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Supply Chain Coordination with a Risk-Averse Retailer and the Call Option Contract in the Presence of a Service Requirement

Han Zhao [1], Hui Wang [1], Wei Liu [2,*], Shiji Song [3] and Yu Liao [4]

1 Department of Finance and Audit, Army Logistics Academy, Chongqing 401311, China; zhao-h15@mail.tsinghua.cn (H.Z.); wang-h12@mail.tsinghua.cn (H.W.)
2 Department of Basic Science, Wuhan Donghu University, Wuhan 430212, China
3 Department of Automation, Tsinghua University, Beijing 100084, China; shijis@mail.tsinghua.edu.cn
4 Department of Basic Science, Army Logistics University, Chongqing 401311, China; yul@mail.tsinghua.edu.cn
* Correspondence: liuwei05@whu.edu.cn

Abstract: This paper investigates a supply chain consisting of a single risk-neutral supplier and a single risk-averse retailer with the call option contract and a service requirement, where the retailer's objective is to maximize the Conditional Value-at-Risk about profit. The optimal ordering quantity of the retailer and the optimal production quantity of the supplier are derived with the call option contract in the presence of a service requirement. Furthermore, by investigating the effect of the service level and the risk aversion on the supply chain, it is found that the retailer's optimal Conditional Value-at-Risk is non-increasing in the service requirement and increasing in the risk aversion, while the supplier's optimal expected profit is non-decreasing in the service and decreasing in the risk aversion. In addition, this paper demonstrates the impact of contract parameters on the service-constrained supply chain, and finds that the retailer's optimal Conditional Value-at-Risk may be increasing, constant or decreasing in unit exercise price. Finally, with the call option contract, a distribution-free coordination condition is derived to achieve the Pareto improvement under Conditional Value-at-Risk criterion in the presence of a service requirement.

Keywords: supply chain coordination; conditional value-at-risk; service requirement; call option contract

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1. Introduction

Today's global market environment is full of uncertainties, coupled with hastened technology advancement and speedy changing consumer preferences. In such a market environment, the accurate prediction of the demand becomes extremely difficult. To improve effective competitiveness, firms are forced to develop the capability of responding flexibly to rapidly changing market conditions. This is particularly true for enterprises dealing with short life cycle products (such as fashion apparel, fresh food, electronics products, high-tech products, etc.) with comparatively long order or production lead-times, short selling seasons, low salvage value, and high uncertainty of customer demand [1]. To hedge against the loss associated with high understock and overstock risks, downstream firms have to order less but more frequently to accommodate demand volatility (e.g., [2,3]), and upstream firms need to have flexible capacity to cater for the irregular orders, which results in a sharp increase in supply cost [4]. These results inevitably lead to conflicts arise between the two parties, and spoil the supply chain performance tremendously. In addition, supply chains have been becoming more and more complex and geographically dispersed, involving a network of upstream and downstream firms around the world, and new risks have emerged in matching demand and supply [5].

To resolve the issue of supply chain inefficiency, call option contracts are efficient instruments to facilitate flexible ordering (e.g., [6–8]), and is becoming increasingly desirable in short life cycle products management. The call option contracts are operated by making use of two parameters, namely, the option price and the exercise price. The option price is a reservation fee paid by downstream firms to upstream firms for reserving one unit of the product before demand uncertainty is resolved, and the exercise price is an additional exercise fee paid by downstream firms to upstream firms for exercising one unit of the call option after demand is revealed. The call option contracts can help downstream firms ensure supply to meet uncertain market demand, and also can provide the flexibility on the number of products to purchase after further demand information is revealed. With call option contracts, upstream firms can receive a reservation fee paid by downstream firms for purchasing call option contracts before demand uncertainty is resolved. In addition, call option contracts can stimulate downstream firms to order more products, which also benefit upstream firms. Thus, the call option contract is frequently adopted to hedge various risks in many firms. It is reported that by employing call option contract to tackle the joint risk in demand, availability and cost for key components, during the period of 2000–2006, HP has achieved nearly $425 million in cumulative cost savings [9]. Furthermore, approximately 35% of HP's procurement value has been achieved by adopting option contract [10]. By using either firm order or exercising call option, China Telecom procures over 100 billion RMB worth of products from his suppliers annually [11]. These examples veal that the call option contract is of great applicability.

With more and more fierce business competition, the service level (the probability of meeting the customer demand) is the key factor that exerts an effect on the purchase choice of consumers. High service level can help firms maintaining the market share in the existing market and gaining a competitive edge in the new market. In recent years, to promote sales, more and more firms are promising a high service level. For example, Dayton Hudson promises a required 100% service level to consumers. Pharmed Group (PMG) commits to a minimum 98% order fulfillment rate. Costless Express guarantees a required 100% fill rate to their customer orders [12]. The service requirement, a constraint on the probability of meeting the customer demand, is becoming a practical tool in firms, and also a challenge faced by firms. Specifically, for short life cycle products, firms face a trade-off between the high service target and the high supply chain risk. Thus, how to set an appropriate service target is critical for the firms. Clearly, it is essential to explore the impact of the service requirement on the supply chain.

In most supply chain models, agents are assumed to be risk-neutral, and seek to maximize expected profit. However, many experimental studies under uncertainty (e.g., [13,14]) have asserted that most decision makers often defy risk neutrality assumption and their decision making behaviors deviate from maximizing expected profit, which is referred to as "decision bias" by Fisher and Raman [15]. Thus, some scholars have advocated relaxing risk neutrality assumption, and adopting risk aversion to characterize the decision-making behavior. Since then, risk-averse preferences have drawn plenty of attention and the supply chain models based on risk aversion have been studied in various contexts. However, the risk management of the risk-averse decision makers receives little attention in the existing studies. The recent research on portfolio management shows that the risk-averse investors are willing to balance lower expected profit for downside protection against potential losses (e.g., [16,17]). Thus, how to hedge against the risk-averse members' risk in supply chain develops into an interesting issue. Nevertheless, in the existing research, only a few researchers introduce Conditional Value-at-Risk (CVaR) measure into the risk-averse newsvendor models (e.g., [18–20]). When incorporating risk management and supply chain management, the risk-averse decision makers'decision making strategy and the performance of supply chains are still unclear.

To fill this research gap, this paper jointly considers risk aversion and risk management, and investigates the one risk-neutral supplier and one risk-averse retailer supply chain with the call option contract and a service requirement. Specifically, CVaR is introduced into our supply chain problem to measure and control various risks for the risk-averse retailer. The following questions are addressed in this paper.

(1) How does the risk-averse retailer determine the order quantity to maximize CVaR about profit in a supply chain with the call option contract in the presence of a service requirement?
(2) What is the supplier's optimal production policy with the call option contract and a service requirement?
(3) How do risk aversion, service requirement and contract parameters affect the retailer's optimal order policy, the supplier's optimal production policy and supply chain performance?
(4) What is the condition for supply chain coordination with the call option contract and a service requirement?

The remainder of this paper is structured as follows. Section 2 summarizes the relevant extant literature. The model description is presented in Section 3. In Sections 4 and 5, the retailer's optimal ordering policy and the supplier's optimal production policy are derived, respectively. In Section 6, conditions for supply chain coordination with the call option contract are investigated. Section 7 draws conclusions and identify some directions for further research.

2. Literature Review

The related literature will be reviewed includes three main research streams: call option contracts, supply chain management under CVaR criterion, and service requirement respectively.

