr}^k = 0.3\mu_{jr}^k$, $L_{jr}^k = 0.7\mu_{jr}^k$ and $U_{jr}^k = 1.3\mu_{jr}^k$. The values of mean travel times μ_{jr}^k (in minutes) between two consecutive stops are shown next to the corresponding segment in Figure 1. The importance of minimizing transfer waiting time β is set to 0.5. Accordingly, the importance of minimizing initial waiting time $\widetilde{\beta}$ is equal to 0.5. The other parameters involved in the model are presented in Table 1.

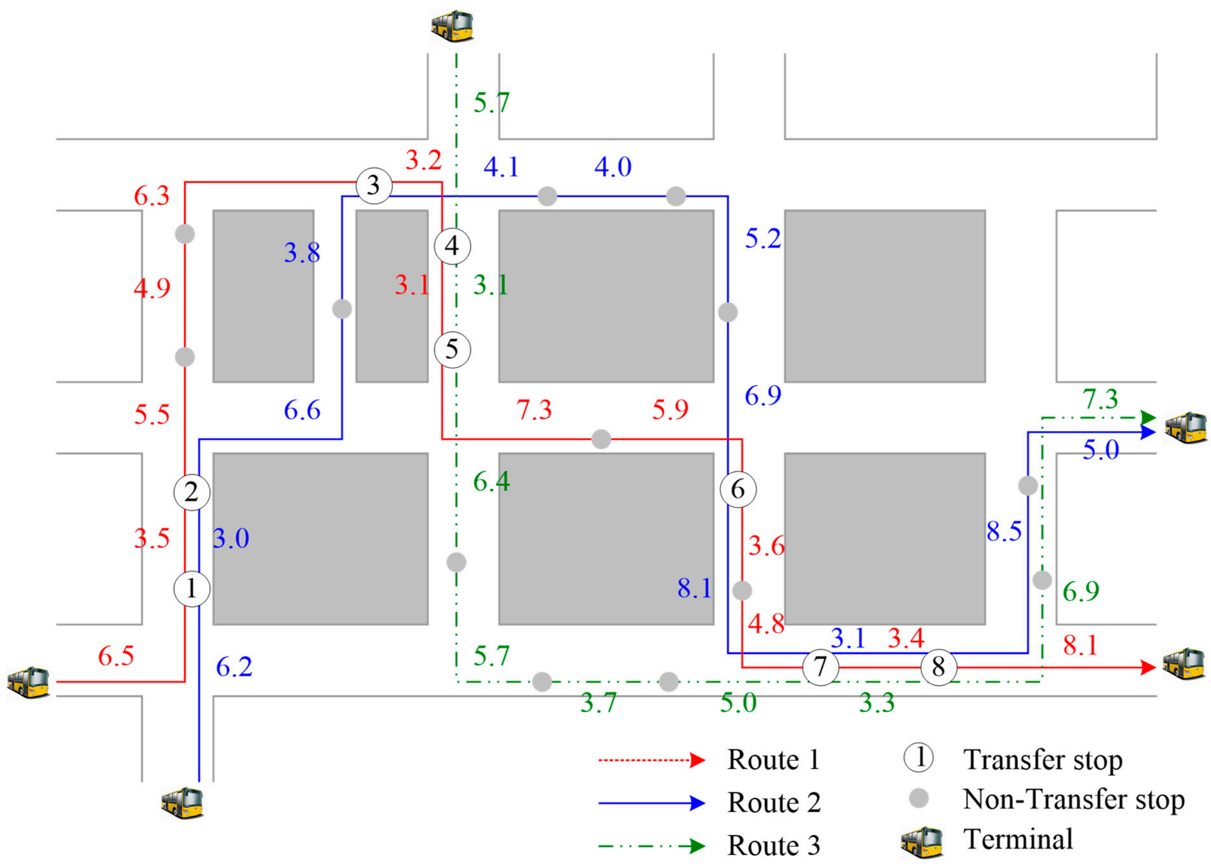

Figure 1. Bus network.

