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A Thermal-Fluid-Solid Coupling Computation Model of Initiation Pressure Using Supercritical Carbon Dioxide Fracturing

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Abstract: With the characteristics of low fracturing pressure, little damage to the reservoirs, and assuming the role of carbon storage, supercritical carbon dioxide (SC-CO₂) fracturing is suitable for the development of unconventional oil and gas resources. Based on the tensile failure mechanism of rocks, this paper establishes a thermal-fluid-solid coupling initiation pressure model for SC-CO₂ fracturing. Using this model, the changes in formation temperature and pore pressure near a wellbore caused by invasion of CO₂ into the formation are analyzed, as well as the impact of these changes on the tangential stress of reservoir rocks. The field data of SC-CO₂ fracturing in a sandstone gas well are used to validate the reliability of the model. The results show that SC-CO₂ fracturing can significantly reduce the initiation pressure, which decreases with the increase in fracturing fluid injection rate. The minimum value of tangential stress is located at the well wall, and the direction of tangential stress caused by formation temperature and pore pressure is opposite, with the former greater than the latter. The increase in Poisson's ratio, the increase in elastic modulus and the decrease in bottom hole temperature can reduce the initial fracturing pressure of the reservoir. The computation model established in this paper provides an effective method for understanding the reservoir fracturing mechanism under the condition of SC-CO₂ invasion.

Keywords: SC-CO₂; hydraulic fracturing; initiation pressure; tangential stress; thermal-fluid-solid coupling



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1. Introduction

Unconventional oil and gas resources in China are characterized by tight reservoir, complex geological structure, and high clay mineral content, which increases the difficulty of unconventional oil and gas field development [1–4]. Practice shows that large-scale hydraulic fracturing is one of the most effective technical means in unconventional oil and gas development fields [5,6]. However, the traditional hydraulic fracturing poses problems such as high water consumption, reservoir damage, and flowback fluid pollution [7], which restrict the scale and effectiveness of reservoir fracturing, and hinder the process of commercial development of unconventional oil and gas resources in China. Therefore, it is urgent to explore a new environmentally friendly water free fracturing technology.

SC-CO₂ fracturing is a new technology that uses SC-CO₂ (critical point: temperature 304.1 K, pressure 7.38 MPa) as the fracturing fluid to fracture the reservoir [8,9]. During

fracturing, liquid CO₂ is injected into the wellbore at a high pressure and low temperature and heated by the formation around the wellbore when flowing down the wellbore. When it reaches the bottom of the wellbore, it is usually in a supercritical state [10]. In this case, CO₂ assumes the characteristics of high density, low viscosity, strong permeability, etc. Compared with water-based fracturing fluid, it is easier to induce secondary fractures and form a complex fracture network [11,12], which makes promising application prospects in fracturing and stimulation of unconventional oil and gas reservoirs. In addition, SC-CO₂ fracturing has advantages in reducing crude oil viscosity, competing with methane in terms of adsorption, clay expansion inhibition, water resources conservation, and effective burial of CO₂ [13]. Therefore, it is a green and efficient fracturing technology.

The mechanism of reservoir initiation is the inception and primary point of the basic theoretical research of SC-CO₂ fracturing which can be traced back to the 1950s. At present, it is principally based on Hubbert–Willis model and Haimson–Fairhurst model. Hubbert et al. [14] proposed a near-wellbore stress model (Hubbert–Willis model) affected by wellbore pressure based on triaxial compression experiments in 1957 and provided criteria for wellbore fracture mode, but did not take into consideration the influence of radial seepage. In 1967, Haimson et al. [15,16] introduced the seepage term into the wellbore stress model and deduced the calculation formula of the starting pressure and fracture width. Huang [17] further proposed the criteria for the initiation and propagation of vertical and horizontal fractures, and concluded that lithology, interface, and fracturing fluid can affect the initiation and propagation of fractures. In 2000, Hossain et al. established the initiation criterion of hydraulic fracturing for perforated wells by using coordinate transformation and superposition principle [18]. Subsequently, scholars gradually carried out research on the effects of the well deviation and orientation [19], completion and perforation [20,21], formation structure and bedding structure [22,23], and reservoir anisotropy and natural fracture distribution [24,25] on the initiation pressure and orientation. However, when SC-CO₂ is used as a fracturing fluid, its high permeability and interaction with rock significantly affects the stress state around the well, thereby changing the fracturing pressure. Laboratory experiments show that SC-CO₂ fracturing has lower initiation pressure and more complex fracture network systems than conventional hydraulic fracturing [26,27]. Chen et al. [28] established a prediction model for the initiation pressure of SC-CO₂ fracturing considering such factors as SC-CO₂ viscosity, fluid compressibility, and pressurization rate, and conducted parameter sensitivity analysis. Xiao et al. [29] coupled wellbore flow equation, wellbore pressurization rate, and initiation pressure equation, proposed a CO₂ fracturing initiation pressure model considering real bottom hole temperature and pressure conditions, and proposed the optimal injection rate range. Zhong et al. [30] established a thermal-fluid-solid coupling model considering wellbore temperature pressure field and stress field around the well and analyzed the change rule of SC-CO₂ fracturing time with factors such as initial injection rate, injection pressure, and injection temperature.

In summary, the research on the fracturing mechanism of SC-CO₂ fracturing is predominantly confined to laboratory experiments, and a theoretical system of reservoir fracturing considering the impact of CO₂ invasion has yet to be formed. In view of the shortcomings of the existing theoretical models, this paper attempts to establish a thermal-fluid-solid coupling SC-CO₂ fracturing initiation pressure model, taking into full account of the impact of SC-CO₂ on the near-wellbore formation temperature, pore pressure, and rock tangential stress. By analyzing the stress weakness of the wellbore and near-wellbore formation, the authors aim to calculate the initiation pressure and provide theoretical guidance for on-site construction.

2. Theoretical Model

2.1. Tangential Stress Model around Well

The fracture is related to tensile failure during fracturing. Taking the compressive stress as positive, the condition for formation fracturing is as follows:

$$\sigma_{\theta} \leq -\sigma_t \quad (1)$$

where σ_{θ} is effective tangential stress, MPa; σ_t is the tensile strength of reservoir rock, MPa.

CO₂ can invade the formation after reaching the bottom of the well for its strong permeability, and then change the pore pressure around the well and affect the physical properties of the rock, thus change the tangential stress around the well. At the same time, the temperature difference between the fluid and the reservoir also causes the change in the tangential stress around the well.