The first stream is the literature on supply chain with call option contracts. Barnes-Schuster et al. [21] investigated the value of the call option contract in a two-period supply chain, and show that if the exercise price is piecewise linear, the supply chain can be coordinated. When the product demand and the spot price are random, Fu et al. [22] analyzed the effects of call option contract on the supply chain, and derive the optimal procurement solution by a shortest-monotone path algorithm. Zhao et al. [7] took a cooperative game approach and discuss that the issues concerning implementation of the coordinating option contract form, considering decision makers' risk preferences and negotiating powers. With yield and market demand uncertainty, Hu et al. [1] studied the optimal ordering policy of the retailer and the corresponding production decision of the manufacturer with an option contract, and demonstrate that both the retailer and the manufacturer can be better off. Chen et al. [8] considered the supply chain with call option contract and one risk-averse retailer, obtained the optimal ordering policy and the optimal production policy, and examined the impact of price, risk aversion, and cost parameters on the optimal ordering policy. Under random yield and spot market, Luo et al. [23] examined the supply chain members' optimal decisions and derive the condition for supply chain coordination with the call option contract. Wang et al. [24] derived that the firm's joint optimal decisions for both single- and multi-periods with call option contracts in the presence of customer returns. Under the mean-variance framework, Zhuo et al. [25] investigated the conditions that the supply chain is coordinated by the call option contract. Wan and Chen [26] explored a finite-horizon replenishment problem with call, put and bidirectional option contracts in the context of a spot market. Huang et al. [27] investigated a vendor-managed inventory supply chain by introducing a composed option and cost-sharing contract, and showed that Pareto improvement can be achieved with this composite contract. Fan et al. [28] established a Stackelberg game model via CVaR minimization. Furthermore, they examined the influences of adjusting both option price and option exercise price on the benefits and risks at the supplier and buyer sides. Liu et al. [29] explored the coordination of both the supplier-led and the retailer-led supply chains with the call option contract. Furthermore, they studied the option pricing, ordering, and producing problems under the CVaR criteria.

The second stream is the literature on supply chain under CVaR criterion. Yang et al. [30] studied the performance of the supply chain when the retailer is risk-averse with several single contract. Wu et al. [31] obtained closed-form solutions for the optimal ordering policy of the risk-averse manufacturer with the supply contract. Chen et al. [32] considered multiple risk-averse supplier and one risk-averse retailer supply chain, and study the performance and stability of the contract. Li et al. [33] investigated one risk-averse manufacturer and one risk-neutral retailer supply chain. Furthermore, they studied the condition for supply chain coordination with the risk-sharing contract. Wang et al. [34] consider one risk-neutral supplier and two competing risk-averse retailers supply chain with the call option contract, and determine the equilibrium option order quantity of the retailer and the condition that the supply chain is coordinated. Under CVaR criterion, Xie et al. [35] showed that the supply chain could be coordinated by buy-back contracts, wholesale price contracts and revenue-sharing contracts, respectively. Zhao et al. [36] investigated the ability of a combined contract to improve the efficiency of a supply chain, and find that the combined contract has some advantages over individual contract in terms of supply chain coordination and profit allocation. Chen et al. [37] showed that when the risk-averse manufacturer's confidence level is small enough, the quantity discount contract can coordinate the supply chain. Under the carbon emissions tax regulation, Zhao et al. [38] derived that the call option contract can benefit the risk-neutral supplier and one risk-averse retailer, improve the supply chain performance, and decrease invalid carbon emissions. Liu et al. [39] showed that the combined contract can coordinate the supply chain, and find that when the confidence level is below a threshold, the unique coordinating wholesale price exists.

The third stream is the literature on supply chain with a service requirement. Ernst and Powell [40] propose a plausible model based on service-sensitive demand under a distribution system. In addition, they derive a credible solution to the problem of manufacturer-retailer cooperation. Sethi et al. [41] study a two-stage supply chain with a demand information update and a service requirement. Furthermore, they derive that the optimal first-stage order quantity is increasing in the target service level, while the optimal expected profit is decreasing in the target service level. In addition, they analyze the channel coordination issue with and without an order cancelation. With and without the call option contract, Chen and Shen [42] give the optimal production policy of the supplier and the optimal ordering policy of the retailer in the presence of a service requirement, and show that with the option contract, the supply chain performance is improved. Jha and Shanker [43] formulate an integrated production-inventory model, and determine the optimal production-inventory policy by minimizing the joint total expected cost. Sawik [44] considers supply chain disruption risks, and obtains the risk-averse solutions that optimize the worst-case performance of a supply chain with the two different service levels measures. Sethi et al. [45] extend their model to the case where a service constraint is imposed for each procurement stage, and determine the optimal ordering policy with a demand information update and two service constraints. Furthermore, they extend their model to a multi-period problem. Hu and Feng [12] use the revenue sharing contract to study a service-constrained supply chain with supply and demand uncertainty, and analyze the condition that the supply chain is coordinated. Chen et al. [46] characterize the optimal decision strategy for the supplier and the retailer with and without the bidirectional option contract, and examine the effect of the bidirectional option contract and service requirement on the supply chain. He et al. [47] formulate several game-theoretic models with a service requirement. In addition, under these models, they compare firms' equilibrium order/production quantity and profits. Chen et al. [48] investigate the optimal operational decisions for the supplier and the retailer with and without the put option contract, and find that with the put option contract, high service level always benefit the supplier.

Table 1 compares the contribution of different authors. According to Table 1, call option contracts, service requirement and CVaR have been extensively studied. However, little research has jointly considered call option contracts, service requirement and risk aversion (by CVaR). Zhao et al. [7] and Chen et al. [8] considered decision makers' risk aversion, but do not consider the service requirement and CVaR. Chen and Shen [42] and Chen et al. [46] considered the call option contract and the service requirement, but do not consider decision makers' risk attitude (by CVaR). This paper is an extension of the work of Zhao et al. [7], Chen et al. [8], Chen and Shen [42], and Chen et al. [46], which jointly considered these three factors concurrently. Specifically, (1) this paper provides insights regarding the impact of service requirement and risk aversion (by CVaR) on supply chain; (2) this paper provides suggestions for how to effectively coordinate the channel with the service requirement and the call option contract.

Table 1. Comparison with contributions of different authors.

Author(s)	Call Option Contract	Service Requirement	Risk Aversion	CVaR
Barnes-Schuster et al. [21]	√			
Fu et al. [22]	√			
Zhao et al. [7]	√		√	
Hu et al. [1]	√			
Chen et al. [8]	√		√	
Luo et al. [23]	√			
Wang et al. [24]	√			
Zhuo et al. [25]	√			
Wan and Chen [26]	√			
Huang et al. [27]	√			
Fan et al. [28]	√		√	√
Liu et al. [29]	√		√	√
Yang et al. [30]			√	√
Wu et al. [31]			√	√
Chen et al. [32]			√	√
Li et al. [33]			√	√
Wang et al. [34]	√		√	√
Xie et al. [35]			√	√
Zhao et al. [36]			√	√
Chen et al. [37]			√	√
Zhao et al. [38]	√		√	√
Liu et al. [39]			√	√
Ernst and Powell [40]		√		
Sawik [44]		√	√	
Sethi et al. [41]		√		
Chen and Shen [42]	√	√		
Jha and Shanker [43]		√		
Sethi et al. [45]		√	√	
Hu and Feng [12]			√	
Chen et al. [46]	√	√		
He et al. [47]		√		
Chen et al. [48]		√		
This paper	√	√	√	√

3. Model Description

Consider a two-echelon supply chain where the supplier is risk-neutral and the retailer is risk-averse. The risk-neutral supplier produces the short life cycle product. The risk-averse retailer orders the call option contract from the supplier and sells to the customers with random demand in the selling season. Since the product's life cycle is shortening and subject to high demand uncertainty, product second purchasing and return are not allowed. Furthermore, to promote the sales of products, the retailer guarantees a service target to the customers. The model notions in this article are summarized in Table 2.