Table 1. Parameters in Numerical Experiments.

r	D_{1r}	D_{2r}	D_{3r}	D_{4r}	S_r^{min}	S_r^{max}
1	06:01:00	06:14:00	06:27:00	NA	−6	6
2	06:02:00	06:14:00	06:26:00	NA	−5	5
3	06:00:00	06:10:00	06:20:00	06:30:00	−4	4

Note: NA is not applicable.

The sample size N in the Monte Carlo simulation is 100. The solution algorithm was programmed using the language YALMIP [33] in MATLAB (R2016a) which is coupled with the IBM ILOG CPLEX 12.8 solver. All computations were performed on a desktop computer with an Intel (R) Core i7-10700 CPU @ 2.90 GHz and 16.0 GB of RAM running Microsoft Windows 10.

5.2. Optimization Results

Four scenarios, that is, Scenarios 1–4, were conducted to validate the effectiveness of the proposed model and to explore the influence of the weight values on the optimal results. The objective function value for each scenario is tabulated in Table 2. The values of the weight λ for Scenarios 1–4 were set to 0, 1, 3 and 5, respectively. When the weight λ is equal to zero, the optimal terminal departure offset times minimize the average rate of increase in total waiting time but cannot effectively control the variability of the rate of increase in total waiting time.

Table 2. Effects of Weight λ.

Scenario	1	2	3	4
λ	0	1	3	5
$\sum_{l\in \boldsymbol{L}} \eta_l \cdot wp_l$	−0.8048	−0.7815	−0.7379	−0.7155
$\sum_{l\in \boldsymbol{L}} \eta_l \cdot \widetilde{w}p_l$	−0.2238	−0. 2238	−0. 2238	−0.2238
$\sum_{l\in \boldsymbol{L}} \eta_l \cdot p_l$	−0.5143	−0.5027	−0.4809	−0.4696
$\sum_{l\in \boldsymbol{L}} \eta_l \cdot \left\| p_l - \sum_{l'\in \boldsymbol{L}} \eta_{l'} \cdot p_{l'} \right\|$	0.0317	0.0199	0.0066	0.0036

As shown in Table 2, transfer waiting times in Scenarios 1–4 have declined by 80.48%, 78.15%, 73.79%, and 71.55%, respectively. Namely, the transfer waiting time is greatly reduced by timetable coordination. The negative values of the rates of increase in initial waiting time in Scenarios 1–4 imply that the waiting time of non-transfer passengers is also decreased due to terminal departure time offsets. In summary, the timetable coordination method is effective in improving the bus service for both transfer and non-transfer passengers.

Compared with the optimization results in Scenario 1, the average rate of increase in total waiting time in Scenario 4 increases by 8.69%, and the average absolute deviation value of the rate of increase in total waiting time decreases by 88.64%, implying that the variability of the total waiting time can be greatly reduced at the cost of increasing the average value of total waiting time. Passengers will experience a more stable waiting situation but bear a slightly higher waiting time. In effect, for daily commuters, definite waiting time implies efficient travel schedule. The multiple-objective optimization model proposed in this study is therefore applicable in practical bus timetable optimization problems. Furthermore, Scenarios 2–4 were conducted to provide information regarding the trade-off between the average value of total waiting time and its average absolute deviation value.

The optimal offset time solutions for Scenarios 1–4 are presented in Table 3, illustrating how the extent to which the variability of passenger waiting time is considered impacts the terminal departure times. It is apparent in Table 3 that slight offset time differences are capable of incurring remarkably different waiting situations.

Table 3. Offset Time Solutions for Scenarios 1–4.

Scenario		1			2			3			4	
λ		0			1			3			5	
r	1	2	3	1	2	3	1	2	3	1	2	3
x_{1r}	4	5	4	4	5	4	4	5	4	4	5	4
x_{2r}	6	1	−4	6	2	−4	6	2	−4	6	2	−4
x_{3r}	−6	−5	−4	−6	−5	−4	−6	−5	−4	−6	−5	2
x_{4r}	NA	NA	−4	NA	NA	−4	NA	NA	−4	NA	NA	−4

Note: NA is not applicable.