According to the analysis above, the tangential stress around the well during SC-CO₂ fracturing is mainly caused by four basic loads: external radial stress, wellbore pressure, pore pressure, and thermal effect. According to the superposition principle [24], the expression of tangential stress around the well is:

$$\sigma_{\theta} = \sigma_{\theta 1} + \sigma_{\theta 2} + \sigma_{\theta 3} + \sigma_{\theta 4} \quad (2)$$

where

$$\begin{cases} \sigma_{\theta 1} = \frac{\sigma_H + \sigma_h}{2} \left(1 + \frac{R_w^2}{r^2} \right) - \frac{\sigma_H - \sigma_h}{2} \left(1 + 3 \frac{R_w^4}{r^4} \right) \cos 2\theta \\ \sigma_{\theta 2} = -p_w \frac{R_w^2}{r^2} \end{cases} \quad (3)$$

where $\sigma_{\theta 1}$ is the tangential stress caused by external radial stress, MPa; $\sigma_{\theta 2}$ is the tangential stress caused by the wellbore pressure, MPa; $\sigma_{\theta 3}$ is the tangential stress caused by fluid seepage effect in the wellbore, MPa; $\sigma_{\theta 4}$ is the tangential stress caused by thermal effect, MPa; σ_H , σ_h is the maximum and minimum horizontal principal stress, respectively, MPa; R_w is the borehole radius, m; r is the distance from the borehole in the horizontal direction, m; p_w is the bottom hole pressure, MPa; and θ is the well circumference angle, (°).

The mud cake cannot be formed inside the well wall for CO₂ has little wall building property [31]. At the same time, the bottom hole pressure increases during injection, and the pore pressure increases caused by invasion of SC-CO₂ into the formation, thus the tangential stress changes. The pore pressure change Δp_p is defined as

$$\Delta p_p(r, t) = p_p(r, t) - p_{po} \quad (4)$$

where $p_p(r, t)$ is the formation pore pressure around the well, MPa; p_{po} is the original formation pressure, MPa; and t is the time, s.

The tangential stress distribution formula caused by the change in pore pressure is

$$\begin{cases} \sigma_{\theta 3} = -\frac{2\eta}{r^2} \int_{R_w}^r r' \Delta p_p(r', t) dr' + 2\eta \Delta p_p(r, t) \\ \eta = \frac{G}{\lambda + 2G} \alpha = \frac{1-2\nu}{2(1-\nu)} \alpha \end{cases} \quad (5)$$

where η is the stress coefficient of porous elastic medium; G and γ are the Lamé coefficient of rock; ν is Poisson's ratio; α is the Biot coefficient; and r' is the variable in the integral.

The thermal stress is generated when there is a temperature difference between the fluid and the reservoir, which causes rock damage [32,33]. During SC-CO₂ fracturing, there is a temperature difference between the injected low temperature CO₂ and the reservoir when it reaches the bottom of the well, and the resulting thermal stress affects the tangential stress around the well. Define the temperature change field ΔT of rock around the well as

$$\Delta T(r, t) = T(r, t) - T_0 \quad (6)$$

where $T(r, t)$ is the formation temperature around the well, K; and T_0 is the original formation temperature, K.

Then the tangential stress caused by temperature difference in the wellbore is

$$\sigma_{\theta 4} = -\frac{E}{3(1-\nu)}\alpha_m\left[\frac{1}{r^2}\int_{R_w}^r \Delta T(r', t)r'dr' - \Delta T(r, t)\right] \quad (7)$$

where E is the elastic modulus of reservoir rock, MPa; and α_m is the volume thermal expansion coefficient of rock, K^{-1} .

If $r = R_w$, the effective tangential stress at the wellbore under the influence of multiple factors is

$$\sigma_{\theta} = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - p_w - 2\eta(p_{po} - p_w) + \frac{E}{3(1-\nu)}\alpha_m(T_w - T_0) \quad (8)$$

where T_w is the temperature when the fluid reaches the bottom hole, K.

Fracturing occurs when the effective tangential stress in the borehole wall reaches the tensile strength of the reservoir rock. Therefore, it can be calculated that the fracture initiation pressure p_f is

$$p_f = p_w = \frac{3\sigma_h - \sigma_H - 2\eta p_{po} + \frac{E}{3(1-\nu)}\alpha_m(T_w - T_0) + \sigma_t}{2 - 2\eta} \quad (9)$$

2.2. Reservoir Temperature and Pore Pressure Model

It can be seen from Equations (5) and (7) that the tangential stress of the near-wellbore formation is related to formation temperature and pore pressure. Therefore, a reservoir temperature and pressure model with mechanical and thermal coupling is established to analyze the initiation conditions of fractures after CO_2 invasion.

Based on the mass conservation equation and energy conservation equation of fluid in pores, it can be concluded:

$$\begin{cases} \frac{\partial(\phi\rho_f)}{\partial t} + \nabla \cdot (\rho_f J_v) = 0 \\ \rho_s c_{ps} \frac{\partial T}{\partial t} - \nabla \cdot (k_T \nabla T) - Q_s = 0 \end{cases} \quad (10)$$

where

$$J_v = -\frac{k}{\mu}\nabla p + K_T \nabla T \quad (11)$$

where ρ_f is the density of carbon dioxide, kg/m^3 ; ϕ is porosity; ρ_s is the density of reservoir rock, kg/m^3 ; c_{ps} is the specific heat capacity of reservoir rock, $J/(kg \cdot K)$; k_T is the thermal conductivity coefficient, $W/(m \cdot K)$; Q_s is the heat source intensity, $J/(m^3 \cdot s)$; and J_v is the fluid velocity, m/s .

After CO_2 invades the reservoir, the fluid velocity is affected by temperature and pressure. The velocity divergence is

$$\nabla J_v = -\frac{k}{\mu}\nabla^2 p + K_T \nabla^2 T \quad (12)$$

where K is permeability, μm^2 ; μ is viscosity, $\mu Pa \cdot s$; p is pore pressure, MPa; and K_T is the thermal permeability coefficient, $m^2/(s \cdot K)$.