Table 2. The notations.

Notation	Description
p	Unit retail price,
o	Unit option price,
e	Unit exercise price,
c	Unit production cost,
s	Unit salvage value, $p > o + e > c > s$,
h	Supplier's unit shortage cost for each exercised call option which cannot be immediately filled, $c < h$,
q	Option order quantity,
Q	Production quantity,
D	Random demand, $\mathrm{E}(D) = \mu$
$f(x)$	Probability density function of D,
$F(x)$	Distribution function of D, $F(0) = 0$ and $F'(x) = f(x)$,
$\overline{F}(x)$	Tail distribution of $F(x)$, i.e., $\overline{F}(x) = 1 - F(x)$,
α	Service requirement, $0 \leq \alpha \leq 1$,
η	Risk aversion coefficient, $0 < \eta \leq 1$,
$\pi_r(D;q)$	Retailer's profit without a service requirement,
$\mathrm{CVaR}_\eta(\pi_r(D;q))$	Retailer's CVaR without a service requirement,
$\pi_s(D;Q)$	Supplier's profit without a service requirement,
$\mathrm{E}_D[\pi_s(D;Q)]$	Supplier's expected profit without a service requirement,
q^β	Optimal option order quantity maximizing CVaR about profit without a service requirement,
q^*	Optimal option order quantity maximizing CVaR about profit,
Q^*	Optimal production quantity.

The event sequence of the model is as follows. At the beginning of the production period, the retailer purchases call options for q units at unit option price o resulting in a cost of oq. To promote sales, the retailer should order enough quantity so that the probability of satisfying the customers' demand is not less than α. Each call option gives the retailer the right to buy one unit at exercise price e after the random demand has been observed. During the production period, based on the stochastic demand and the retailer's order call option quantity, the supplier produces the short life cycle product up to Q at unit production cost c resulting in a cost of cQ. During the selling period, the retailer decides how many units to exercise the call option according to the service requirement and market demand realization. The excess products owned by the supplier will be salvaged after the selling period.

The information available is assumed to be symmetric. The unit shortage cost h denotes the cost involved in expediting production. To avoid impractical cases, it is assumed that $s + o < c < h < o + e < p$. As in [8], it is reasonable to assume that the supplier is risk-neutral when it can diversify its risk through cooperation with many smaller independent retailers, and the retailer is assumed to be risk averse and try to maximize the CVaR of his profit. The CVaR's definition is as follows (e.g., [36,37,49]).

$$\mathrm{CVaR}_\eta(\pi_r(D;q)) = \max_{v \in R}\left\{ v - \frac{1}{\eta}\mathrm{E}_D[v - \pi_r(D;q)]^+ \right\}, \tag{1}$$

where $\pi_r(D;q)$ is the retailer's random profit function, ξ is a real number that represents the target level of the profit, E_D is the expectation operator, and $\eta \in (0,1]$ is the retailer's risk aversion. When $0 < \eta < 1$, the retailer is risk-averse. A lower value for η denotes a higher level of risk aversion. When $\eta = 1$, it is clear that the value of CVaR is equal to the expected profit. Then the retailer is risk-neutral.

4. Risk-Averse Retailer's Optimal Ordering Policy

This section considers the risk-averse retailer's optimal ordering policy under the call option contract in the presence of a service requirement.

Given the decision setting in Section 3, the profit function of the retailer is:

$$\pi_r(D;q) = \begin{cases} (p-e)D-oq, & 0 \leq D \leq q, \\ (p-o-e)q, & D < q. \end{cases} \tag{2}$$

Thus, from Equations (1) and (2), the retailer's optimal ordering decision under the call option contract with a service requirement is

$$\begin{aligned} &\max \mathrm{CVaR}_\eta(\pi_r(D;q)) \\ &s.t. \begin{cases} q \geq 0, \\ \mathrm{P}(q \geq x) \geq \alpha. \end{cases} \end{aligned} \tag{3}$$

$\mathrm{P}(q \geq x) \geq \alpha$ implies that $q \geq F^{-1}(\alpha)$ and $q^\alpha = F^{-1}(\alpha)$. It is clear that q^α is increasing in α.

The following theorem states the retailer's optimal order quantity with the call option contract and a service requirement.

Theorem 1. *With the call option contract and a service requirement α, the retailer's optimal order quantity q^* is unique and satisfies:*

$$q^* = \begin{cases} q^\alpha, & \eta\beta < \alpha, \\ q^\beta, & \eta\beta \geqslant \alpha, \end{cases} \tag{4}$$

where $\beta = \frac{(p-o-e)}{p-e}$ and $q^\beta = F^{-1}(\eta\beta)$.

Proof. See Appendix A. □

From the proof of Theorem 1, $\beta = \frac{(p-o-e)}{p-e}$, which denotes the maximum service level with the call option contract and without a service requirement. Theorem 1 shows that with the call option contract, when $\eta\beta < \alpha$, the service requirement is binding and $q^* = q^\alpha$, which is consistent with those of Chen and Shen [42]. When $\eta\beta \geqslant \alpha$, the service requirement is not binding and $q^* = q^\beta$. Since q^β is related to η, then the risk-neutral model presented in Chen and Shen [42] is a special case of our model. Thus, $q^* = \max(q^\alpha, q^\beta)$ and q^* is non-decreasing in the service requirement α. From Theorem 1, when $\eta\beta < \alpha$, q^* is constant in η; when $\eta\beta \geqslant \alpha$, q^* is increasing in η. Thus, q^* is also non-decreasing in the risk aversion η, and the risk-averse retailer will order less than or equal to the risk-neutral retailer, which is consistent with those of Chen et al. [8]. When $e = 0$, $q^* = \max(F^{-1}(\alpha), F^{-1}(\frac{(p-o)\eta}{p}))$, so the classic newsboy model is a special case of this model.

From Equations (2)–(4), it is calculated that the retailer's maximum CVaR, denoted $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$, is

$$\mathrm{CVaR}_\eta(\pi_r(D;q^*)) = (p-o-e)q^* - \frac{p-e}{\eta}\int_0^{q^*}(q^*-x)f(x)\mathrm{d}x. \tag{5}$$

The following corollary explores how would the maximum CVaR of the retailer $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ change when the service requirement increases.

Corollary 1. *The maximum CVaR of the retailer $CVaR_\eta(\pi_r(D;q^*))$ is a non-increasing function of the service requirement α with the call option contract.*

Proof. See Appendix A. □

Corollary (1) demonstrates that with the call option contract, the service requirement makes a significant impact on the maximum CVaR of the retailer. If the risk-averse retailer has a desire to achieve higher CVaR, the customer has to face a lower service level. If the customer requires a higher service level, the risk-averse retailer has to achieve lower CVaR. Thus, choosing an appropriate service target is important for the retailer. Please note that when the retailer is risk-neutral, i.e., $\eta = 1$, the optimal expected profit of the retailer $\mathrm{E}_D[\pi_r(D;q^*)]$ is also a non-increasing function of the service requirement α, which is consistent with those of Chen and Shen [42] and Chen et al. [46].

Next, this paper investigates the effect of the risk aversion on the retailer's maximum CVaR about profit.