To evaluate the effect of weights β and $\widetilde{\beta}$, we further conducted Scenarios 5 and 6 where the significances of minimizing transfer waiting time were set to 0.75 and 1, respectively. As detailed in Table 4, when weight β grows to 1 (i.e., Scenario 6), the optimal departure time offsets are different from the optimal solutions deduced in Scenarios 4 and 5, giving rise to a greater reduction of transfer waiting time.

Table 4. Effects of Weight β.

Scenario		**4**			**5**			**6**	
λ		5			5			5	
β		0.5			0.75			1	
$\sum_{l \in L} \eta_l \cdot wp_l$		−0.7155			−0.7112			−0.7168	
$\sum_{l \in L} \eta_l \cdot \widetilde{w}p_l$		−0.2238			−0. 2238			−0.1832	
r	1	2	3	1	2	3	1	2	3
x_{1r}	4	5	4	4	5	4	4	5	**3**
x_{2r}	6	2	−4	6	2	−4	6	2	**2**
x_{3r}	−6	−5	2	−6	−5	−4	**−2**	−5	**4**
x_{4r}	NA	NA	−4	NA	NA	−4	NA	NA	−4

Note: NA is not applicable.

5.3. Common Sub-Routes

As depicted in Figure 1, there are generally overlapped sub-routes in a bus network. The consecutive stops along the common sub-route are alternative transfer stops for passengers. Whether a common sub-route can be simplified into a single stop to decrease the complexity of the transfer optimization problem for a large real bus network?

In this section, we treated transfer stops 2, 5, and 8 in Figure 1 as non-transfer stops, as shown in Figure 2, and obtained optimal timetable alterations in such a simplified case with the model proposed in Section 3 and different settings for weights β and $\widetilde{\beta}$. The setting for other parameters is consistent with that in Section 5.1. The optimal solutions are summarized in Table 5.