For the same formation, assuming that the thermal conductivity coefficient k_T is constant [34], and there is no internal heat source in the reservoir, then:

$$\frac{\partial T}{\partial t} = \frac{k_T}{\rho_0 c_p} \cdot \nabla^2 T \quad (13)$$

In this paper, only the reservoir temperature at the distance r from the borehole is required. Therefore, Equation (13) degenerates into a one-dimensional heat conduction equation, that is, a parabolic equation. The analytical solution form can be obtained by

$$T = T_w - (T_w - T_0) \operatorname{erf} \frac{r}{2\sqrt{K_{\text{tdf}}t}} \quad (14)$$

Substitute Equation (12) into continuity equation, then:

$$\phi \frac{\partial \rho_f}{\partial t} - \frac{k\rho_f}{\mu} \nabla^2 p + \frac{\rho_f \rho_0 c_p K_T}{k_T} \frac{\partial T}{\partial t} - \frac{k}{\mu} \nabla p \nabla \rho_f + K_T \nabla T \nabla \rho_f = 0 \quad (15)$$

where $\frac{k\rho_f}{\mu} \nabla^2 p$, $\frac{k}{\mu} \nabla p \nabla \rho_f$ and $k_T \nabla T \nabla \rho_f$ are small quantities which can be ignored, and set the coupling coefficient $c_{\text{cou}} = \frac{\rho_0 c_p K_T}{k_T}$, then:

$$\phi \frac{\partial \rho_f}{\partial t} + c_{\text{cou}} \frac{\partial T}{\partial t} = 0 \quad (16)$$

Equation (13), Equation (16), and CO₂ physical property equation are comprehensively solved to obtain the change relationship of reservoir temperature and pore pressure with time and location.

The boundary conditions are

$$\begin{cases} p(r = R_w, t > 0) = p_w \\ T(r = R_w, t > 0) = T_w \\ p(r \rightarrow \infty, t > 0) = p_{\text{po}} \\ T(r \rightarrow \infty, t > 0) = T_0 \\ p(r, t = 0) = p_0 \\ T(r, t = 0) = T_0 \end{cases} \quad (17)$$

3. Model Solving and Verification

3.1. Model Solving Method

Due to the compressibility of CO₂, its physical parameters are sensitive to the changes in temperature and pressure. At the same time, the physical parameters of CO₂ also change the distribution of reservoir temperature and pore pressure near the well, thus affecting the local stress state of the reservoir. CO₂ interacts with rock after invasion into reservoir, and affects the physical and mechanical properties of rock [35,36]. Therefore, when calculating the fracture initiation pressure, it is necessary to comprehensively consider the following parameters such as influence of CO₂ on the physical and mechanical properties of rocks, fluid seepage and heat transfer, CO₂ physical property changes, and other factors before conducting a coupling solution. The calculation process is shown in Figure 1. In this paper, the Span Wagner equation [37] and Vesovic equation [38,39] with internationally recognized high accuracy are used to calculate the thermophysical properties (density, specific heat capacity, etc.) and migration properties (viscosity, thermal conductivity, etc.) of CO₂ to ensure the accuracy of the model.

3.2. Model Validation

This paper uses the SC-CO₂ fracturing field data from a tight sandstone gas well to verify the model (Table 1). The field data show that the fracturing occurred at the initial stage of injection, the initial pressure decreases from 66.02 MPa to 48.62 MPa, the bottom hole temperature was 368.97 K, and the original formation temperature was 382.15 K. The fracture initiation pressure calculated by the model in this paper is 48.94 MPa, and the absolute error is about 0.32 MPa, which is within 1%, indicating that the model in this paper has high calculation accuracy.

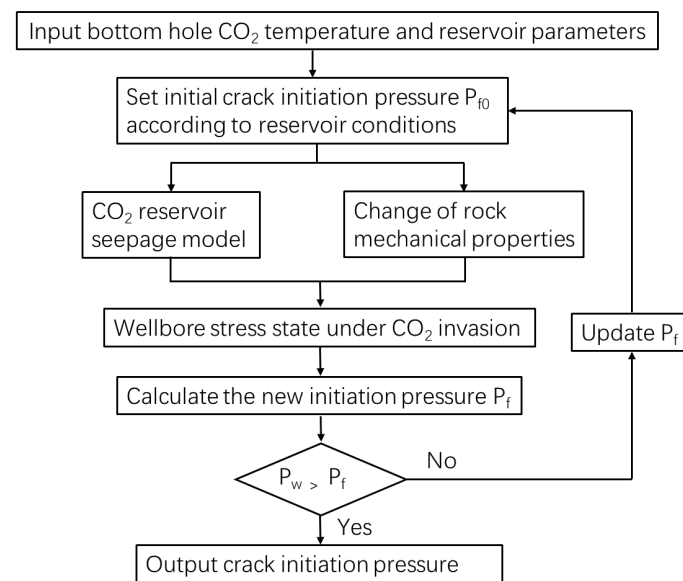


Figure 1. Coupling calculation process.

Table 1. Basic parameters.

Parameter	Value
Well depth/m	3 250
Borehole diameter/mm	121.36
Formation pressure/MPa	28.16
Inlet temperature/K	258
Wellhead pressure/MPa	8
Ground temperature gradient/(K·100 m ^{−1})	2.8
Formation thermal expansion coefficient/K ^{−1}	3 × 10 ^{−5}
Thermal permeability coefficient/(m ² ·(s·K) ^{−1})	6.0 × 10 ^{−11}
Original fracture initiation pressure/MPa	66.02
Formation tensile strength/MPa	11.256
Elastic modulus/GPa	22.04
Poisson's ratio	0.312
Porosity	0.05
Maximum horizontal stress gradient/(g·cm ^{−3})	1.9
Minimum horizontal stress gradient/(g·cm ^{−3})	1.5
Vertical stress gradient/(g·cm ^{−3})	2.3
Coupling coefficient	1.2375 × 10 ^{−3}

4. Results

4.1. Distribution of Formation Temperature and Pressure

During the fracturing and injection of SC-CO₂, the bottom hole temperature gradually decreases and is lower than the formation temperature, while the bottom hole pressure is higher than the pore pressure, so CO₂ is easily invades the formation, affecting the formation temperature and pore pressure near the well [40,41]. Previous research shows that the fracturing fluid injection rate is the main control factor affecting the temperature and pressure of CO₂ in the wellbore [40]. The bottom hole temperature decreases with the increase in injection rate. This paper refers to the calculation of bottom hole temperature and pressure in reference [29] and analyzes the distribution rule of formation temperature and pore pressure near the well under different injection rate (0.5 m³/min, 2.0 m³/min) (Figures 2 and 3).