Corollary 2. *The maximum CVaR of the retailer $CVaR_\eta(\pi_r(D;q^*))$ is an increasing function of the risk aversion η with the call option contract.*

Proof. See Appendix A. □

Please note that a higher value for η corresponds to a lower level of risk aversion. When the level of risk aversion increases, the retailer becomes more conservative, and will order less products to hedge against the potential loss or risk. Thus, the maximum CVaR of the retailer will decrease, which is intuitive.

Now, this paper states how would the retailer's maximum CVaR about profit change when unit option price o or unit exercise price e increases.

Corollary 3. *The maximum CVaR of the retailer $CVaR_\eta(\pi_r(D;q^*))$ has the following relationships with contract parameters:*

(i) *$CVaR_\eta(\pi_r(D;q^*))$ is decreasing in o,*

(ii) *When $\eta\beta \geqslant \alpha$, $CVaR_\eta(\pi_r(D;q^*))$ is decreasing in e. When $\eta\beta < \alpha$, if $\alpha < \eta$, then $CVaR_\eta(\pi_r(D;q^*))$ is decreasing in e; if $\alpha = \eta$, then $CVaR_\eta(\pi_r(D;q^*))$ is constant in e; otherwise, $CVaR_\eta(\pi_r(D;q^*))$ is increasing in e.*

Proof. See Appendix A. □

Corollary 3(i) means that the risk-averse retailer's maximum CVaR decreases in unit option price o, which is consistent with the case in which the retailer is risk-neutral. In addition, Corollary 3(i) indicates that o can be adopted to split the channel profit between the supplier and the retailer, which provides a practical tool for managers to adjust the profit allocation. However, Corollary 3(ii) identifies that the risk-averse retailer's maximum CVaR can be increasing or decreasing in unit exercise price e, which is not intuitive. This result is not concordant with the case where the retailer is risk neutral.

5. Risk-Neutral Supplier's Optimal Production Policy

This section considers the risk-neutral supplier's optimal production policy with the call option contract and a service requirement. Due to the short life cycle product's intrinsic attributes, there is no chance for the risk-neutral supplier to enlarge production capacity. Thus, before the selling period begins, based on the stochastic demand and the retailer's optimal order call option quantity q^*, the supplier must decide the production quantity of the short life cycle product Q. It is clear that $0 \leq Q \leq q^*$. Since the supplier is risk-neutral, then its CVaR is equal to its expected profit.

The expected profit of the risk-neutral supplier is

$$\mathrm{E}_D[\pi_s(D;Q)] = oq^* + e\mathrm{E}_D[\min(D,q^*)] + s\mathrm{E}_D[Q-(\min(D,q^*))^+] - h\mathrm{E}_D[(\min(D,q^*)-Q)^+] - cQ. \tag{6}$$

Equation (6) can be rewritten as

$$\mathrm{E}_D[\pi_s(D;Q)] = (o+e-h)q^* + (h-e)\int_0^{q^*} F(x)\mathrm{d}x + (h-c)Q - (h-s)\int_0^{Q} F(x)\mathrm{d}x. \tag{7}$$

Thus, with the call option contract and a service requirement, the optimization problem faced the supplier is given as follows:

$$\max_{0 \le Q \le q^*} \mathrm{E}_D[\pi_s(D;Q)]. \tag{8}$$

Let Q^* be the optimal solution to Equation (8). The following theorem is derived.

Theorem 2. *With the call option contract and a service requirement, the optimal production policy of the risk-neural supplier is*

$$Q^* = \begin{cases} Q^s & \text{if } Q^s \le q^*, \\ q^* & \text{if } q^* < Q^s, \end{cases} \tag{9}$$

where $Q^s = F^{-1}(\frac{h-c}{h-s})$.

Proof. See Appendix A. □

According to Theorem 2, the optimal production quantity of the risk-neural supplier with the call option contract and a service requirement is an interval. If $Q^s \le q^*$, then $Q^* = Q^s$. In this case, the supplier will increase its production quantity to increase its expected profit. If $q^* < Q^s$ and $\alpha < \eta\beta$, then $Q^* = q^\beta$. In this case, the service requirement is not binding and the supplier will decrease its production quantity up to q^β. If $q^* < Q^s$ and $\alpha \ge \eta\beta$, then $Q^* = q^\alpha$. In this case, the service requirement is binding and the supplier will decrease its production quantity up to q^α. In contrast to the model presented in Zhao et al. [7] and Chen et al. [8], the model studied in this paper with the service requirement extends the scope of their research.

From Equations (7) and (9), it is derived that the optimal expected profit of the risk-neural supplier, denoted $\mathrm{E}_D[\pi_s(D;Q^*)]$, is

$$\mathrm{E}_D[\pi_s(D;Q^*)] = (o+e-h)q^* + (h-e)\int_0^{q^*} F(x)\mathrm{d}x + (h-c)Q^* - (h-s)\int_0^{Q^*} F(x)\mathrm{d}x. \tag{10}$$

Now this section investigates the relationship between the service requirement α and the risk-neural supplier's optimal expected profit with the call option contract, which yields the following corollary.

Corollary 4. *Under CVaR criterion, the risk-neural supplier's optimal expected profit* $\mathrm{E}_D[\pi_s(D;Q^*)]$ *is a non-decreasing function of the service requirement* α *with the call option contract.*

Proof. See Appendix A. □

This corollary reveals that the service requirement has a substantial impact on the risk-neural supplier's optimal expected profit and is beneficial to the supplier. This is because the high service level increases the market demand, contributes to the expansion of market share, and further increases the supplier's expected profit.

Next this section explores the relationship between the risk aversion η and the risk-neural supplier's optimal expected profit with the call option contract. The following corollary is derived.

Corollary 5. *Under CVaR criterion, the risk-neural supplier's optimal expected profit* $\mathrm{E}_D[\pi_s(D;Q^*)]$ *is a non-decreasing function of the risk aversion* η *with the call option contract and a service requirement.*

Proof. See Appendix A. □

This corollary shows that the risk-neural supplier's optimal expected profit decreases in the level of risk aversion, which implies that the risk aversion never benefits to the supplier. This is because as the level of risk aversion increases, the optimal ordering quantity of the retailer will decrease, which incurs that the expected profit of the supplier cannot increase.

To gain more management insights, this paper considers the impact of changes in unit option price o and unit exercise price e on the risk-neural supplier's optimal expected profit. The result is demonstrated in the following corollary.

Corollary 6. *When $\eta\beta < \alpha$, under CVaR criterion, the risk-neural supplier's optimal expected profit $\mathrm{E}_D[\pi_s(D;Q^*)]$ is strictly increasing in o and e.*

Proof. See Appendix A. □

Corollary 6 indicates that when the service service requirement is binding, i.e, $\eta\beta < \alpha$, as the unit option price o or unit exercise price e increases, $\mathrm{E}_D[\pi_s(D;Q^*)]$ will increase. This result is consistent with the case where the retailer is risk neutral. In addition, Corollaries 3 and 6 imply that o can be used to split the channel profit between the supplier and the retailer, which provides an important practical tool for managers to adjust the profit allocation.

6. Supply Chain Coordination

This section address the supply chain coordination with the call option contract in the presence of a service requirement.