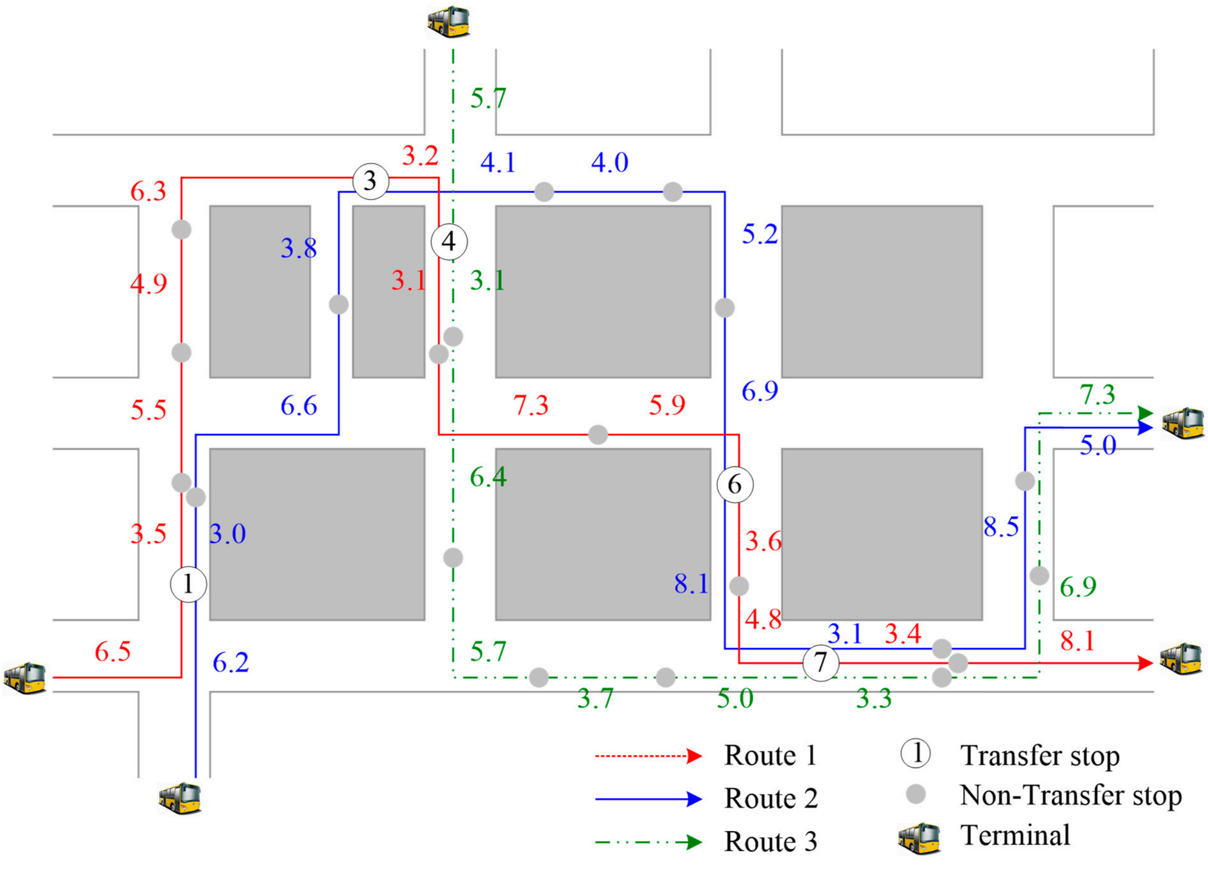

Figure 2. Bus network in simplified cases.

Table 5. Optimal Solutions in Simplified Cases.

Scenario		7			8			9	
λ		5			5			5	
β		0.5			0.75			1	
r	1	2	3	1	2	3	1	2	3
x_{1r}	4	5	4	4	5	4	4	5	3
x_{2r}	6	2	−4	6	2	−4	6	2	2
x_{3r}	−6	−5	**1**	−6	−5	0	**−3**	−5	**3**
x_{4r}	NA	NA	−4	NA	NA	−4	NA	NA	−4

Note: NA is not applicable.

As shown in Table 5, the optimal solutions in Scenario 7 (i.e., a simplified case) are slightly different from the optimal ones deduced in Scenario 4. However, the computation time for Scenario 7 is reduced to 75% of the time spent on Scenario 4 owing to the simplified network. Similarly, Scenarios 5 and 8 have almost identical solutions, except x_{33}. The solutions between Scenarios 6 and 9, where the significance of minimizing transfer waiting time β is equal to 1, are also slightly distinct.

The results in Table 5 suggest that the impact of simplifying a common sub-route into a single transfer stop on timetable coordination does exist, but the deviation in optimal timetables incurred by network simplification seems to be acceptable. In other words, simplifying a common sub-route into a single stop seems to be a tangible way to decrease the model complexity and speed up the computation for a real bus network. This finding, however, needs further exploration and validation, implying a topic of future research.

6. Conclusions

This study proposes a stochastic MILP model to deal with off-peak period transfer problems in a bus network, allowing for the randomness of bus travel time. The model is designed to arrive at coordinated timetables to reduce waiting time and its variability by searching for the optimal offset times of scheduled terminal departures for a fixed group of bus trips. It can also be used to solve the first-bus transfer problem via adding linear constraints that force a first bus to arrive at each stop within an allowable time period. A solution method based on the Monte Carlo simulation is developed to solve the stochastic MILP model. Numerical experiments are performed to explore the effectiveness and applicability of the model proposed in this study. The results show that the timetables after coordination can substantially improve the bus service for transfer and non-transfer passengers compared with the existing timetables. In addition, based on numerical experiments, this study attempts to answer a novel but practical question about whether it is acceptable to simplify a common sub-route into a single node in a transfer problem.