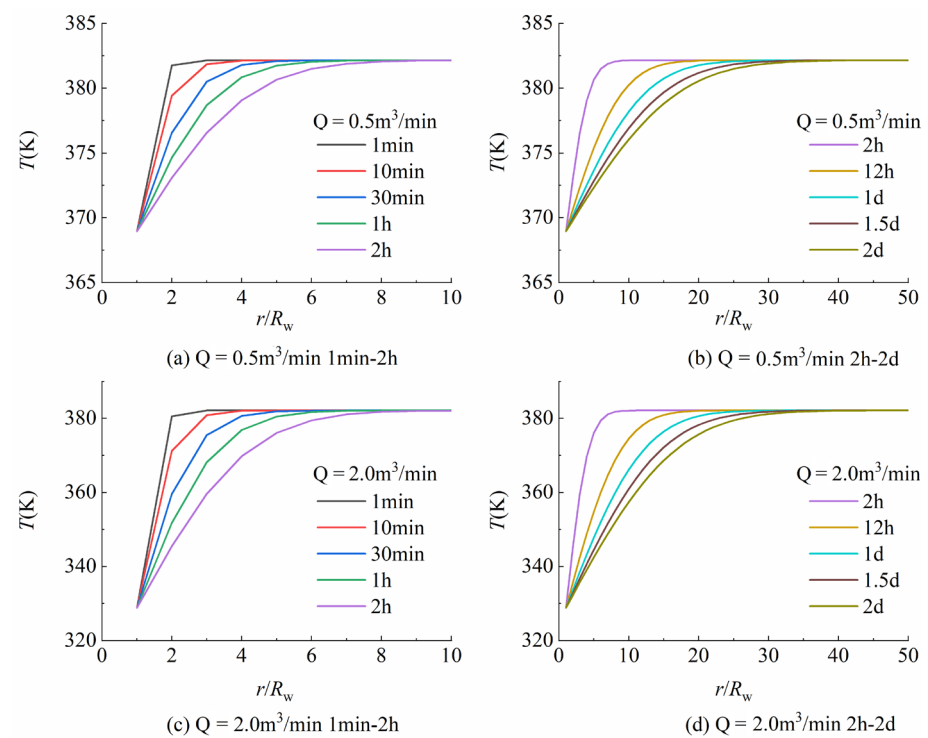


Figure 2. Distribution of formation temperature near well under different injection rate.

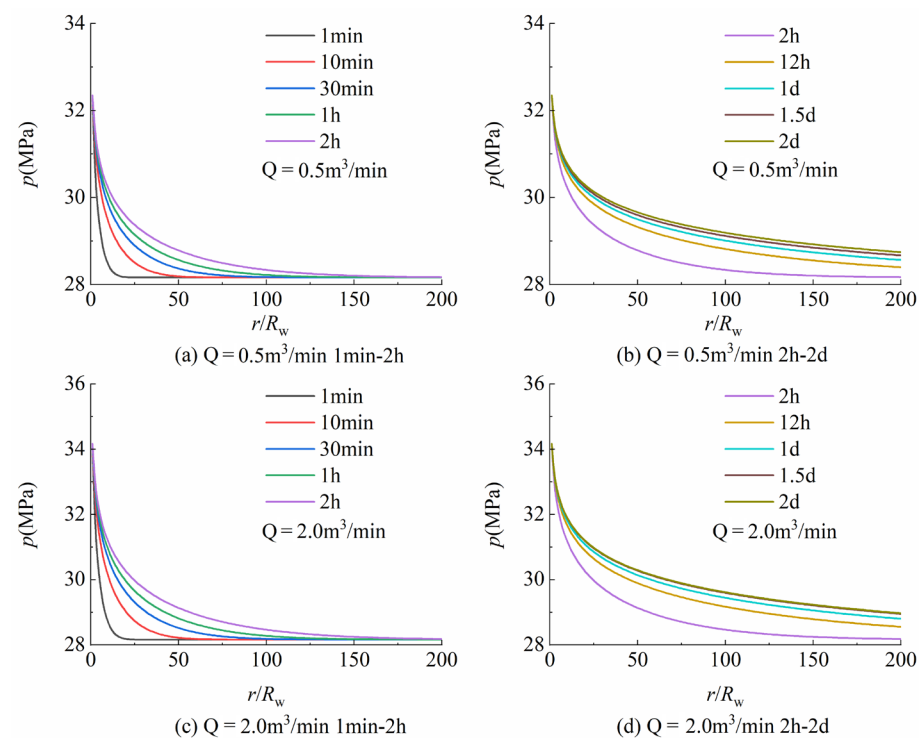


Figure 3. Distribution of pore pressure near well under different injection rate.

It can be seen from Figure 2 that when the injection rate increases from $0.5 \text{ m}^3/\text{min}$ to $2.0 \text{ m}^3/\text{min}$, the bottom hole temperature decreases from 368.97 K to 328.83 K , a decrease of 40.14 K . Under a different injection rate, the formation near the bottom of the well has an evident temperature drop with the increase in time, and the influence range gradually expands. In addition, it can be seen from Figure 2 that the injection rate has little influence on the range of formation temperature drop.

Figure 3 shows the distribution of pore pressure near the wellbore formation with time and radial distance under different injection rate ($0.5 \text{ m}^3/\text{min}$, $2.0 \text{ m}^3/\text{min}$). It can be seen from Figure 3 that under the same wellhead pressure, when the injection rate increases from $0.5 \text{ m}^3/\text{min}$ to $2.0 \text{ m}^3/\text{min}$, the bottom hole pressure increases from 32.35 MPa to 34.17 MPa, an increase of 1.82 MPa. This is because the pressure increment caused by the increase in gravity is greater than the frictional pressure drop increment caused by the increase in injection rate. Under a different injection rate, the formation pore pressure near the bottom hole increases with the increase in time, and the influence range increases. In addition, it can be found that the sweep velocity of bottom hole pressure is significantly higher than that of bottom hole temperature by comparing Figures 2 and 3.

4.2. Distribution of In Situ Stress

The near-wellbore temperature and pore pressure are substituted into Equations (5) and (7) to obtain the stress changes caused by formation temperature and pore pressure (Figure 4). It can be seen from Figure 4a that the thermal stress near the well wall ($\sigma_{\theta 4}$) is negative, which can effectively reduce the tangential stress at the well wall, thus reducing the fracture initiation pressure. Maximum temperature stress ($|\sigma_{\theta 4}|$) is 5.24 MPa, which is located at the well wall. The absolute value of temperature stress gradually decreases and changes to a positive value with the increase in radial distance; the negative range of temperature stress increases with the increase in time.