To provide a benchmark, the integrated supply chain is considered. In the integrated case, the supplier and the retailer are taken as an entity and owned by one risk neutral firm. This paper assumes that the integrated firm's production quantity is Q_0. Then the integrated firm's profit function, denoted $\pi_s(D;Q_0)$, is

$$\pi_s(D;Q_0) = p\min(Q_0, x) + s(Q_0 - x)^+ - h(x - Q_0)^+ - cQ_0.$$

Then the expected profit of the integrated firm, denoted $\mathrm{E}_D[\pi_s(D;Q_0)]$, is

$$\mathrm{E}_D[\pi_s(D;Q_0)] = (p + h - c)Q_0 - h\mu - (p + h - s)\int_0^{Q_0} F(x)\mathrm{d}x. \tag{11}$$

Thus, with a service requirement, the optimization problem faced the integrated firm is given as follows:

$$\begin{aligned} &\max \quad \mathrm{E}_D[\pi_s(D;Q_0)] \\ &s.t. \begin{cases} Q_0 \geq 0, \\ \mathrm{P}(Q_0 \geq x) \geq \alpha. \end{cases} \end{aligned} \tag{12}$$

$\mathrm{P}(Q_0 \geq x) \geq \alpha$ indicates that $Q \geq F^{-1}(\alpha)$ and $Q^\alpha = F^{-1}(\alpha)$. Clearly, Q^α is increasing in α. Let Q_0^* be the optimal solution to Equation (12). The optimal production policy of the integrated firm with a service requirement is demonstrated in the following theorem.

Theorem 3. *With a service requirement α, the optimal production quantity of the integrated firm Q_0^* is unique and satisfies:*

$$Q_0^* = \begin{cases} Q_0^\alpha, & \gamma < \alpha, \\ Q_0^\gamma, & \gamma \geqslant \alpha, \end{cases} \tag{13}$$

where $Q_0^\gamma = F^{-1}(\gamma)$ and $\gamma = \frac{p+h-c}{p+h-s}$.

Proof. See Appendix A. □

According the proof of Theorem 3, $\gamma = \frac{p+h-c}{p+h-s}$, which denotes the maximum service level without a service requirement in the integrated case. According to Theorem 3, The optimal production quantity of the integrated firm is an interval and related to α, when $\gamma < \alpha$, the service requirement is binding and $Q_0^* = Q_0^\alpha$; when $\gamma \geq \alpha$, the service requirement is not binding and $Q_0^* = Q_0^\gamma$. Clearly, $Q_0^* = \max(Q_0^\alpha, Q_0^\gamma)$ and Q_0^* is a non-decreasing function of the service requirement α.

According to Equation (11) and Theorem 3, it is derived that

$$\mathrm{E}_D[\pi_s(D; Q_0)] = (p + h - c)Q_0^* - h\mu - (p + h - s)\int_0^{Q_0^*} F(x)\mathrm{d}x. \quad (14)$$

Below this paper explores the coordination condition for the supply chain with the call option contract and a service requirement when the retailer is risk averse. Comparing Theorems 1–3 provides the following theorem on supply chain coordination.

Theorem 4. *Under the CVaR criterion, the supply chain with the call option contract and a service requirement can be coordinated when* $\alpha = \gamma = \frac{p+h-c}{p+h-s}$ *and* $\frac{h-c}{h-s} = \frac{(p-o-e)\eta}{p-e}$ *are satisfied.*

Proof. See Appendix A. □

Theorem 4 states that the coordination condition of the channel with the call option contract and a service requirement is determined by the relationship between the risk aversion (η), service requirement (α), unit option price (o), unit exercise price (e), unit retail price (p), unit production cost (c) and unit shortage cost (h). When $\eta = 1$, our model reduces to the risk-neutral one. When $\alpha = 0$, our model reduces to the model without a service requirement. Clearly, when $\eta \neq 1$ and $\alpha \neq 0$, the coordination condition becomes complicated. Please note that this coordination condition is not related to demand uncertainty, which is very important in practice and makes the implementation of the coordination call option contract easier. In addition, from Theorem 4, when the downstream retailer is risk-averse and operates under CVaR, the risk aversion has a significant effect on supply chain coordination. In practical situations, the risk aversion can be determined through many experiments (e.g., [50]).

It is shown that the total expected profit of the supply chain with coordination is higher than the baseline case that the supply chain is not coordinated. According to Corollaries 3 and 6, it is obtained that the unit option price o split the channel profit between the supplier and the retailer. In contrast to the non-coordinating contract, from Theorem 4, there always exists a *Pareto* coordination contract. Thus, the supplier should push for coordination so that both the supplier and the retailer will see increased expected profit.

7. Conclusions

This paper investigates a supply chain consisting of a single risk-neutral supplier and a single risk-averse retailer, where the retailer's objective is to maximize the CVaR about profit. In contrast to existing research that considers this supply chain setup, this paper assumes that the retailer commits to a service requirement. To the best of our knowledge, previous works do not address the supply chain problem when jointly considering call option contracts, service requirement and risk aversion (by CVaR). Specifically, with the call option contract, the risk-averse retailer's optimal ordering policy and the risk-neutral supplier's optimal production policy under the CVaR criterion in the presence of a service requirement are derived. Furthermore, this paper finds that the service requirement and risk aversion (by CVaR) promotes the firms to rethink operational decisions. In contrast to Zhao et al. [7] and Chen et al. [8], who studied the supply chain problem composed of a risk-neutral supplier and a risk-neutral retailer without a service requirement, our results indicate that the service requirement has a great impact on the supply chain. In comparison with Chen and Shen [42] and Chen et al. [46], who studied the supply chain consisting of a risk-neutral supplier and a risk-neutral retailer with the service requirement, our results indicate that the risk aversion (by CVaR) has a great impact on the supply chain. In addition, this paper shows that the call option contract can improve the supply chain performance when the retailer is risk averse.

Our research provides several interesting managerial implications. First, with the service requirement, there are unique optimal solutions for both the ordering policy of the retailer and and the production policy of the supplier when the retailer is risk-averse (by CVaR). Specifically, the optimal ordering quantity of the retailer is an interval and depends on the service requirement, while the optimal production quantity of the supplier is also an interval and related to the service requirement. This observation provides some insights into the optimal strategies of the retailer and the supplier with the call option contact in the presence of the service requirement. Second, this paper finds that under CVaR criterion, the risk-averse retailer's maximum CVaR decreases in unit option price, while the supplier's optimal expected profit increases as unit option price increases, which means that the option price can be used to adjust the profit allocation between the supplier and retailer. Finally, our research demonstrates that under CVaR criterion, the call option contract can coordinate the service-constrained supply chain, and the coordination condition is independent of random market demand, which is very important in practice and makes the implementation of the coordination call option contract easier.

This paper provides some fruitful future research directions. First, this paper only considers a one-period two-party supply chain with a single risk-neutral supplier and a single risk-averse retailer. This framework could be extended to include multiple risk-averse suppliers, multiple risk-averse retailers or both. Considering multiple players at each level of the supply chain introduces competition and cooperation which could lead to useful results that mimic practice more closely. Second, in the multi-period case, the risk aversion may change in different periods and affect the optimal ordering quantity and optimal production quantity. How to determine the optimal ordering policy and production policy in each period will be a direction for future research. Third, the supply uncertainty is ignored to facilitate the analysis. When considering the supply uncertainty, both the demand uncertainty and the supply uncertainty impact the supply chain performance, so the complexity of analysis is increased largely. Finally, this paper only considers the effect of the call option contract on the supply chain. It will be interesting to investigate the impact of different types of option contracts on the supply chain and explore which type of option contracts is the best choice in the presence of a service requirement.