When the model is used to coordinate bus timetables for a large-scale bus network, the computation burden may be a bottleneck due to the use of the branch and bound method. Hybrid algorithms incorporating meta-heuristic methods and problem decomposition to use optimization solvers are a viable solution in this circumstance, which is a worthy topic of future research. Another extension of this study may be investigating how to take into account the impacts of scheduled terminal departure time offsets on the bus operation of each single route (e.g., bus bunching) when doing timetable coordination in a bus network.

Author Contributions: Methodology, X.D. and T.L.; Writing—original draft, X.D.; Writing—review and editing, T.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Beijing Natural Science Foundation, grant number L201008, and the National Natural Science Foundation of China, grant number 52002008.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data used in numerical experiments to support the findings of this study are included within the article.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Land Transport Authority of Singapore. Public Transport Ridership. 2020. Available online: https://www.lta.gov.sg/content/dam/ltagov/who_we_are/statistics_and_publications/statistics/pdf/PT_Ridership_2015_2019.pdf (accessed on 17 June 2022).
2. Espino, R.; Román, C. Valuation of transfer for bus users: The case of Gran Canaria. *Transp. Res. Part A Policy Pract.* **2020**, *137*, 131–144. [CrossRef]
3. Wu, W.; Liu, R.; Jin, W.; Ma, C. Stochastic bus schedule coordination considering demand assignment and rerouting of passengers. *Transp. Res. Part B Methodol.* **2019**, *121*, 275–303. [CrossRef]
4. Liu, T.; Ceder, A. Integrated public transport timetable synchronization and vehicle scheduling with demand assignment: A bi-objective bi-level model using deficit function approach. *Transp. Res. Part B Methodol.* **2018**, *117*, 935–955. [CrossRef]
5. Klemt, W.D.; Stemme, W. Schedule synchronization for public transit networks. In *Computer-Aided Transit Scheduling*; Springer: Berlin/Heidelberg, Germany, 1988; pp. 327–335.
6. Schröder, M.; Solchenbach, I. *Optimization of Transfer Quality in Regional Public Transit*; Berichte des Fraunhofer ITWM: Kaiserslautern, Germany, 2006.
7. Khani, A.; Shafahi, Y. Transfer optimization in transit networks: Headway and departure time coordination. In Proceedings of the 14th International IEEE Conference on Intelligent Transportation Systems, Washington, DC, USA, 5–7 October 2011. [CrossRef]
8. Parbo, J.; Nielsen, O.A.; Prato, C.G. User perspectives in public transport timetable optimization. *Transp. Res. Part C Emerg. Technol.* **2014**, *48*, 269–284. [CrossRef]
9. Shafahi, Y.; Khani, A. A practical model for transfer optimization in a transit network: Model formulations and solutions. *Transp. Res. Part A Policy Pract.* **2010**, *44*, 377–389. [CrossRef]
10. Saharidis, K.D.G.; Dimitropoulos, C.; Skordilis, E. Minimizing waiting times at transitional nodes for public bus transportation in Greece. *Oper. Res.* **2014**, *14*, 341–359. [CrossRef]
11. Abdolmaleki, M.; Masoud, N.; Yin, Y. Transit timetable synchronization for transfer time minimization. *Transp. Res. Part B Methodol.* **2020**, *131*, 143–159. [CrossRef]
12. Chakroborty, P.; Deb, K.; Subrahmanyam, P.S. Optimal scheduling of urban transit system using genetic algorithms. *J. Transp. Eng.* **1995**, *121*, 544–553. [CrossRef]
13. Chen, C.; Yan, S.; Tseng, C. Inter-city bus scheduling for allied carriers. *Transportmetrica* **2010**, *6*, 161–185. [CrossRef]