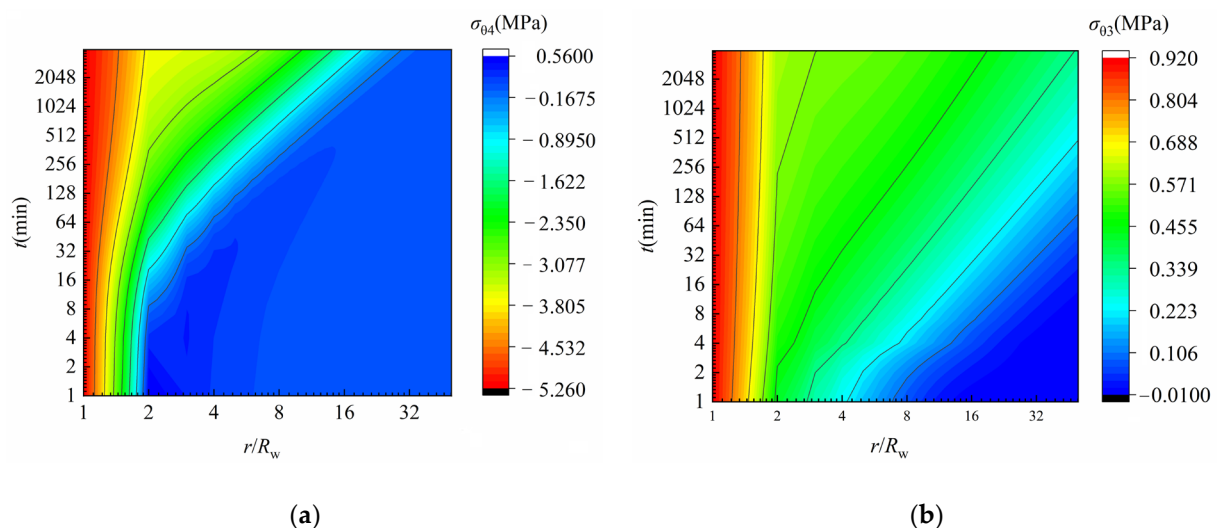


Figure 4. Rock stress changes caused by formation temperature and pore pressure ($Q = 0.5 \text{ m}^3/\text{min}$). (a) Formation temperature; (b) pore pressure.

It can be seen from Figure 4b that the tangential stress caused by pore pressure near the wellbore ($\sigma_{\theta 3}$) is a positive value, indicating that the increase in pore pressure can cause the increase in tangential stress and the increase in fracture initiation pressure. The maximum value of $\sigma_{\theta 3}$ also appears at the well wall, which is 0.92 MPa. In the process of CO_2 fracturing, the effect of pore pressure change on the tangential stress at the well wall is less than that of the reservoir temperature; as the radial distance increases, $\sigma_{\theta 3}$ gradually decreases and turns into a negative value. The range of positive values of $\sigma_{\theta 3}$ increases gradually with the increase in time. In addition, it can be found from Figure 4b that the tangential stress concentration area caused by pore pressure is concentrated near the well wall.

In general, the stress caused by the change in formation temperature during SC- CO_2 fracturing is greater than that caused by the change in pore pressure. This is because the injected low temperature CO_2 can reach the supercritical state when reaching the bottom of

the well, but the temperature is lower than the reservoir temperature, resulting in greater thermal stress.

Figure 5 shows the distribution of tangential stress in the near-wellbore formation. It can be seen from the figure that the minimum tangential stress is 41.16 MPa, which is located at the well wall. With the increase in the radial distance, the tangential stress first increases, then decreases and gradually plateaus. There is a stress concentration area within the range of 2~4 times the borehole diameter. With the increase in time, the stress concentration area is gradually released, which is the result of the comprehensive effect of formation temperature and pore pressure changes near the well.

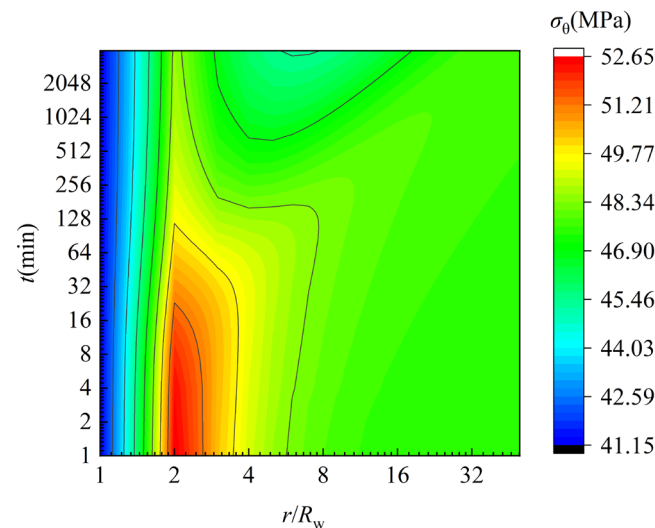


Figure 5. Distribution of formation tangential stress ($Q = 0.5 \text{ m}^3/\text{min}$).

4.3. Influencing Factors of Fracture Initiation Pressure

It can be seen from Equation (10) that there are many factors that affect the fracture initiation pressure of SC- CO_2 fracturing. In this paper, the effects of rock mechanical properties (Poisson's ratio, elastic modulus) and bottom hole temperature are analyzed.

4.3.1. Poisson's Ratio

After the invasion of CO_2 , the axial strain of reservoir rock increases, leading to a decrease in Poisson's ratio. Ding et al. [30] found through shale core experiments that the Poisson's ratio of reservoir rock decreases from 0.280 to 0.257, a decrease of 8.2 % after the invasion of CO_2 .

Porous elastic stress coefficient η and thermal stress $\sigma_{\theta T}$ are affected by Poisson's ratio. It can be seen from Equations (5) and (7) that the former decreases, while the latter increases with the increase in Poisson's ratio. Figure 6 shows the variation in the porous elastic stress coefficient and the fracture initiation pressure with Poisson's ratio under different Biot coefficients.

It can be seen from Figure 6 that η and pf increase with the increase in Biot coefficient α . Under different Biot coefficients, η and pf reduce with the increase in Poisson's ratio of rock. Combined with the research results of Ding et al. [34], the Poisson's ratio of rocks after CO_2 invasion is reduced, so the variation in CO_2 into the formation may increase the initial fracturing pressure of the reservoir.