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Appendix A

Proof of Theorem 1. Define an auxiliary function

$$H(q,v) = v - \frac{1}{\eta}\mathrm{E}[v - \pi_r(D,q)]^+.$$

From Equations (2), it is obtained that

$$H(q,v) = v - \frac{1}{\eta}\int_0^q [v-(p-e)x+oq]^+ f(x)\mathrm{d}x - \frac{1}{\eta}\int_q^\infty [v-(p-o-e)q]^+ f(x)\mathrm{d}x. \tag{A1}$$

It follows from Zhao et al. [38] that if $\mathrm{P}(q \geq x) \geq \alpha$, then the optimal solution to $\max\limits_{q\geq 0} \mathrm{CVaR}_\eta(\pi_r(D;q))$ is equal to that to $\max\limits_{q\geq 0}\left[\max\limits_{v\in R} H(q,v)\right]$. Thus, for any given q, to solve the optimization problem $\max\limits_{v\in R} H(Q,v)$, this paper considers the following five cases:

(i) $v \leq -oq$.

In this case,

$$H(q,v) = v. \tag{A2}$$

It is clear that $\frac{\partial H(q,v)}{\partial v} = 1$, and $H(q,v)$ is increasing in v.

(ii) $-oq < v \leq (p-o-e)q$.

Here,

$$H(q,v) = v - \frac{1}{\eta}\int_0^{\frac{v+oq}{p-e}} [v-(p-e)x+oq] f(x)\mathrm{d}x. \tag{A3}$$

It is easy to calculate that

$$\frac{\partial H(q,v)}{\partial v} = 1 - \frac{1}{\eta}F\left[\frac{v+oq}{p-e}\right], \tag{A4}$$

and $\frac{\partial^2 H(q,v)}{\partial v^2} < 0$, which implies $H(Q,v)$ is a concave function of v. From Equation (A4), it is obtained that $\frac{\partial H(q,v)}{\partial v}\Big|_{v=-oq} = 1 > 0$ and $\frac{\partial H(Q,v)}{\partial v}\Big|_{v=(p-o-e)q} = 1 - \frac{1}{\eta}F(q)$. If $1 - \frac{1}{\eta}F\left[\frac{v+oq}{p-e}\right] = 0$, then there is a unique v^* that satisfies $\frac{\partial H(Q,v)}{\partial v}\Big|_{v=v^*} = 0$ and $v^* = (p-e)F^{-1}(\eta) - oq$. It is derived that $\frac{\mathrm{d}H(q,v^*)}{\mathrm{d}q} < 0$, which indicates the optimal solution is not $v^* = (p-e)F^{-1}(\eta) - oq$, and $1 - \frac{1}{\eta}F\left[\frac{v+oq}{p-e}\right] > 0$.

(iii) $(p-o-e)q < v$.

In this case,

$$H(q,v) = v - \frac{1}{\eta}\int_0^q [v-(p-e)x+oq] f(x)\mathrm{d}x - \frac{1}{\eta}\int_q^\infty [v-(p-o-e)q] f(x)\mathrm{d}x. \tag{A5}$$

Then

$$\frac{\partial H(q,v)}{\partial v} = 1 - \frac{1}{\eta} \leq 0, \tag{A6}$$

which indicates $H(q,v)$ is a decreasing function of v and the optimal solution is $v^* = (p-o-e)q$.

Thus, combining above three cases and Equation (A1), it is obtained that

$$H(q,v^*) = (p-o-e)q - \frac{p-e}{\eta}\int_0^q (q-x)f(x)\mathrm{d}x. \tag{A7}$$

From Equation (1), $\mathrm{CVaR}_\eta(\pi_r(D;q)) = H(q,v^*)$.

The next section explores the optimal solution of the optimization problem $\max\limits_{q\geq 0} \mathrm{CVaR}_\eta(\pi_r(D;q))$ without a service requirement. Then

$$\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q))}{\partial q} = p - o - e - \frac{p-e}{\eta}F(q), \tag{A8}$$

and

$$\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(D;q))}{\partial q^2} = -\frac{p-e}{\eta}f(q) < 0, \tag{A9}$$

which indicates $\mathrm{CVaR}_\eta(\pi_r(D;q))$ is concave in q. Since $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q))}{\partial q}\Big|_{q=0} = (p-o-e) > 0$ and $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q))}{\partial q}\Big|_{q\to+\infty} = p-o-e-\frac{p-e}{\eta} < 0$, there is a unique optimal solution q^β that satisfies $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q))}{\partial q}\Big|_{q=q^\beta} = 0$, i.e., $q^\beta = F^{-1}\left[\frac{(p-o-e)\eta}{p-e}\right] = F^{-1}(\eta\beta)$. Since $q^\alpha = F^{-1}(\alpha)$, it is derived that the optimal solution of Equation (3) q^* is

$$q^* = \begin{cases} q^\alpha, & \eta\beta < \alpha, \\ q^\beta, & \eta\beta \geqslant \alpha. \end{cases}$$

□

Proof of Corollary 1. From Theorem 1, if $\eta\beta \leq \alpha$, then $q^* = q^\alpha$. It is obtained that $\mathrm{dCVaR}_\eta(\pi_r(D;q^*))/\mathrm{d}\alpha = \left[p-o-e-\frac{p-e}{\eta}F(q^\alpha)\right]\mathrm{d}q^\alpha/\mathrm{d}\alpha = \left[\frac{p-e}{\eta}(F(q^\beta)-F(q^\alpha))\right]\mathrm{d}q^\alpha/\mathrm{d}\alpha$. Since $\mathrm{d}q^\alpha/\mathrm{d}\alpha > 0$ and $F(q^\beta) < F(q^\alpha)$, it is derived that when $\eta\beta \leq \alpha$, $\mathrm{dCVaR}_\eta(\pi_r(D;q^*))/\mathrm{d}\alpha = \mathrm{dCVaR}_\eta(\pi_r(D;q^\alpha))/\mathrm{d}\alpha < 0$. If $\eta\beta > \alpha$, then $q^* = q^\beta$. It is clear that q^* is constant in α. According to Equation (5), it is calculated that when $\eta\beta > \alpha$, $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is constant in α. Thus, $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is a non-increasing function of the service requirement α. □

Proof of Corollary 2. According to Equation (5), when $\eta\beta \leq \alpha$, $\mathrm{dCVaR}_\eta(\pi_r(D;q^*))/\mathrm{d}\eta = \mathrm{dCVaR}_\eta(\pi_r(D;q^\alpha))/\mathrm{d}\eta = \frac{p-e}{\eta^2}\int_0^{q^\alpha}(q^\alpha - x)f(x)\mathrm{d}x > 0$. When $\eta\beta > \alpha$, $\mathrm{dCVaR}_\eta(\pi_r(D;q^*))/\mathrm{d}\eta = \mathrm{dCVaR}_\eta(\pi_r(D;q^\beta))/\mathrm{d}\eta = \left[p-o-e-\frac{p-e}{\eta}F(q^\beta)\right]\mathrm{d}q^\beta/\mathrm{d}\eta + \frac{p-e}{\eta^2}\int_0^{q^\beta}(q^\beta - x)f(x)\mathrm{d}x = \frac{p-e}{\eta^2}\int_0^{q^\beta}(q^\beta - x)f(x)\mathrm{d}x > 0$. Thus, it is clear that $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is an increasing function of the risk aversion η. □