14. Chu, J.C.; Korsesthakarn, K.; Hsu, Y.-T.; Wu, H.Y. Model and a solution algorithm for planning transfer synchronization of bus timetables. *Transp. Res. Part E Logist. Transp. Rev.* **2019**, *131*, 247–266. [CrossRef]
15. Ceder, A.; Tal, O. Designing synchronization into bus timetables. *Transp. Res. Rec.* **2001**, *1760*, 28–33. [CrossRef]
16. Ibarra-Rojas, J.; Rios-Solis, Y.A. Synchronization of bus timetabling. *Transp. Res. Part B Methodol.* **2012**, *46*, 599–614. [CrossRef]
17. Ibarra-Rojas, J.; López-Irarragorri, F.; Rios-Solis, Y.A. Multiperiod bus timetabling. *Transp. Sci.* **2015**, *50*, 805–822. [CrossRef]
18. Bookbinder, J.H.; Désilets, A. Transfer optimization in a transit network. *Transp. Sci.* **1992**, *26*, 106–118. [CrossRef]
19. Cevallos, F.; Zhao, F. Minimizing transfer times in a public transit network with a genetic algorithm. *Transp. Res. Rec.* **2006**, *1971*, 74–79. [CrossRef]
20. Wu, Y.; Tang, J.; Yu, Y.; Pan, Z. A stochastic optimization model for transit network timetable design to mitigate the randomness of travelling time by adding slack time. *Transp. Res. Part C Emerg. Technol.* **2015**, *52*, 15–31. [CrossRef]
21. Kim, M.; Schonfeld, P. Integration of conventional and flexible bus services with timed transfers. *Transp. Res. Part B Methodol.* **2014**, *68*, 76–97. [CrossRef]
22. Gong, M.; Hu, Y.; Chen, Z.; Li, X. Transfer-based customized modular bus system design with passenger-route assignment optimization. *Transp. Res. Part E Logist. Transp. Rev.* **2021**, *153*, 102422. [CrossRef]
23. Wang, Y.; Zhou, Y.; Yan, X. Autonomous bus timetable synchronization for maximizing smooth transfers with passenger assignment. *Expert Syst. Appl.* **2022**, *193*, 116430. [CrossRef]
24. Guo, X.; Wu, J.; Sun, H.; Liu, R.; Gao, Z. Timetable coordination of first trains in urban railway network: A case study of Beijing. *Appl. Math. Model.* **2016**, *40*, 8048–8066. [CrossRef]
25. Kang, L.; Zhu, X. A simulated annealing algorithm for first train transfer problem in urban railway networks. *Appl. Math. Model.* **2016**, *40*, 419–435. [CrossRef]
26. Kang, L.; Zhu, X.; Sun, H.; Puchinger, J.; Ruthmair, M.; Hu, B. Modeling the first train timetabling problem with minimal missed trains and synchronization time differences in subway networks. *Transp. Res. Part B Methodol.* **2016**, *93*, 17–36. [CrossRef]
27. Kang, L.; Li, H.; Sun, H.; Wu, J.; Cao, Z.; Buhigiro, N. First train timetabling and bus service bridging in intermodal bus-and-train transit networks. *Transp. Res. Part B Methodol.* **2021**, *149*, 443–462. [CrossRef]
28. Liu, T.; Cats, O.; Gkiotsalitis, K. A review of public transport transfer coordination at the tactical planning phase. *Transp. Res. Part C Emerg. Technol.* **2021**, *133*, 103450. [CrossRef]
29. Gkiotsalitis, K.; Cats, O.; Liu, T. A review of public transport transfer synchronisation at the real-time control phase. *Transp. Rev.* **2023**, *43*, 88–107. [CrossRef]

30. Ceder, A.; Marguier, P.H.J. Passenger waiting time at transit stops. *Traffic Eng. Control* **1985**, *26*, 327–329.
31. Mak, W.K.; Morton, D.P.; Wood, R.K. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Oper. Res. Lett.* **1999**, *24*, 47–56. [CrossRef]
32. Ma, Z.; Ferreira, L.; Mesbah, M.; Zhu, S. Modeling distribution of travel time variability for bus operations. *J. Adv. Transp.* **2016**, *50*, 6–24. [CrossRef]
33. Löfberg, J. YALMIP: A toolbox for modeling and optimization in MATLAB. In Proceedings of the IEEE International Conference on Robotics and Automation, Taipei, Taiwan, 26 April–1 May 2004. [CrossRef]