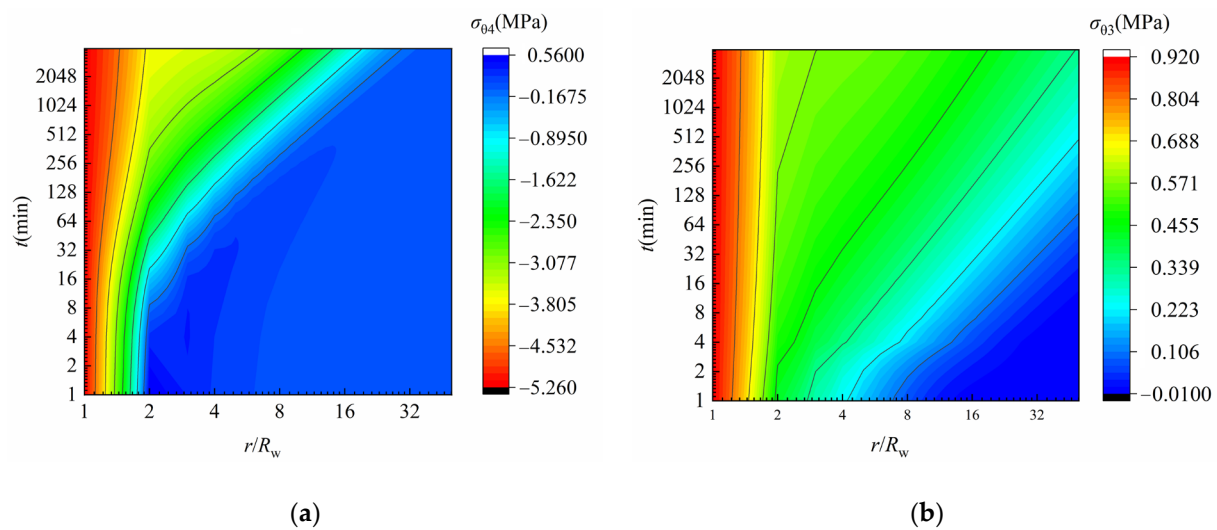


Figure 6. Change in elastic stress coefficient and fracture initiation pressure with Poisson's ratio. (a) Elastic stress coefficient; (b) fracture initiation pressure.

4.3.2. Elastic Modulus

Ding et al. found that the strain difference that the core can withstand decreases and the elastic modulus increases after intrusion of CO₂. According to Equation (10), when the bottom hole fluid temperature is low, the initiation pressure decreases with the increase in elastic modulus. Figure 7 shows the change in the fracture initiation pressure with the elastic modulus. It can be seen from the figure that the fracture initiation pressure decreases linearly with the increase in elastic modulus.

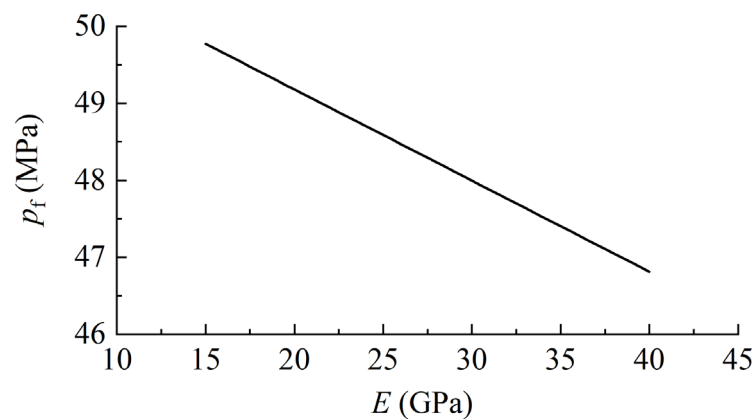


Figure 7. Change in initiation pressure with elastic modulus.

4.3.3. Bottom Hole Temperature

Bottom hole temperature is an important factor affecting the fracture initiation pressure of SC-CO₂ fracturing reservoirs. The change in initiation pressure with Poisson's ratio at different bottom hole temperatures is shown in Figure 8. In the figure, the injection rate is 0.5 m³/min and 2.0 m³/min, respectively, and the corresponding bottom hole temperature difference and formation temperature difference are 13.18 K (bottom hole temperature 368.97 K) and 53.32 K (bottom hole temperature 328.83 K), respectively.

It can be seen from Figure 8 that under different temperature differences, the reservoir fracture initiation pressure decreases linearly with the increase in Poisson's ratio. The reservoir pressure decreases with the increase in temperature difference between the bottom hole and the reservoir.

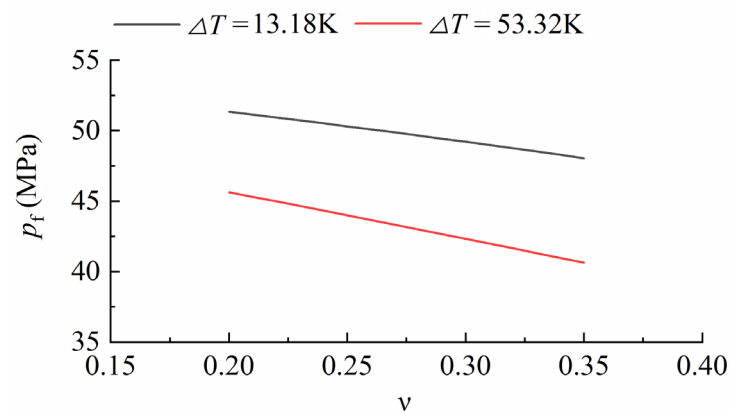


Figure 8. Effect of temperature difference between fluid and formation on initiation pressure.

According to the analysis above, the Poisson's ratio and elastic modulus of reservoir rock change distinctly after contacting with CO₂. In order to comprehensively analyze the influence of changes in Poisson's ratio and elastic modulus of rock on the initiation pressure, the contour of the initiation pressure changes with Poisson's ratio and elastic modulus when the bottom hole temperature is 368.97 K as provided in Figure 9. It can be seen from the figure that the increase in Poisson's ratio and elastic modulus can lead to the decrease in reservoir initiation pressure. In practical application, the change in initiation pressure can be judged according to the increment of Poisson's ratio and elastic modulus after CO₂ invasion.

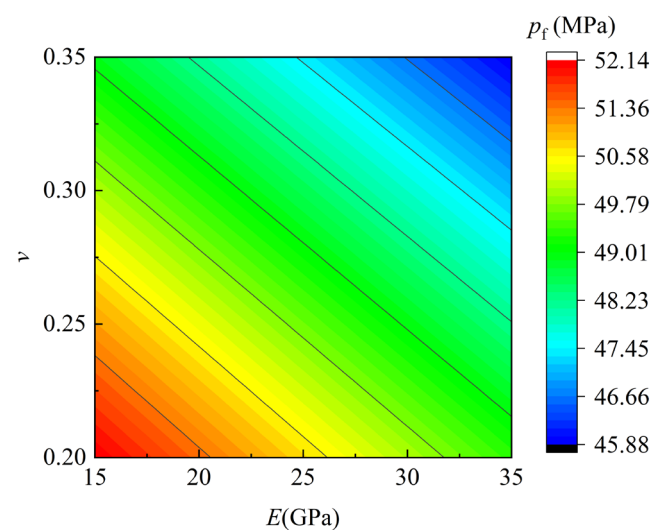


Figure 9. Effect of Poisson's ratio and elastic modulus on initiation pressure.