Proof of Corollary 3. When $\eta\beta \leq \alpha$, $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial o} = \frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^\alpha))}{\partial o} = -q^\alpha < 0$. When $\eta\beta > \alpha$, $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial o} = \frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^\beta))}{\partial o} = -q^\beta + \left[p-o-e-\frac{p-e}{\eta}F(q^\beta)\right]\mathrm{d}q^\beta/\mathrm{d}o = -q^\beta < 0$. Thus, it is easy to obtain that $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is decreasing in o. It is calculated that $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e} = -q^* + \frac{1}{\eta}\int_0^{q^*}(q^*-x)f(x)\mathrm{d}x + \left[p-o-e-\frac{p-e}{\eta}\int_0^{q^*}f(x)\mathrm{d}x\right]\mathrm{d}q^*/\mathrm{d}e$. When $\eta\beta > \alpha$, $q^* = q^\beta$ and $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e} = -q^\beta + \frac{1}{\eta}\int_0^{q^\beta}(q^\beta - x)f(x)\mathrm{d}x + [p-o-e-\frac{p-e}{\eta}\int_0^{q^\beta}f(x)\mathrm{d}x]\mathrm{d}q^\beta/\mathrm{d}e = -q^\beta + \frac{1}{\eta}\int_0^{q^\beta}F(x)\mathrm{d}x$. It is obtained that $\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e \partial q^\beta} = -1 + \frac{1}{\eta}F(q^\beta) = \beta - 1 < 0$. Since $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^\beta))}{\partial e}\Big|_{q^\beta=0} = 0$, then $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e} < 0$. Thus, it is derived that $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is decreasing in e. When $\eta\beta \leq \alpha$, $q^* = q^\alpha$ and $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e} = -q^\alpha + \frac{1}{\eta}\int_0^{q^\alpha}(q^\alpha - x)f(x)\mathrm{d}x = -q^\alpha + \frac{1}{\eta}\int_0^{q^\alpha}F(x)\mathrm{d}x$. Let $V(q^\alpha) = -q^\alpha + \frac{1}{\eta}\int_0^{q^\alpha}F(x)\mathrm{d}x$. It is clear that $V(0) = 0$. Then $\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(D;q^*))}{\partial e \partial q^\alpha} = \frac{\partial V(q^\alpha)}{\partial q^\alpha} = -1 + \frac{1}{\eta}F(q^\alpha) = -1 + \frac{\alpha}{\eta}$. It is obtained that if $\alpha < \eta$, then $\frac{\partial V(q^\alpha)}{\partial q^\alpha} < 0$ and $V(q^\alpha) < 0$. It follows that $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is decreasing in e. If $\alpha = \eta$, then $\frac{\partial V(q^\alpha)}{\partial q^\alpha} = 0$, and $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is constant in e. If $\alpha > \eta$, then $\frac{\partial V(q^\alpha)}{\partial q^\alpha} > 0$ and $V(q^\alpha) > 0$. It is derived that $\mathrm{CVaR}_\eta(\pi_r(D;q^*))$ is increasing in e. □

Proof of Theorem 2. It follows from Equation (8) that

$$\frac{\partial \mathrm{E}_D[\pi_s(D;Q)]}{\partial Q} = h - c - (h-s)F(Q), \tag{A10}$$

and

$$\frac{\partial^2 \mathrm{E}_D[\pi_s(D;Q)]}{\partial Q^2} = -(h-s)f(Q) < 0. \tag{A11}$$

Thus, $\mathrm{E}_D[\pi_s(D;Q)]$ is a concave function of Q, which implies that the supplier has a unique optimal production quantity with the call option contract and a service requirement. Let $\frac{\partial \mathrm{E}_D[\pi_s(D;Q)]}{\partial Q} = 0$, then the optimal production quantity of the risk-neutral supplier without constraints is $Q^s = F^{-1}\left(\frac{h-c}{h-s}\right)$. Considering the constraint of $0 \le Q \le q^*$, with the call option contract and a service requirement, the optimal production policy of the risk-neural supplier is

$$Q^* = \begin{cases} Q^s & \text{if } Q^s \le q^*, \\ q^* & \text{if } q^* < Q'. \end{cases}$$

□

Proof of Corollary 4. According to Theorem 2 and Equation (10), it is calculated that $\mathrm{E}_D[\pi_s(D;Q^*)] = (o+e-h)q^* + (h-e)\int_0^{q^*} F(x)\mathrm{d}x + (h-c)Q^* - (h-s)\int_0^{Q^*} F(x)\mathrm{d}x$. Then

$$\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\alpha = [o+e-h+(h-e)F(q^*)]\mathrm{d}q^*/\mathrm{d}\alpha + [h-c-(h-s)F(Q^*)]\mathrm{d}Q^*/\mathrm{d}\alpha. \tag{A12}$$

From Theorem 1, the following four cases are considered.

(i) $Q^s \le q^*$ and $\alpha < \eta\beta$.

In this case, it is obtained that $Q^* = Q^s$ and $q^* = q^\beta$. Since both Q^s and q^β are constant in α. It follows from Equation (A12) that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is also constant in α.

(ii) $Q^s \le q^*$ and $\alpha \ge \eta\beta$.

Here, it is clear that $Q^* = Q^s$ and $q^* = q^\alpha$. Since Q^s is constant in α and q^α is increasing in α, from Equation (A12), then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\alpha = [o+e-h+(h-e)\alpha]\mathrm{d}q^\alpha/\mathrm{d}\alpha$. Since $o+e-h+(h-e)\alpha > 0$ and $\mathrm{d}q^\alpha/\mathrm{d}\alpha > 0$, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\alpha > 0$, which indicates that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is increasing in α.

(iii) $Q^s > q^*$ and $\alpha < \eta\beta$.

In this case, it is derived that $Q^* = q^* = q^\beta$. Since q^β is constant in α. It follows from Equation (A12) that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is also constant in α.

(iv) $Q^s > q^*$ and $\alpha \ge \eta\beta$.

Here, it is obtained that $Q^* = q^* = q^\alpha$. Since q^α is increasing in α, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\alpha = [o+e-h+(h-e)\alpha]\mathrm{d}q^\alpha/\mathrm{d}\alpha + \frac{1}{h-s}[F(Q^s) - F(q^\alpha)]\mathrm{d}q^\alpha/\mathrm{d}\alpha$. Since $o+e-h+(h-e)\alpha > 0$, $F(Q^s) > F(q^\alpha)$ and $\mathrm{d}q^\alpha/\mathrm{d}\alpha > 0$, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\alpha > 0$, which implies that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is increasing in α.

Thus, combining the above four cases, the supplier's optimal expected profit $\mathrm{E}_D[\pi_s(D;Q^*)]$ is a non-decreasing function of the service requirement α with the call option contract. □

Proof of Corollary 5. According to proof of Corollary 4, $\mathrm{E}_D[\pi_s(D;Q^*)] = (o+e-h)q^* + (h-e)\int_0^{q^*} F(x)\mathrm{d}x + (h-c)Q^* - (h-s)\int_0^{Q^*} F(x)\mathrm{d}x$. It is derived that

$$\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta = [o+e-h+(h-e)F(q^*)]\mathrm{d}q^*/\mathrm{d}\eta + [h-c-(h-s)F(Q^*)]\mathrm{d}Q^*/\mathrm{d}\eta. \tag{A13}$$

According to Theorems 1 and 2, the four distinct cases are considered.

(i) $Q^s \le q^*$ and $\alpha < \eta\beta$.