5. Application Example

Based on the analysis above, the initiation pressure at the injection rate of 0.5 m³/min and 2.0 m³/min is calculated, respectively (Table 2). It can be seen from Table 2 that SC-CO₂ fracturing can effectively reduce the initial pressure of the reservoir. When the injection rate increases from 0.5 m³/min to 2.0 m³/min, the initial pressure decreases from 48.94 MPa to 40.99 MPa, reducing by nearly 8 MPa. It should be noted that although increasing the injection rate can reduce the fracture initiation pressure, it also brings large energy consumption. In the fracture initiation stage, it is not appropriate to excessively pursue the low fracture initiation pressure under high injection rate.

Table 2. Calculation results of initiation pressure at the injection rate of 0.5 m³/min and 2.0 m³/min.

Q (m ³ ·min ^{−1})	Bottom Hole Temperature /K	Initiation Pressure /MPa	Original Initiation Pressure /MPa	Reduction Rate /%
0.5	368.97	48.94	66.02	25.87
2.0	328.83	40.99	66.02	37.91

From the perspective of dynamic fracturing process, low initial pressure is conducive to the formation of fractures, which expands rapidly under the bottom hole pressure. Due to the slow propagation speed of the temperature field, the thermal stress generated by the temperature field can induce the generation and expansion of secondary fractures, and thus create a fracture network system with high conductivity in the reservoir.

6. Conclusions

SC-CO₂ fracturing can effectively reduce the fracture initiation pressure of reservoir. The fracture initiation pressure decreases with the increase in injection rate.

With the injection of CO₂, the influence of bottom hole temperature and pressure on the near-wellbore formation expands radially. The sweep velocity of bottom hole pressure is higher than that of bottom hole temperature. The injection rate mainly affects the near-wellbore formation temperature and pore pressure, but has little influence on the sweep velocity.

The maximum tangential stress caused by formation temperature and pore pressure are both located at the well wall. The former reduces the tangential stress at the well wall, while the latter increases the tangential stress at the well wall. The stress caused by formation temperature change is greater than that caused by pore pressure change. On the whole, the minimum tangential stress is also located at the well wall.

The increase in Poisson's ratio, the increase in elastic modulus and the decrease in bottom hole temperature can all contribute to reducing the initial pressure of the reservoir.

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References

- Hu, J.; Xia, X.; Li, W. Rapid development of unconventional oil and gas: Status, challenges and prospects of shale oil extraction technology. *Front. Energy Res.* **2022**, *10*, 1–4. [\[CrossRef\]](#)
- Li, Q.; Wang, F.; Forson, K.; Zhang, J.; Zhang, C.; Chen, J.; Xu, N.; Wang, Y. Affecting analysis of the rheological characteristic and reservoir damage of CO₂ fracturing fluid in low permeability shale reservoir. *Environ. Sci. Pollut. Res.* **2022**, *29*, 37815–37826. [\[CrossRef\]](#) [\[PubMed\]](#)
- Zhang, W.; Shi, X.; Jiang, S.; Cao, Q.; Wang, F.; Wang, Z.; Ge, Y.; Du, Y. Experimental study of hydraulic fracture initiation and propagation in highly saturated methane-hydrate-bearing sands. *J. Nat. Gas Sci. Eng.* **2020**, *79*, 1–12. [\[CrossRef\]](#)
- Li, Q.; Wu, J. Factors affecting the lower limit of the safe mud weight window for drilling operation in hydrate-bearing sediments in the Northern South China Sea. *Geomech. Geophys. Geo-Energy Geo-Resour.* **2022**, *82*, 1–21. [\[CrossRef\]](#)
- Xue, M.; Cheng, Y.; Yan, C.; Li, Y.; Li, Q.; Han, S. Optimization of multistage hydraulic fracturing method of horizontal wells in shale gas reservoirs. *Pet. Geol. Oilfield Dev. Daqing* **2020**, *39*, 152–159.
- Chen, Z.; Li, S.; Chen, Z.; Wang, H. Hydraulic fracture initiation and extending tests in deep shale gas formations and fracturing design optimization. *Pet. Drill. Tech.* **2020**, *48*, 70–76.