In this case, it is calculated that $Q^* = Q^s$ and $q^* = q^\beta$. Clearly, Q^s is independent of η and q^β is increasing in η. From Equation (A13), it is derived that $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta = [o+e-h+(h-e)F(q^\beta)]\mathrm{d}q^\beta/\mathrm{d}\eta = [o+e-h+(h-e)\eta\beta]\mathrm{d}q^\beta/\mathrm{d}\eta$. Since $o+e-h+(h-e)\eta\beta > 0$ and $\mathrm{d}q^\beta/\mathrm{d}\eta > 0$, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta > 0$, which means that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is increasing in η.

(ii) $Q^s \le q^*$ and $\alpha \ge \eta\beta$.

Here, it is easy to obtain that $Q^* = Q^s$ and $q^* = q^\alpha$. Since Q^s and q^α are independent of η, it follows from Equation (A13) that $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta = 0$, which indicates that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is independent of η.

(iii) $Q^s > q^*$ and $\alpha < \eta\beta$.

In this case, it is calculated that $Q^* = q^* = q^\beta$. Since q^η is increasing in η, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta = [o+e-h+(h-e)\eta\beta]\mathrm{d}q^\beta/\mathrm{d}\eta + \frac{1}{h-s}[F(Q^s) - F(q^\beta)]\mathrm{d}q^\beta/\mathrm{d}\eta$. Since $o+e-h+(h-e)\eta\beta > 0$, $F(Q^s) > F(q^\beta)$ and $\mathrm{d}q^\beta/\mathrm{d}\eta > 0$, then $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}\eta > 0$, which implies that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is increasing in η.

(iv) $Q^s > q^*$ and $\alpha \ge \eta\beta$.

Here, it is derived that $Q^* = q^* = q^\alpha$. Since q^α is independent of α, it follows from Equation (A13) that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is also independent of α.

Combining the four cases, the supplier's optimal expected profit $\mathrm{E}_D[\pi_s(D;Q^*)]$ is a non-decreasing function of the risk aversion η with the call option contract and a service requirement. □

Proof of Corollary 6. Since $\mathrm{E}_D[\pi_s(D;Q^*)] = (o+e-h)q^* + (h-e)\int_0^{q^*} F(x)\mathrm{d}x + (h-c)Q^* - (h-s)\int_0^{Q^*} F(x)\mathrm{d}x$, then it is calculated that

$$\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}o = q^* + [o+e-h+(h-e)F(q^*)]\mathrm{d}q^*/\mathrm{d}o + [h-c-(h-s)F(Q^*)]\mathrm{d}Q^*/\mathrm{d}o, \tag{A14}$$

and

$$\begin{aligned}\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}e =& q^* - \int_0^{q^*} F(x)\mathrm{d}x + [o+e-h+(h-e)F(q^*)]\mathrm{d}q^*/\mathrm{d}e \\ &+ [h-c-(h-s)F(Q^*)]\mathrm{d}Q^*/\mathrm{d}e.\end{aligned} \tag{A15}$$

From Theorems 1 and 2, this paper consider the two distinct cases:

(i) $Q^s \le q^*$ and $\alpha \ge \eta\beta$.

Here, it is obtained that $Q^* = Q^s$ and $q^* = q^\alpha$. Clearly, Q^s and q^α are independent of o and e. It follows from Equation (A14) that $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}o = q^\alpha > 0$, which indicates that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is strictly increasing in o. From Equation (A15), it is derived that $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}e = q^\alpha - \int_0^{q^\alpha} F(x)\mathrm{d}x > 0$, which implies that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is strictly increasing in e.

(ii) $Q^s > q^*$ and $\alpha \ge \eta\beta$.

In this case, it is derived that $Q^* = q^* = q^\alpha$. Since q^α is independent of α, then from Equations (A14) and (A15), it is calculated that $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}o = q^\alpha > 0$ and $\mathrm{dE}_D[\pi_s(D;Q^*)]/\mathrm{d}e = q^\alpha - \int_0^{q^\alpha} F(x)\mathrm{d}x > 0$, which indicate that $\mathrm{E}_D[\pi_s(D;Q^*)]$ is strictly increasing in o and e. From the above two cases, when $\alpha \ge \eta\beta$, the supplier's optimal expected profit $\mathrm{E}_D[\pi_s(D;Q^*)]$ is increasing in o and e. □

Proof of Theorem 3. From Equation (11), it is obtained that $\mathrm{dE}_D[\pi_s(D;Q_0)]/\mathrm{d}Q_0 = (p+h-c) - (p+h-s)F(Q_0)$ and $\mathrm{d}^2\mathrm{E}_D[\pi_s(D;Q_0)]/\mathrm{d}Q_0^2 = -(p+h-s)f(Q_0) < 0$. Thus, $\mathrm{E}_D[\pi_s(D;Q_0)]$ is concave in Q_0. Let $\mathrm{dE}_D[\pi_s(D;Q_0)]/\mathrm{d}Q_0 = 0$, it is derived that the optimal solution of Equation (12) without the constraint is $Q_0^\gamma = F^{-1}\left(\frac{p+h-c}{p+h-s}\right) = F^{-1}(\gamma)$. Since $Q_0^\alpha = F^{-1}(\alpha)$, according to Equation (12), it is obtained that with the service requirement, the integrated firm's optimal production strategy is

$$Q_0^* = \begin{cases} Q_0^\alpha, & \gamma < \alpha, \\ Q_0^\gamma, & \gamma \geqslant \alpha, \end{cases}$$

□

Proof of Theorem 4. From Equation (14), it is derived that

$$\mathrm{dE}_D[\pi_s(D; Q_0^*)]/\mathrm{d}\alpha = [(p+h-c) - (p+h-s)F(Q_0^*)]\mathrm{d}Q_0^*/\mathrm{d}\alpha. \tag{A16}$$

According to Theorem 3, if $\gamma \geqslant \alpha$, then $Q_0^* = Q_0^\gamma$. Since Q_0^γ is constant in α, from Equation (A16), then $\mathrm{dE}_D[\pi_s(D; Q_0^*)]/\mathrm{d}\alpha = 0$. If $\lambda < \alpha$, then $Q_0^* = Q_0^\alpha$. It is derived that $\mathrm{dE}_D[\pi_s(D; Q_0^*)]/\mathrm{d}\alpha = (p+h-s)[F(Q_0^\gamma) - F(Q_0^\alpha)]\mathrm{d}Q_0^\alpha/\mathrm{d}\alpha$. Since $F(Q_0^\gamma) < F(Q_0^\alpha)$ and $\mathrm{d}Q_0^\alpha/\mathrm{d}\alpha > 0$, it is calculated that $\mathrm{dE}_D[\pi_s(D; Q_0^*)]/\mathrm{d}\alpha < 0$. Thus, the optimal expected profit of the integrated firm is non-increasing in α. Clearly, the integrated firm's optimal service level is $\alpha = \gamma = \frac{p+h-c}{p+h-s}$. This result shows that when $\alpha = \frac{p+h-c}{p+h-s}$, the supply chain's service requirement can be coordinated. In addition, the supplier's optimal production quantity and the retailer's optimal ordering quantity should be coordinated. According to Theorems 1 and 2, the supply chain can be coordinated when $\eta\beta = \frac{h-c}{h-s}$ is satisfied. Since $\frac{p+h-c}{p+h-s} > \frac{h-c}{h-s}$, it follows that $\gamma > \frac{h-c}{h-s} = \eta\beta$. Since supply chain coordination needs the service requirement to satisfy $\alpha = \gamma$, then the retailer's optimal order quantity and the supplier's optimal production quantity are concordant with the optimal production quantity of the integrated firm. □

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