7. Valder, J.F.; McShane, R.R.; Barnhart, T.B.; Wheeling, S.L.; Carter, J.M.; Macek-Rowland, K.M.; Delzer, G.C.; Thamke, J.N. *Analytical Framework to Estimate Water Use Associated with Continuous Oil and Gas Development*; US Geological Survey: Reston, VA, USA, 2019; pp. 1–17.
8. Zhang, C.P.; Liu, S.; Ma, Z.Y.; Ranjith, P.G. Combined micro-proppant and supercritical carbon dioxide (SC-CO₂) fracturing in shale gas reservoirs: A review. *Fuel* **2021**, *305*, 121431. [\[CrossRef\]](#)
9. Li, X.J.; Li, G.; Wang, H.; Tian, S. A wellbore flow model and coupling solution for supercritical CO₂ fracturing. *J. China Univ. Pet.* **2018**, *42*, 87–94.
10. Xiao, C.; Ni, H.; Shi, X. Unsteady model for wellbore pressure transmission of carbon dioxide fracturing considering limited-flow outlet. *Energy* **2021**, *239*, 122289. [\[CrossRef\]](#)
11. Lu, Y.; Liao, Y.; Tang, J.; Zhang, X.; Han, S.; Ling, Y. Experimental study on fracture initiation pressure and morphology in shale using supercritical CO₂ fracturing. *J. China Coal Soc.* **2018**, *43*, 175–180.
12. Chen, H.; Hu, Y.; Liu, J.; Liu, F.; Liu, Z.; Kang, Y.; Wang, X. Surface characteristics analysis of fractures induced by supercritical CO₂ and water through three-dimensional scanning and scanning electron micrography. *J. Rock Mech. Geotech. Eng.* **2021**, *13*, 1047–1058. [\[CrossRef\]](#)
13. Wang, H.; Li, G.; He, Z.; Shen, Z.; Li, X.; Zhang, Z.; Wang, M.; Yang, B.; Zheng, Y.; Shi, L. Analysis of mechanisms of supercritical CO₂ fracturing. *Rock Soil Mech.* **2018**, *39*, 3589–3596.
14. Hubbert, M.K.; Willis, D.G. Mechanics of hydraulic fracturing. *Trans. AIME* **1957**, *210*, 153–168. [\[CrossRef\]](#)
15. Haimson, B.; Fairhurst, C. Initiation and extension of hydraulic fractures in rocks. *Soc. Pet. Eng. J.* **1967**, *7*, 310–318. [\[CrossRef\]](#)
16. Haimson, B.; Fairhurst, C. In-situ stress determination at great depth by means of hydraulic fracturing. In Proceedings of the 11th U.S. Symposium on Rock Mechanics (USRMS), Berkeley, CA, USA, 16–19 June 1969; pp. 559–584.
17. Huang, R. Fracture initiation and propagation in hydraulic fracturing. *Pet. Explor. Dev.* **1981**, *8*, 62–74.
18. Hossain, M.; Rahman, M.; Rahman, S. Hydraulic fracture initiation and propagation: Roles of wellbore trajectory, perforation and stress regimes. *J. Pet. Sci. Eng.* **2000**, *27*, 129–149. [\[CrossRef\]](#)
19. Chen, M.; Chen, Z.; Huang, R. Hydraulic fracturing of highly deviated wells. *J. Univ. Pet.* **1995**, *19*, 30–35.
20. Li, G.; Liu, L.; Huang, Z.; Niu, J. Study of effect of hydraulic perforating on formation fracturing pressure. *J. China Univ. Pet.* **2006**, *30*, 42–45.
21. Fallahzadeh, S.A.H.; Shadizadeh, R.S.; Pourafshary, P. Dealing with the challenges of hydraulic fracture initiation in deviated-cased perforated boreholes. In Proceedings of the SPE Trinidad and Tobago Energy Resources Conference, Port of Spain, Trinidad and Tobago, 27–30 June 2010; pp. 1–15.
22. Ma, T.; Zhang, Q.B.; Chen, P.; Yang, C.; Zhao, J. Fracture pressure model for inclined wells in layered formations with anisotropic rock strengths. *J. Pet. Sci. Eng.* **2017**, *149*, 393–408. [\[CrossRef\]](#)
23. Liang, C.; Yi-Yu, L.; Zhao-Long, G.; Hong, D.; Ding-Yun, Z. Initiation pressure calculation model and judgment criterion for hydraulic fracturing of inclined coal seam. *Rock Soil Mech.* **2015**, *36*, 444–450.
24. Jin, Y.; Chen, M.; Zhang, Z. Hydraulic fracturing initiation pressure models for directional wells in naturally fractured formation. *Acta Pet. Sin.* **2006**, *27*, 124–126.
25. Zhao, J.; Ren, L.; Hu, Y.; Wang, L. A calculation model of breakdown pressure for perforated wells in fractured formations. *Acta Pet. Sin.* **2012**, *33*, 841–845.
26. Liu, G.-J.; Xian, X.-F.; Ping, Z.; Zhang, L.; Liu, Q.-L.; Zhao, Y.; Zhang, S. Experimental study on the supercritical CO₂ fracturing of shale. *J. China Coal Soc.* **2017**, *42*, 694–701.
27. Zhang, X.; Lu, Y.; Tang, J.; Zhou, Z.; Liao, Y. Experimental study on fracture initiation and propagation in shale using supercritical carbon dioxide fracturing. *Fuel* **2017**, *190*, 370–378. [\[CrossRef\]](#)
28. Chen, L.; Tian, S.; Li, G.; Xin, F. Initiation pressure models for supercritical CO₂ fracturing and sensitivity analysis. *Rock Soil Mech.* **2015**, *36*, 125–131.
29. Xiao, C.; Ni, H.; Shi, X.; Wang, R. A fracture initiation model for carbon dioxide fracturing considering the bottom hole pressure and temperature condition. *J. Pet. Sci. Eng.* **2020**, *184*, 106541. [\[CrossRef\]](#)
30. Zhong, G.; Zuo, L.; Jiang, T.; Wang, H.; Li, S.; Song, W. Fracture initiation pressure prediction for shale gas well fracturing with super-critical carbon dioxide. *Fault-Block Oil Gas Field* **2020**, *27*, 710–714.
31. Sun, X.; Sun, B.; Wang, Z. Fissure temperature field model of supercritical CO₂ fracturing. *Acta Pet. Sin.* **2015**, *36*, 1586–1592.
32. Li, M.; Ni, H.; Wang, R.; Song, W. The effect of thermal stresses on the relation between rock failure and temperature and pressure of supercritical carbon dioxide jet. *Greenh. Gases Sci. Technol.* **2018**, *8*, 218–237. [\[CrossRef\]](#)
33. Li, X.; Li, G.; Yu, W.; Wang, H.; Sepehrnoori, K.; Chen, Z.; Sun, H.; Zhang, S. Thermal effects of liquid/supercritical carbon dioxide arising from fluid expansion in fracturing. *SPE J.* **2018**, *23*, 2026–2040. [\[CrossRef\]](#)
34. Ding, L.; Ni, H.; Li, M.; Li, W.; Song, W.; Guo, X. Wellbore collapse pressure analysis under supercritical carbon dioxide drilling condition. *J. Pet. Sci. Eng.* **2018**, *161*, 458–467. [\[CrossRef\]](#)
35. Viète, D.R.; Ranjith, P.G. The effect of CO₂ on the geomechanical and permeability behaviour of brown coal: Implications for coal seam CO₂ sequestration. *Int. J. Coal Geol.* **2006**, *66*, 204–216. [\[CrossRef\]](#)
36. Sun, X.; Ni, H.; Qiao, H.; Wang, X.; Ma, B.; Wang, R.; Shen, Z.; Zhao, M. Experimental study on the mechanism of carbon dioxide removing formation paraffin deposits. *J. Nat. Gas Sci. Eng.* **2016**, *32*, 59–65. [\[CrossRef\]](#)

37. Span, R.; Wagner, W. A new equation of state for carbon dioxide covering the fluid region from the triple-point temperature to 1100 K at pressures up to 800 MPa. *J. Phys. Chem. Ref. Data* **1996**, *25*, 1509–1596. [[CrossRef](#)]
38. Vesovic, V.; Wakeham, W.A.; Olchowky, G.A.; Sengers, J.V.; Watson, J.T.R.; Millat, J. The transport properties of carbon dioxide. *J. Phys. Chem. Ref. Data* **1990**, *19*, 763–808. [[CrossRef](#)]
39. Fenghour, A.; Wakeham, W.A.; Vesovic, V. The viscosity of carbon dioxide. *J. Phys. Chem. Ref. Data* **1998**, *27*, 31–44. [[CrossRef](#)]
40. Song, W.; Ni, H.; Wang, R.; Sun, B.; Shen, Z. Pressure transmission in the tubing of supercritical carbon dioxide fracturing. *J. CO₂ Util.* **2017**, *21*, 467–472. [[CrossRef](#)]
41. Li, X.; Li, G.; Wang, H.; Tian, S.; Song, X.; Lu, P.; Wang, M. A unified model for wellbore flow and heat transfer in pure CO₂ injection for geological sequestration, EOR and fracturing operations. *Int. J. Greenh. Gas Control.* **2017**, *57*, 102–115. [[CrossRef](#)]

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